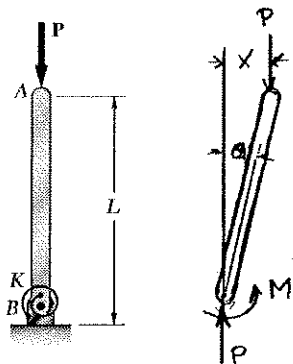


Chapter 10

Problem 10.1

10.1 Knowing that the torsional spring at B is of constant K and that the bar AB is rigid, determine the critical load P_{cr} .



Let θ be the angle change of bar AB .

$$M = K\theta, \quad x = L \sin \theta \approx L\theta$$

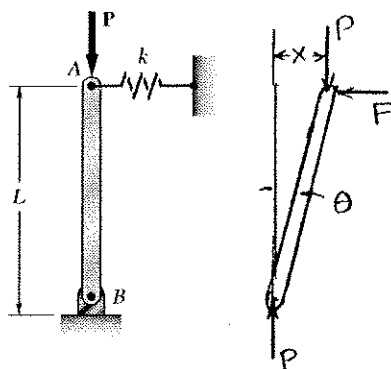
$$\sum M_B = 0: \quad M - Px = 0 \quad K\theta - PL\theta = 0$$

$$(K - PL)\theta = 0$$

$$P_{cr} = K/L \quad \rightarrow$$

Problem 10.2

10.2 Knowing that the spring at A is of constant k and that the bar AB is rigid, determine the critical load P_{cr} .



Let θ be the angle change of bar AB .

$$F = kx = kL \sin \theta$$

$$\sum M_B = 0: \quad Fh \cos \theta - Px = 0$$

$$kL^2 \sin \theta \cos \theta - PL \sin \theta = 0$$

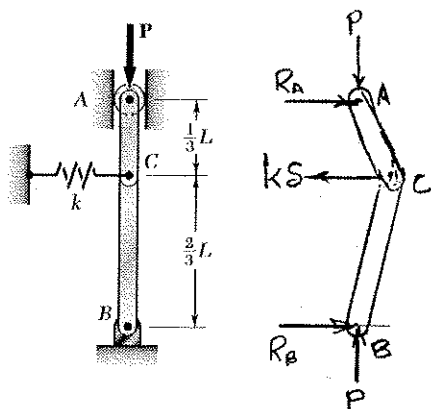
Using $\sin \theta \approx \theta$ and $\cos \theta \approx 1$, $kL^2 \theta - PL \theta = 0$

$$(kL^2 - PL)\theta = 0$$

$$P_{cr} = kL \quad \rightarrow$$

Problem 10.3

10.3 Two rigid bars AC and BC are connected as shown to a spring of constant k . Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.



Let S be the deflection of point C .

Using free body AC and $+\circlearrowleft \Sigma M_C = 0$:

$$-\frac{1}{3}LR_A + PS = 0 \quad R_A = \frac{3PS}{L}$$

Using free body BC and $+\circlearrowleft \Sigma M_C = 0$:

$$\frac{2}{3}LR_B - PS = 0 \quad R_B = \frac{3PS}{2L}$$

Using both free bodies together,

$$+\rightarrow \Sigma F_x = 0: R_A + R_B - kS = 0$$

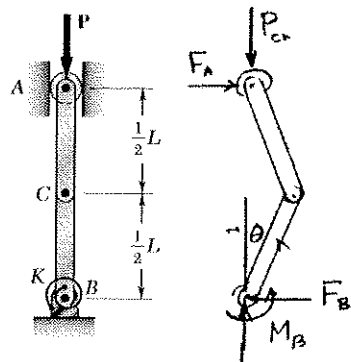
$$\frac{3PS}{L} + \frac{3PS}{2L} - kS = 0$$

$$\left(\frac{9}{2} \frac{P}{L} - k\right) S = 0$$

$$P_{cr} = \frac{2kL}{9}$$

Problem 10.4

10.4 Two rigid bars AC and BC are connected by a pin at C as shown. Knowing that the torsional spring at B is of constant K , determine the critical load P_{cr} for the system.



Let θ be the angle change of each bar.

$$M_B = K\theta$$

$$+\circlearrowleft \Sigma M_B = 0: K\theta - F_A L = 0 \quad F_A = \frac{K\theta}{L}$$

Bar AC $+\circlearrowleft \Sigma M_C = 0$:

$$P_{cr} \frac{1}{2}L\theta - \frac{1}{2}L F_A = 0$$

$$P_{cr} = \frac{F_A}{\theta} \quad P_{cr} = \frac{K}{L}$$

Problem 10.9

10.9 Determine the critical load of a wooden meter stick which has a 7×24 -mm rectangular cross section. Use $E = 12$ GPa.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (24)(7)^3 = 686 \text{ mm}^4 = 686 \times 10^{-12} \text{ m}^4$$

$$L = 1.00 \text{ m}$$

$$E = 12 \times 10^9 \text{ Pa}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (12 \times 10^9)(686 \times 10^{-12})}{(1.00)^2}$$

$$P_{cr} = 81.2 \text{ N} \quad \blacktriangleleft$$

Problem 10.10

10.10 Determine the critical load of a round wooden dowel that is 0.9 m long and has a diameter of (a) 10 mm, (b) 15 mm. Use $E = 12$ GPa.

$$(a) \quad c = \frac{1}{2} d = 5 \text{ mm} \quad I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (5)^4 = 490.87 \text{ mm}^4$$

$$I = 490.87 \times 10^{-12} \text{ m}^4 \quad E = 12 \times 10^9 \text{ Pa} \quad L = 0.9 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (12 \times 10^9)(490.87 \times 10^{-12})}{(0.9)^2}$$

$$P_{cr} = 71.8 \text{ N} \quad \blacktriangleleft$$

$$(b) \quad c = \frac{1}{2} d = 7.5 \text{ mm} \quad I = \frac{\pi}{4} c^4 = \frac{\pi}{4} (7.5)^4 = 2.485 \times 10^3 \text{ mm}^4$$

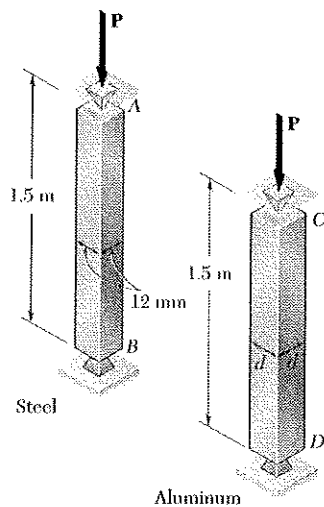
$$I = 2.485 \times 10^{-9} \text{ m}^4 \quad E = 12 \times 10^9 \text{ Pa} \quad L = 0.9 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (12 \times 10^9)(2.485 \times 10^{-9})}{(0.9)^2}$$

$$P_{cr} = 363 \text{ N} \quad \blacktriangleleft$$

Problem 10.11

10.11 Determine the dimension d so that the aluminum and steel struts will have the same weight, and compute the critical load for each strut.



$$\text{Steel: } E = 200 \text{ GPa} \\ \gamma = 7860 \text{ kg/m}^3$$

$$\text{Aluminum: } E = 70 \text{ GPa} \\ \gamma = 2800 \text{ kg/m}^3$$

$$\text{Length: } L = 1.5 \text{ m}$$

$$\text{Weight of steel strut: } W_s = \gamma L d_s^2$$

$$W_s = (7860)(9.81)(1.5)(0.012^2) = 16.655 \text{ N}$$

$$\text{Weight of aluminum strut: } W_a = \gamma L d^2 = (2800)(9.81)(1.5)d^2 = 41202 d^2$$

$$\text{For } W_a = W_s, \quad 41202 d^2 = 16.655 \quad d^2 = 0.404227 \times 10^{-3} \text{ m}^2$$

$$d = 20.1 \text{ mm}$$

$$\text{Steel strut: } I = \frac{1}{12} d_s^4 = \frac{1}{12} (0.012)^4 = 1728 \times 10^{-12} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (1728 \times 10^{-12})}{(1.5)^2}$$

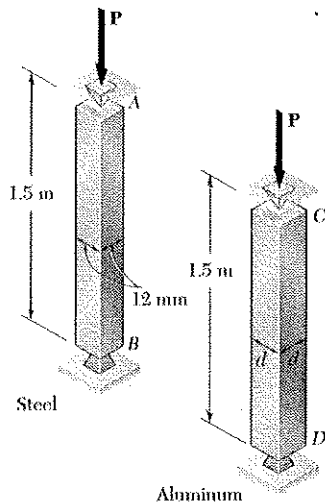
$$P_{cr} = 1516 \text{ N}$$

$$\text{Aluminum strut: } I = \frac{1}{12} d^4 = \frac{1}{12} (0.0201)^4 = 13602 \times 10^{-12} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (70 \times 10^9) (13602 \times 10^{-12})}{(1.5)^2}$$

$$P_{cr} = 4177 \text{ N}$$

Problem 10.12



10.12 Determine (a) the critical load for the steel strut, (b) the dimension d for which the aluminum strut will have the same critical load. (c) Express the weight of the aluminum strut as a percent of the weight of the steel strut.

Steel: $E = 200 \text{ GPa}$
 $\gamma = 7860 \text{ kg/m}^3$

Aluminum: $E = 70 \text{ GPa}$
 $\gamma = 2800 \text{ kg/m}^3$

Length: $L = 1.5 \text{ m}$

(a) Steel strut:

$$I = \frac{1}{12} d_s^4 = \frac{1}{12} (0.012)^4 = 1728 \times 10^{-12} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (1728 \times 10^{-12})}{(1.5)^2} \quad P_{cr} = 1516 \text{ N}$$

Weight: $W_s = \gamma_s L d_s^2 = (7860)(9.81)(1.5)(0.012)^2 = 16.655 \text{ N}$

(b) Aluminum strut: $P_{cr} = \frac{\pi^2 EI}{L^2}$

$$I = \frac{P_{cr} L^2}{\pi^2 E} = \frac{(1516)(1.5)^2}{\pi^2 (70 \times 10^9)} = 3290 \times 10^{-12} \text{ m}^4 = 3290 \text{ mm}^4$$

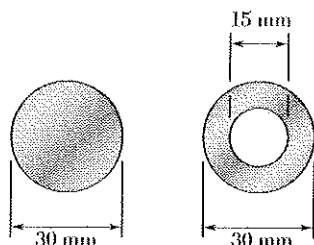
$$I = \frac{1}{12} d^4 \quad d = \sqrt[4]{12 I} = \sqrt[4]{(12) 3290} \quad d = 14.1 \text{ mm}$$

Weight: $W_a = \gamma_a L d^2 = (2800)(9.81)(1.5)(0.0141)^2 = 8.191 \text{ N}$

(c) Weight ratio as a percent: $\frac{W_a}{W_s} \times 100\%$

$$\frac{8.191}{16.655} \times 100\% \quad 49.2\%$$

Problem 10.13



10.13 A compression member of 1.5-m effective length consists of a solid 30-mm-diameter brass rod. In order to reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (a) the percent reduction in the critical load, (b) the value of the critical load for the hollow rod. Use $E = 105 \text{ GPa}$.

$$(a) \quad P_{cr} = \frac{\pi^2 EI}{L^2}$$

P_{cr} is proportional to I .

For solid rod, $c = \frac{1}{2}d$, $I_s = \frac{\pi}{4}c^4$

$$c = \frac{1}{2}(30) = 15 \text{ mm} \quad I_s = \frac{\pi}{4}(15)^4 = 39.761 \times 10^3 \text{ mm}^4 = 39.761 \times 10^{-9} \text{ m}^4$$

For hollow rod, $c_o = \frac{1}{2}d_o$ $I_h = \frac{\pi}{4}(c_o^4 - c_i^4)$

$$\begin{aligned} \frac{(P_{cr})_h}{(P_{cr})_s} &= \frac{I_h}{I_s} = \frac{c_o^4 - c_i^4}{c^4} = 1 - \left(\frac{c_i}{c_o}\right)^4 = 1 - \left(\frac{d_i}{d_o}\right)^4 \\ &= 1 - \left(\frac{15}{30}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16} \end{aligned}$$

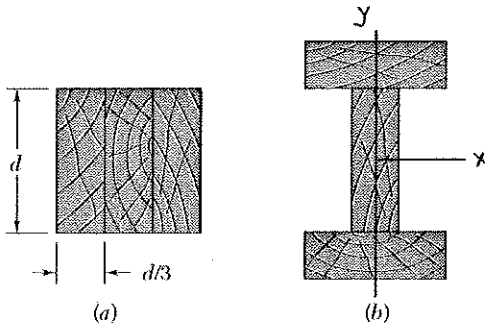
Percent reduction in P_{cr} : $\frac{1}{16} \times 100\% = 6.25\%$

$$(b) \quad P_{cr} = \frac{15}{16} \frac{\pi^2 EI_s}{L^2} = \frac{15}{16} \frac{\pi^2 (105 \times 10^9)(39.761 \times 10^{-9})}{(1.5)^2} = 17.17 \times 10^3 \text{ N}$$

$$P_{cr} = 17.17 \text{ kN}$$

Problem 10.14

10.14 A column of effective length L can be made by gluing together identical planks in either of the arrangements shown. Determine the ratio of the critical load using the arrangement a to the critical load using the arrangement b .



Arrangement (a).

$$I_a = \frac{1}{12} d^4$$

$$P_{cr,a} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d^4}{12 L_e^2}$$

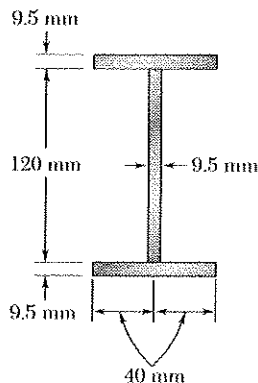
Arrangement (b). $I_{min} = I_y = \frac{1}{12} \left(\frac{d}{3} d^3 \right) + \frac{1}{12} \left(d \left(\frac{d}{3} \right)^3 \right) + \frac{1}{12} \left(\frac{d}{3} d^3 \right) = \frac{19}{324} d^4$

$$P_{cr,b} = \frac{\pi^2 EI}{L_e^2} = \frac{19 \pi^2 E d^4}{324 L_e^2}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = \frac{1}{12} \cdot \frac{324}{19} = \frac{27}{19}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = 1.421 \quad \blacktriangleleft$$

Problem 10.15



$$P = 16 \text{ kN}$$

10.15 A column of 6-m effective length is to be made from three plates as shown. Using $E = 200 \text{ GPa}$, determine the factor of safety with respect to buckling for a centric load of 16 kN.

Moment of inertia is smallest about the vertical centroidal axis.

$$I_{\min} = 2 \left[\frac{1}{12} (9.5) (80)^3 \right] + \frac{1}{12} (120) (9.5)^3$$

$$= 819.24 \times 10^3 \text{ mm}^4 = 819.24 \times 10^{-9} \text{ m}^4$$

$$E = 200 \times 10^9 \text{ Pa}$$

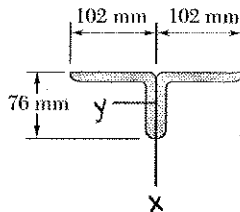
$$P_{cr} = \frac{\pi^2 E I_{\min}}{L^2} = \frac{\pi^2 (200 \times 10^9) (819.24 \times 10^{-9})}{(6)^2}$$

$$= 44.92 \times 10^3 \text{ N} = 44.92 \text{ kN}$$

$$F.S. = \frac{P_{cr}}{P} = \frac{44.92}{16}$$

$$F.S. = 2.81$$

Problem 10.16



10.16 and 10.17 A compression member of 3.6 m effective length is made by welding together two $102 \times 76 \times 6.4 \text{ mm}$ steel angles as shown. Using $E = 200 \text{ GPa}$, determine the allowable centric load for the member if a factor of safety of 2.5 is required.

For $102 \times 76 \times 6.4 \text{ mm}$ steel angle,

$$A = 1100 \text{ mm}^2, \quad \bar{I}_x = 1.17 \times 10^6 \text{ mm}^4, \quad \bar{y} = 31.6 \text{ mm}$$

$$\bar{I}_y = 0.563 \times 10^6 \text{ mm}^4, \quad \bar{x} = 18.6 \text{ mm}$$

For welded member,

$$I_x = 2 \left[1.17 \times 10^6 + (1100) (31.6)^2 \right] = 4.5368 \times 10^6 \text{ mm}^4$$

$$I_y = 2 \left[0.563 \times 10^6 \right] = 1.126 \times 10^6 \text{ mm}^4 = I_{\min}$$

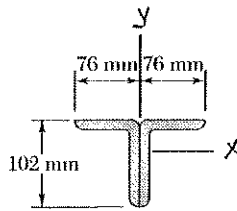
$$L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 E I_{\min}}{L^2} = \frac{\pi^2 (200 \times 10^9) (1.126 \times 10^{-6})}{(3.6)^2} = 171500 \text{ N}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{171500}{2.5}$$

$$P_{all} = 68.6 \text{ kN}$$

Problem 10.17



10.16 and 10.17 A compression member of 3.6 m effective length is made by welding together two $102 \times 76 \times 6.4$ mm steel angles as shown. Using $E = 200$ GPa, determine the allowable centric load for the member if a factor of safety of 2.5 is required.

For $102 \times 76 \times 6.4$ mm steel angle,

$$A = 1100 \text{ mm}^2, \quad \bar{I}_x = 1.17 \times 10^6 \text{ mm}^4, \quad \bar{y} = 31.6 \text{ mm}$$

$$\bar{I}_y = 0.563 \times 10^6 \text{ mm}^4, \quad \bar{x} = 18.6 \text{ mm}$$

For welded member,

$$I_x = 2[1.17 \times 10^6] = 2.34 \times 10^6 \text{ mm}^4$$

$$I_y = 2[0.563 \times 10^6 + (1100)(18.6)^2] = 1.887 \times 10^6 \text{ mm}^4 = I_{\min}$$

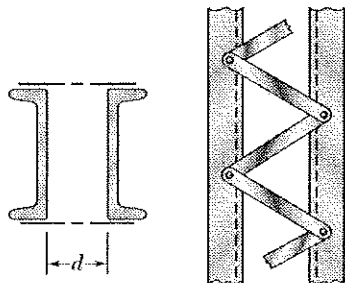
$$L = 3.6 \text{ m}$$

$$P_{\text{cr}} = \frac{\pi^2 E I_{\min}}{L^2} = \frac{\pi^2 (200 \times 10^9) (1.887 \times 10^{-6})}{(3.6)^2} = 287407 \text{ N}$$

$$P_{\text{all}} = \frac{P_{\text{cr}}}{\text{F.S.}} = \frac{287407}{2.5}$$

$$P_{\text{all}} = 115 \text{ kN}$$

Problem 10.18



10.18 A single compression member of 8.2-m effective length is obtained by connecting two C 200 $\times$ 17.1 steel channels with lacing bars as shown. Knowing that the factor of safety is 1.85, determine the allowable centric load for the member. Use $E = 200$ GPa and $d = 100$ mm.

For C 200 $\times$ 17.1 steel channel, $A = 2170 \text{ mm}^2$

$$\bar{I}_x = 13.4 \times 10^6 \text{ mm}^4, \quad \bar{I}_y = 0.538 \times 10^6 \text{ mm}^4$$

$$\bar{x} = 14.4 \text{ mm}$$

For the fabricated column,

$$I_x = 2\bar{I}_x = (2)(13.4 \times 10^6) = 26.8 \times 10^6 \text{ mm}^4$$

$$I_y = 2[\bar{I}_y + A(\frac{d}{2} + \bar{x})^2]$$

$$= 2[0.538 \times 10^6 + 2170(\frac{100}{2} + 14.4)^2]$$

$$= 19.0755 \times 10^6 \text{ mm}^4$$

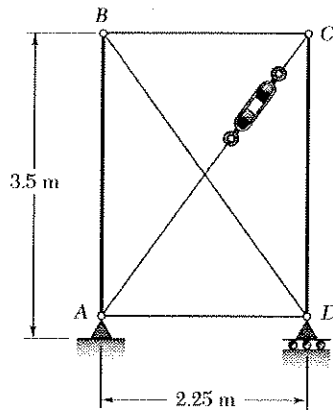
$$I_{\min} = I_y = 19.0755 \times 10^6 \text{ mm}^4 = 19.0755 \times 10^{-6} \text{ m}^4 \quad E = 200 \times 10^9 \text{ Pa}$$

$$P_{\text{cr}} = \frac{\pi^2 E I_{\min}}{L^2} = \frac{\pi^2 (200 \times 10^9) (19.0755 \times 10^{-6})}{(8.2)^2} = 559.99 \times 10^3 \text{ N}$$

$$P_{\text{all}} = \frac{P_{\text{cr}}}{\text{F.S.}} = \frac{559.99 \times 10^3}{1.85} = 303 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 303 \text{ kN}$$

Problem 10.19



10.19 Members AB and CD are 30-mm-diameter steel rods, and members BC and AD are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that a factor of safety with respect to buckling of 2.75 is required, determine the largest allowable tension in AC . Use $E = 200$ GPa and consider only buckling in the plane of the structure.

$$L_{AC} = \sqrt{(3.5)^2 + (2.25)^2} = 4.1608 \text{ m}$$

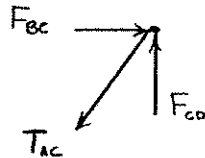
Joint C:

$$\pm \sum F_x = 0: F_{BC} - \frac{2.25}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.84926 F_{BC}$$

$$+\sum F_y = 0: F_{CD} - \frac{3.5}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.1888 F_{CD}$$



Members BC and AD: $I_{BC} = \frac{\pi}{4} \left(\frac{d_{BC}}{2} \right)^4 = \frac{\pi}{4} \left(\frac{22}{2} \right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$

$$L_{BC} = 2.25 \text{ m} \quad F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9) (11.499 \times 10^{-9})}{(2.25)^2} = 4.4836 \times 10^3 \text{ N}$$

$$F_{BC,all} = \frac{F_{BC,cr}}{F.S.} = 1.6304 \times 10^3 \text{ N} \quad T_{AC,all} = 3.02 \times 10^3 \text{ N}$$

Members AB and CD: $I_{CD} = \frac{\pi}{4} \left(\frac{d_{CD}}{2} \right)^4 = \frac{\pi}{4} \left(\frac{30}{2} \right)^4 = 39.761 \times 10^3 \text{ mm}^4 = 39.761 \times 10^{-9} \text{ m}^4$

$$L_{CD} = 3.5 \text{ m} \quad F_{CD,cr} = \frac{\pi^2 E I_{CD}}{L_{CD}^2} = \frac{\pi^2 (200 \times 10^9) (39.761 \times 10^{-9})}{(3.5)^2} = 6.4069 \times 10^3 \text{ N}$$

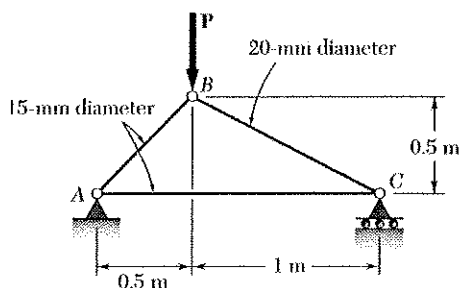
$$F_{CD,all} = \frac{F_{CD,cr}}{F.S.} = 2.3298 \times 10^3 \text{ N} \quad T_{AC,all} = 2.77 \times 10^3 \text{ N}$$

Smaller value for $T_{AC,all}$ governs.

$$T_{AC,all} = 2.77 \text{ kN} \quad \blacktriangleleft$$

Problem 10.20

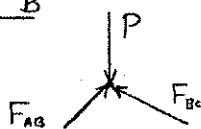
10.20 Knowing that a factor of safety of 2.6 is required, determine the largest load P that can be applied to the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



$$\begin{aligned} \text{BC: } L_{BC} &= \sqrt{1^2 + 0.5^2} = 1.1180 \text{ m} \\ I &= \frac{\pi}{64} d_{BC}^4 = \frac{\pi}{64} (20)^4 = 7.854 \times 10^3 \text{ mm}^4 \\ &= 7.854 \times 10^{-9} \text{ m}^4 \\ P_{cr} &= \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (7.854 \times 10^{-9})}{(1.1180)^2} \\ &= 12.403 \times 10^3 \text{ N} = 12.403 \text{ kN} \\ F_{BC,all} &= \frac{P_{cr}}{F.S.} = \frac{12.403}{2.6} = 4.770 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{AB: } L_{AB} &= \sqrt{0.5^2 + 0.5^2} = 0.70711 \text{ m} \\ I &= \frac{\pi}{64} d^4 = \frac{\pi}{64} (15)^4 = 2.485 \times 10^3 \text{ mm}^4 = 2.485 \times 10^{-9} \text{ m}^4 \\ P_{cr} &= \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (200 \times 10^9) (2.485 \times 10^{-9})}{(0.70711)^2} = 9.8106 \times 10^3 \text{ N} = 9.8106 \text{ kN} \\ F_{AB,all} &= \frac{P_{cr}}{F.S.} = \frac{9.8106}{2.6} = 3.773 \text{ kN} \end{aligned}$$

Joint B



$$\sum F_x = 0: \frac{0.5}{0.70711} F_{AB} - \frac{1.0}{1.1180} F_{BC} = 0$$

$$F_{BC} = 0.79057 F_{AB}$$

$$\sum F_y = 0: \frac{0.5}{0.70711} F_{AB} + \frac{0.5}{1.1180} F_{BC} + P = 0$$

$$0.70711 F_{AB} + (0.44721)(0.79057 F_{AB}) - P = 0$$

$$P = 1.06066 F_{AB}$$

$$P = (1.06066) \frac{F_{BC}}{0.79057} = 1.3416 F_{BC}$$

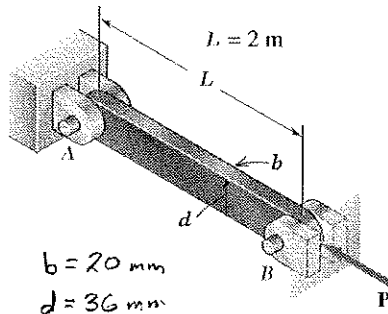
Allowable value for P.

$$P < 1.06066 F_{AB,all} = (1.06066)(3.773) = 4.00 \text{ kN}$$

$$P < 1.3416 F_{BC,all} = (1.3416)(4.770) = 6.40 \text{ kN}$$

$$P_{all} = 4.00 \text{ kN}$$

Problem 10.21



10.21 The uniform aluminum bar AB has a 20×36 -mm rectangular cross section and is supported by pins and brackets as shown. Each end of the bar may rotate freely about a horizontal axis through the pin, but rotation about a vertical axis is prevented by the brackets. Using $E = 70$ GPa, determine the allowable centric load P if a factor of safety of 2.5 is required.

Buckling in horizontal plane.

$$(L_e)_1 = \frac{L}{2} = \frac{2}{2} = 1.00 \text{ m}$$

$$I_1 = \frac{1}{12} d b^3 = \frac{1}{12} (36)(20)^3 = 24 \times 10^3 \text{ mm}^4 = 24 \times 10^{-9} \text{ m}^4$$

$$(P_{cr})_1 = \frac{\pi^2 E I_1}{(L_e)_1^2} = \frac{\pi^2 (70 \times 10^9)(24 \times 10^{-9})}{(1.00)^2}$$

$$= 16.581 \times 10^3 \text{ N} = 16.581 \text{ kN}$$

Buckling in vertical plane. $(L_e)_2 = L = 2 \text{ m}$

$$I_2 = \frac{1}{12} b d^3 = \frac{1}{12} (20)(36)^3 = 77.76 \times 10^3 \text{ mm}^4 = 77.76 \times 10^{-9} \text{ m}^4$$

$$(P_{cr})_2 = \frac{\pi^2 E I_2}{(L_e)_2^2} = \frac{\pi^2 (70 \times 10^9)(77.76 \times 10^{-9})}{(2)^2} = 13.4306 \times 10^3 \text{ N}$$

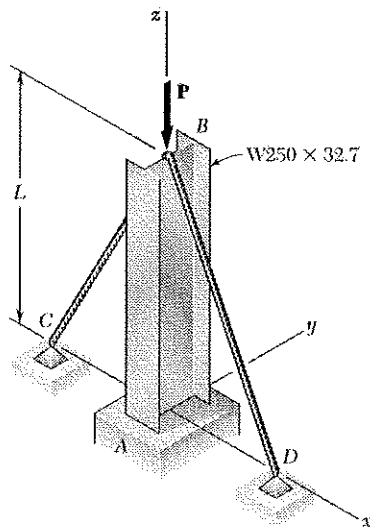
$$= 13.4306 \text{ kN}$$

$$P_{cr} = \min [(P_{cr})_1, (P_{cr})_2] = 13.4306 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{13.4306}{2.5}$$

$$P = 5.37 \text{ kN} \quad \blacktriangleleft$$

Problem 10.22



10.22 Column AB carries a centric load P of magnitude 60 kN. Cables BC and BD are taut and prevent motion of point B in the xz plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length L . Use $E = 200$ GPa.

$$W250 \times 32.7: I_x = 48.9 \times 10^6 \text{ mm}^4, I_y = 4.73 \times 10^6 \text{ mm}^4$$

$$P = 60 \text{ kN}$$

$$P_{cr} = (F.S.)P = (2.2)(60) = 132 \text{ kN}$$

Buckling in xz -plane: $L_e = 0.7L$

$$P_{cr} = \frac{\pi^2 EI_y}{(0.7L)^2} \quad L = \frac{\pi}{0.7} \sqrt{\frac{EI_y}{P_{cr}}}$$

$$L = \frac{\pi}{0.7} \sqrt{\frac{(200 \times 10^9)(4.73 \times 10^{-6})}{132000}} = 12.01 \text{ m}$$

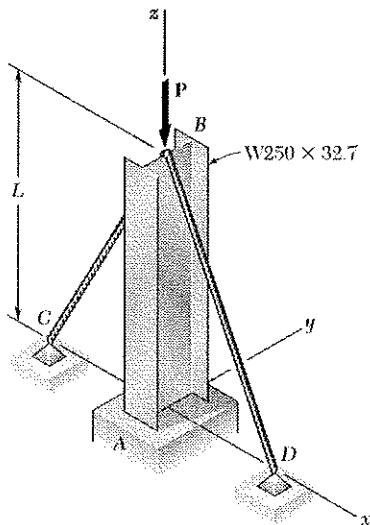
Buckling in yz -plane: $L_e = 2L$

$$P_{cr} = \frac{\pi^2 EI_x}{(2L)^2} \quad L = \frac{\pi}{2} \sqrt{\frac{EI_x}{P_{cr}}} = \frac{\pi}{2} \sqrt{\frac{(200 \times 10^9)(48.9 \times 10^{-6})}{132000}} = 13.52 \text{ m}$$

Smaller value for L governs.

$$L = 12.01 \text{ m}$$

Problem 10.23



10.23 A $W200 \times 31.3$ rolled-steel shape is used with the support and cable arrangement shown in Prob. 10.21. Knowing that $L = 7$ m, determine the allowable centric load P if a factor of safety of 2.2 is required. Use $E = 200$ GPa.

10.22 Column AB carries a centric load P of magnitude 60 kN. Cables BC and BD are taut and prevent motion of point B in the xz plane. Using Euler's formula and a factor of safety of 2.2, and neglecting the tension in the cables, determine the maximum allowable length L . Use $E = 200$ GPa.

$$W200 \times 31.3: I_x = 31.4 \times 10^6 \text{ mm}^4, I_y = 4.1 \times 10^6 \text{ mm}^4$$

$$L = 7 \text{ m}$$

Buckling in xz -plane: $L_e = 0.7L = 4.9 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(4.1 \times 10^{-6})}{(4.9)^2} = 337.07 \text{ kN}$$

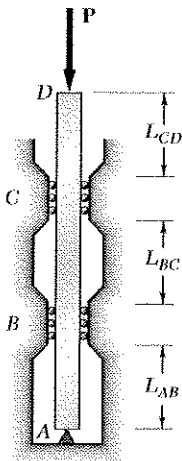
Buckling in yz -plane: $L_e = 2L = 14 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI_x}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(31.4 \times 10^{-6})}{(14)^2} = 316.23 \text{ kN}$$

The smaller value governs. $P_c = 316.23 \text{ kN}$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{316.23}{2.2} = 143.7 \text{ kN}$$

Problem 10.24



10.24 A 25-mm-square aluminum strut is maintained in the position shown by a pin support at A and by sets of rollers at B and C that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 0.9$ m, determine (a) the largest values of L_{BC} and L_{CD} that may be used if the allowable load P is to be as large as possible, (b) the magnitude of the corresponding allowable load. Consider only buckling in the plane of the figure and use $E = 72$ GPa.

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (25)(25)^3 = 32.552 \times 10^3 \text{ mm}^4$$

(a) Equivalent lengths: AB $L_e = 0.7 L_{AB} = 0.63 \text{ m}$

BC $L_e = 0.5 L_{BC}$

$$L_{BC} = \frac{0.63}{0.5} = 1.26 \text{ m}$$

CD $L_e = 2 L_{CD}$

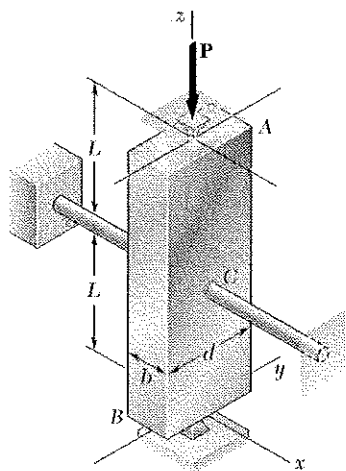
$$L_{CD} = \frac{0.63}{2} = 0.315 \text{ m}$$

(b) $P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{(F.S.) L_e^2} = \frac{\pi^2 (72 \times 10^9) (32.552 \times 10^{-9})}{(3.2)(0.63)^2} = 18213 \text{ N}$

$$P_{all} = 18.2 \text{ kN}$$

Problem 10.25

10.25 Column ABC has a uniform rectangular cross section with $b = 12$ mm and $d = 22$ mm. The column is braced in the xz plane at its midpoint C and carries a centric load P of magnitude 3.8 kN. Knowing that a factor of safety of 3.2 is required, determine the largest allowable length L . Use $E = 200$ GPa.



$$P_{cr} = (F.S.) P = (3.2)(3.8 \times 10^3) = 12.16 \times 10^3 \text{ N}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad L_e = \pi \sqrt{\frac{EI}{P_{cr}}}$$

Buckling in xz -plane. $L = L_e = \pi \sqrt{\frac{EI}{P_{cr}}}$

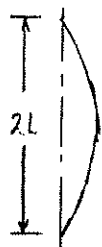


$$I = \frac{1}{12} db^3 = \frac{1}{12} (22)(12)^3 = 3.168 \times 10^3 \text{ mm}^4 = 3.168 \times 10^{-9} \text{ m}^4$$

$$L = \pi \sqrt{\frac{(200 \times 10^9)(3.168 \times 10^{-9})}{12.16 \times 10^3}} = 0.717 \text{ m}$$

Buckling in yz -plane.

$$L_e = 2L \quad L = \frac{L_e}{2} = \frac{\pi}{2} \sqrt{\frac{EI}{P_{cr}}}$$



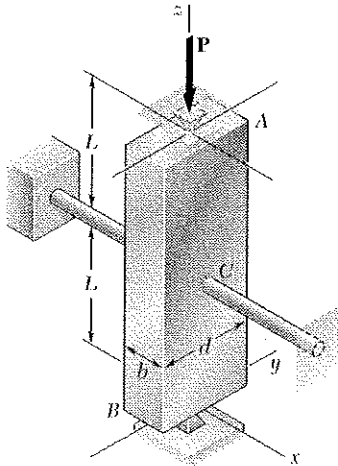
$$I = \frac{1}{12} bd^3 = \frac{1}{12} (12)(22)^3 = 10.648 \times 10^3 \text{ mm}^4 = 10.648 \times 10^{-9} \text{ m}^4$$

$$L = \frac{\pi}{2} \sqrt{\frac{(200 \times 10^9)(10.648 \times 10^{-9})}{12.16 \times 10^3}} = 0.657 \text{ m}$$

The smaller length governs. $L = 0.657 \text{ m} = 657 \text{ mm}$ ◀

Problem 10.26

10.26 Column ABC has a uniform rectangular cross section and is braced in the xz plane at its midpoint C . (a) Determine the ratio b/d for which the factor of safety is the same with respect to buckling in the xz and yz planes. (b) Using the ratio found in part a, design the cross section of the column so that the factor of safety will be 3.0 when $P = 4.4 \text{ kN}$, $L = 1 \text{ m}$, and $E = 200 \text{ GPa}$.



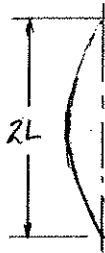
Buckling in xz -plane. $L_e = L$, $I = \frac{db^3}{12}$



$$(P_{cr})_1 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d b^3}{12 L^2}$$

$$(F.S.)_1 = \frac{(P_{cr})_1}{P} = \frac{\pi^2 E d b^3}{12 P L^2}$$

Buckling in yz -plane. $L_e = 2L$, $I = \frac{bd^3}{12}$



$$(P_{cr})_2 = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E b d^3}{12 (2L)^2}$$

$$(F.S.)_2 = \frac{(P_{cr})_2}{P} = \frac{\pi^2 E b d^3}{48 P L^2}$$

(a) Equating the two factors of safety,

$$\frac{\pi^2 E d b^3}{12 P L^2} = \frac{\pi^2 E b d^3}{48 P L^2}$$

$$b^2 = \frac{1}{4} d^2 \quad b/d = 1/2 \quad \blacktriangleleft$$

$$\text{Then, } (F.S.) = \frac{\pi^2 E d^4}{96 P L^2}$$

$$d^4 = \frac{96 (F.S.) P L^2}{\pi^2 E} = \frac{(96)(3.0)(4.4 \times 10^3)(1)^2}{\pi^2 (200 \times 10^9)} = 641.97 \times 10^{-9} \text{ m}^4$$

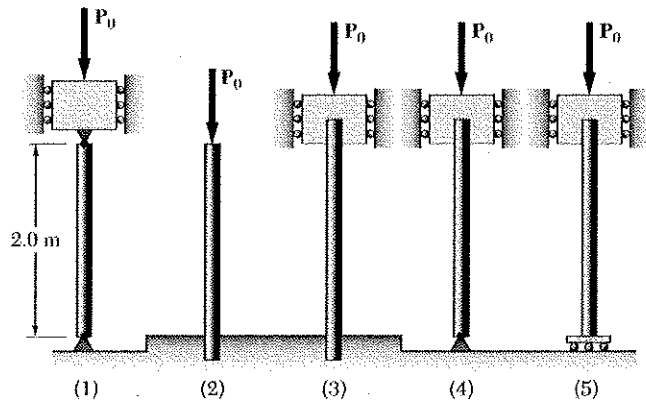
(b) $d = 28.3 \times 10^{-3} \text{ m}$

$$d = 28.3 \text{ mm} \quad \blacktriangleleft$$

$$b = 14.15 \text{ mm} \quad \blacktriangleleft$$

Problem 10.27

10.27 Each of the five struts consists of an aluminum tube that has a 32-mm outer diameter and a 4-mm wall thickness. Using $E = 70$ GPa and a factor of safety of 2.3, determine the allowable load P_0 for each support condition shown.



$$C_o = \frac{1}{2} d_o = \frac{1}{2}(32) = 16 \text{ mm}$$

$$C_i = C_o - t = 16 - 4 = 12 \text{ mm}$$

$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = 35.1858 \times 10^3 \text{ mm}^4$$

$$= 35.1858 \times 10^{-9} \text{ m}^4$$

$$\pi^2 EI = \pi^2 (70 \times 10^9)(35.1858 \times 10^{-9})$$

$$= 24309 \text{ N} \cdot \text{m}^2$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{24309}{L_e^2}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{10569}{L_e^2}$$

$$(1) L_e = (1)(2.0) = 2.0 \text{ m}, \quad P_{all} = 2642 \text{ N}$$

$$(2) L_e = (2)(2.0) = 4.0 \text{ m}, \quad P_{all} = 661 \text{ N}$$

$$(3) L_e = \left(\frac{1}{2}\right)(2.0) = 1.0 \text{ m}, \quad P_{all} = 10569 \text{ N}$$

$$(4) L_e = (0.7)(2.0) = 1.4 \text{ m}, \quad P_{all} = 5392 \text{ N}$$

$$(5) L_e = (1.0)(2.0) = 2.0 \text{ m}, \quad P_{all} = 2642 \text{ N}$$

$$P_{all} = 2.64 \text{ kN} \rightarrow$$

$$P_{all} = 0.661 \text{ kN} \rightarrow$$

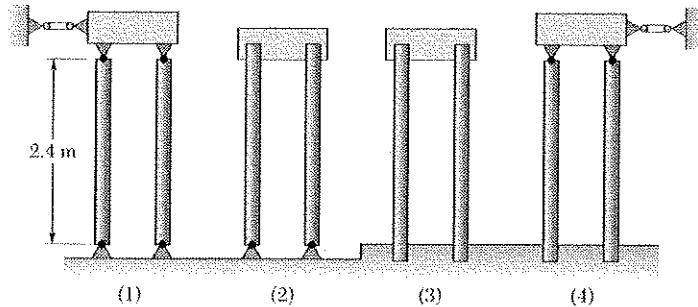
$$P_{all} = 10.57 \text{ kN} \rightarrow$$

$$P_{all} = 5.39 \text{ kN} \rightarrow$$

$$P_{all} = 2.64 \text{ kN} \rightarrow$$

Problem 10.28

10.28 Two columns are used to support a block weighing 14 kN in each of the four ways shown. (a) Knowing that the column of Fig. (1) is made of steel with a 30-mm diameter, determine the factor of safety with respect to buckling for the loading shown. (b) Determine the diameter of each of the other columns for which the factor of safety is the same as the factor of safety obtained in part a. Use $E = 200 \text{ GPa}$.



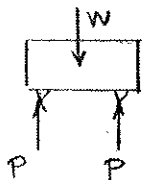
$$(a) \quad I = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} (30)^4$$

$$= 39761 \text{ mm}^4$$

$$L = 2.4 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{\pi^2 (200 \times 10^9) (39761 \times 10^{-12})}{(2.4)^2} = 13.626 \text{ kN for one column.}$$



$$P = \frac{1}{2} W = \frac{14}{2} = 7 \text{ kN.}$$

$$F.S. = \frac{P_{cr}}{P} = \frac{13.626}{7} = 1.95$$

$$P_{cr(1)} = \frac{\pi^2 EI}{L^2}$$

$$P_{cr(\alpha)} = \frac{\pi^2 E I_{\alpha}}{(L_{e,\alpha})^2}$$

$$\frac{P_{cr(\alpha)}}{P_{cr(1)}} = 1$$

$$\frac{I_{\alpha}}{I_{(1)}} \cdot \frac{L^2}{L_{e^2}} = 1 \quad \left(\frac{d_{\alpha}}{d_{(1)}} \right)^4 \left(\frac{L_e}{L} \right)^2 = 1$$

$$d_{\alpha} = d_{(1)} \sqrt[4]{\frac{L_{e(1)}}{L}}$$

$$(b) \quad (2) \quad L_{e(2)} / L = 2.0$$

$$d_{(2)} = 30 \sqrt[4]{2.0} = 42.4 \text{ mm.}$$

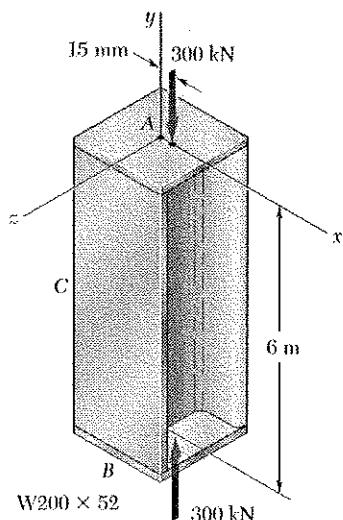
$$(3) \quad L_{e(3)} / L = 1.0$$

$$d_{(3)} = 30 \text{ mm}$$

$$(4) \quad L_{e(4)} / L = 0.7$$

$$d_{(4)} = 30 \sqrt[4]{0.7} = 25.1 \text{ mm}$$

Problem 10.29



10.29 The line of action of the 300-kN axial load is parallel to the geometric axis of the column AB and intersects the x axis at $x = 15$ mm. Using $E = 200$ GPa, determine (a) the horizontal deflection of the midpoint C of the column, (b) the maximum stress in the column.

$$L_e = L = 6 \text{ m}$$

$$e = 15 \text{ mm}$$

$$W_{200 \times 52}$$

$$A = 6660 \text{ mm}^2$$

$$I_y = 17.8 \times 10^6 \text{ mm}^4$$

$$S_y = 175 \times 10^3 \text{ mm}^3$$

$$E = 200 \text{ GPa}$$

$$P_{cr} = \frac{\pi^2 EI_y}{L^2} = \frac{\pi^2 (200 \times 10^9) (17.8 \times 10^{-6})}{(6)^2} = 976 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{300}{976} = 0.3074$$

$$\begin{aligned} (a) \quad y_m &= e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \\ &= (15) \left[\sec\left(\frac{\pi}{2} \sqrt{0.3074}\right) - 1 \right] \\ &= (15) \left[\sec(0.8709) - 1 \right] \\ &= 8.287 \text{ mm} \end{aligned}$$

$$y_m = 8.3 \text{ mm}$$

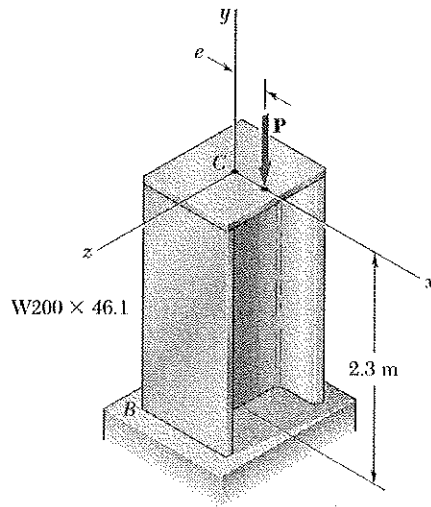
$$(b) \quad M_{max} = P(y_m + e) = (300)(0.015 + 0.008287) = 6.986 \text{ kN}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{300 \times 10^3}{6660 \times 10^{-6}} + \frac{6.986 \times 10^3}{175 \times 10^{-6}}$$

$$\sigma_{max} = 85 \text{ MPa}$$

Problem 10.30

10.30 An axial load P of magnitude 560 kN is applied at a point on the x axis at a distance $e = 6$ mm from the geometric axis of the W200 $\times$ 46.1 rolled-steel column BC . Using $E = 200$ GPa, determine (a) the horizontal deflection of end C , (b) the maximum stress in the column.



$$L_e = 2L = (2)(2.3) = 4.6 \text{ m} \quad e = 6 \times 10^{-3} \text{ m}$$

$$W 200 \times 46.1 \quad A = 5860 \text{ mm}^2 = 5860 \times 10^{-6} \text{ m}^2$$

$$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (15.3 \times 10^{-6})}{(4.6)^2}$$

$$= 1.42727 \times 10^6 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{560 \times 10^3}{1.42727 \times 10^6} = 0.39236$$

$$(a) \quad y_m = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = (6 \times 10^{-3}) \left[\sec\left(\frac{\pi}{2} \sqrt{0.39236}\right) - 1 \right]$$

$$= (6 \times 10^{-3}) \left[\sec(0.98393) - 1 \right] = (6 \times 10^{-3}) (1.8058 - 1)$$

$$= 4.835 \times 10^{-3} \text{ m}$$

$$y_m = 4.84 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad M_{max} = P(y_m + e) = (560 \times 10^3)(4.835 \times 10^{-3} + 6 \times 10^{-3}) = 6.0676 \times 10^3 \text{ N}\cdot\text{m}$$

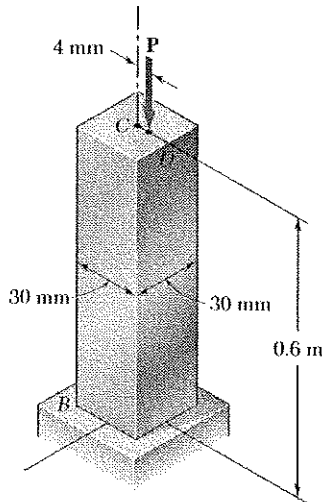
$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

$$\sigma_{max} = \frac{P}{A} + \frac{M}{S_y} = \frac{560 \times 10^3}{5860 \times 10^{-6}} + \frac{6.0676 \times 10^3}{151 \times 10^{-6}} = 135.7 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 135.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 10.31

10.31 An axial load $P = 15$ kN is applied at point D that is 4 mm from the geometric axis of the square aluminum bar BC . Using $E = 70$ GPa, determine (a) the horizontal deflection of end C , (b) the maximum stress in the column.



$$A = (30)^2 = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} (30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4 = 67.5 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} (30) = 15 \text{ mm} = 0.015 \text{ m} \quad e = 4 \times 10^{-3} \text{ m}$$

$$L_e = 2L = (2)(0.6) = 1.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (70 \times 10^9) (67.5 \times 10^{-9})}{(1.2)^2}$$

$$= 32.385 \times 10^3 \text{ N} = 32.385 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{15}{32.385} = 0.46318$$

$$(a) \quad y_m = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = (4 \times 10^{-3}) \left[\sec\left(\frac{\pi}{2} \sqrt{0.46318}\right) - 1 \right]$$

$$= (4 \times 10^{-3}) [\sec(1.06904) - 1] = 4.3166 \times 10^{-3} \text{ m}$$

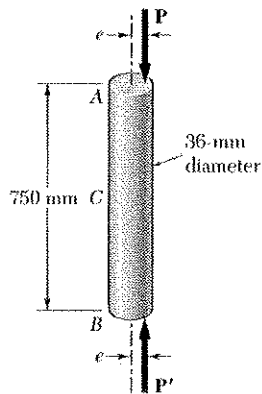
$$y_m = 4.32 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad M_{max} = P(e + y_m) = (15 \times 10^3) (4 \times 10^{-3} + 4.3166 \times 10^{-3}) = 124.75 \text{ N} \cdot \text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{15 \times 10^3}{900 \times 10^{-6}} + \frac{(124.75)(0.015)}{67.5 \times 10^{-9}} = 44.4 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 44.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 10.32



10.32 An axial load P is applied to the 36-mm-diameter steel rod AB as shown. When $P = 90$ kN, it is observed that the horizontal deflection of the midpoint C is 0.8 mm. Using $E = 200$ GPa, determine (a) the eccentricity e of the load, (b) the maximum stress in the rod.

$$c = \frac{1}{2}d = 18 \text{ mm} \quad A = \pi c^2 = 1017.9 \text{ mm}^2$$

$$I = \frac{\pi}{4} c^4 = 82448 \text{ mm}^4$$

$$L_e = L = 0.75 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (82448 \times 10^{-12})}{(0.75)^2} = 289.325 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{90}{289.325} = 0.31107$$

$$(a) \quad y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = e [\sec 0.8761 - 1] \\ = 0.5621 e$$

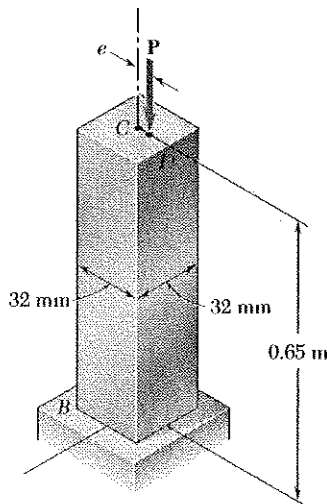
$$e = \frac{y_{max}}{0.5621} = \frac{0.8}{0.5621} = 1.42 \text{ mm}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (90)(1.42 + 0.8) = 199.8 \text{ Nm}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{90 \times 10^3}{1017.9} + \frac{(199.8)(18)(10^3)}{82448} = 132.03 \text{ MPa} \\ = 132 \text{ MPa}$$

Problem 10.33

10.33 An axial load P is applied to the 32-mm-square aluminum bar BC as shown. When $P = 24$ kN, the horizontal deflection at end C is 4 mm. Using $E = 70$ GPa, determine (a) the eccentricity e of the load, (b) the maximum stress in the rod.



$$I = \frac{1}{12} (32)^4 = 87.3813 \times 10^3 \text{ mm}^4 = 87.3813 \times 10^{-9} \text{ m}^4$$

$$A = (32)^2 = 1.024 \times 10^3 \text{ mm}^2 = 1.024 \times 10^{-3} \text{ m}^2$$

$$L_e = 2L = (2)(0.65) = 1.30 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (70 \times 10^9) (87.3813 \times 10^{-9})}{(1.30)^2} = 35.7215 \times 10^3 \text{ N} = 35.7215 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{24}{35.7215} = 0.67186$$

$$(a) \quad y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] = e \left[\sec\left(\frac{\pi}{2} \sqrt{0.67186}\right) - 1 \right] = e [\sec 1.28754 - 1] = 2.5780 e$$

$$e = \frac{y_{max}}{2.5780} = \frac{4}{2.5780} \quad e = 1.552 \text{ mm} \blacktriangleleft$$

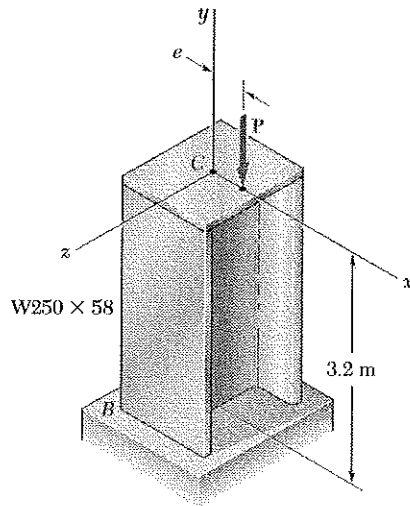
$$(b) \quad M_{max} = P(e + y_{max}) = (24 \times 10^3) [1.552 \times 10^{-3} + 4 \times 10^{-3}] = 133.24 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{24 \times 10^3}{1.024 \times 10^{-3}} + \frac{(133.24)(0.016)}{87.3813 \times 10^{-9}} = 47.8 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 47.8 \text{ MPa} \blacktriangleleft$$

Problem 10.34

10.34 The axial load P is applied at a point located on the x axis at a distance e from the geometric axis of the rolled-steel column BC . When $P = 350$ kN, the horizontal deflection of the top of the column is 5 mm. Using $E = 200$ GPa, determine (a) the eccentricity e of the load, (b) the maximum stress in the column.



$$W 250 \times 58 \quad A = 7420 \text{ mm}^2 = 7420 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.8 \times 10^6 \text{ mm}^4 = 18.8 \times 10^{-6} \text{ m}^4$$

$$S_y = 185 \times 10^3 \text{ mm}^3 = 185 \times 10^{-6} \text{ m}^3$$

$$L_e = 2L = (2)(3.2) = 6.4 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (18.8 \times 10^{-6})}{(6.4)^2}$$

$$= 906 \times 10^3 \text{ N} = 906 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{350}{906} = 0.38631$$

$$(a) \quad y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = e \left[\sec 0.97632 - 1 \right] = 0.78546 e$$

$$e = \frac{y_{max}}{0.78540} = \frac{5}{0.78540}$$

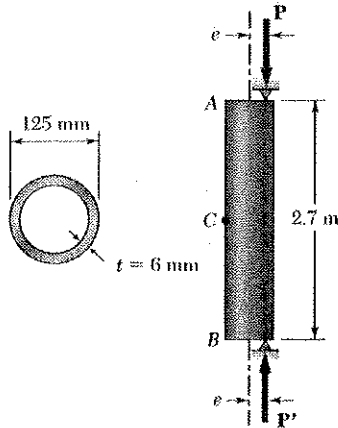
$$e = 6.37 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad M_{max} = P(e + y_{max}) = (350 \times 10^3) (6.37 \times 10^{-3} + 5 \times 10^{-3}) = 3.978 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{350 \times 10^3}{7420 \times 10^{-6}} + \frac{3.978 \times 10^3}{185 \times 10^{-6}} = 68.7 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 68.7 \text{ MPa} \quad \blacktriangleleft$$

Problem 10.35



10.35 A brass pipe having the cross section shown has an axial load P applied 4 mm from its geometric axis. Using $E = 120 \text{ GPa}$, determine (a) the load P for which the horizontal deflection at the midpoint C is 5 mm, (b) the corresponding maximum stress in the column.

$$c_o = \frac{1}{2} d_o = \frac{1}{2} (125) = 62.5 \text{ mm} \quad c_i = c_o - t = 56.5 \text{ mm}$$

$$I = \frac{\pi}{4} (c_o^4 - c_i^4) = \frac{\pi}{4} (62.5^4 - 56.5^4) = 3.9807 \times 10^6 \text{ mm}^4$$

$$L_e = L = 2.7 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (120 \times 10^9) (3.9807 \times 10^{-6})}{(2.7)^2} = 646.71 \text{ kN}$$

$$(a) \quad y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{e + y_{max}} \quad \frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2$$

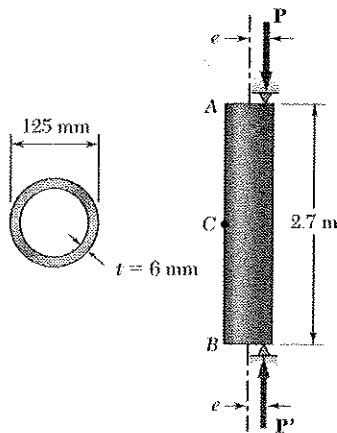
$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{4}{5 + 4} \right]^2 = 0.4996 \quad P = 0.4996 P_{cr} = 323 \text{ kN}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (323)(5 + 4) = 2907 \text{ kN}\cdot\text{mm}$$

$$A = \pi (c_o^2 - c_i^2) = \pi (62.5^2 - 56.5^2) = 2243 \text{ mm}^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{323000}{2243} + \frac{(2907000)(62.5)}{3.9807 \times 10^6} = 189.7 \text{ MPa}$$

Problem 10.36



10.36 Solve Prob. 10.35, assuming that the axial load P is applied 8 mm from the geometric axis of the column.

10.35 A brass pipe having the cross section shown has an axial load P applied 4 mm from its geometric axis. Using $E = 120$ GPa, determine (a) the load P for which the horizontal deflection at the midpoint C is 5 mm, (b) the corresponding maximum stress in the column.

$$c_o = \frac{1}{2} d_o = \frac{1}{2}(125) = 62.5 \text{ mm} \quad c_i = c_o - t = 56.5 \text{ mm}$$

$$I = \frac{\pi}{4} (c_o^4 - c_i^4) = \frac{\pi}{4} (62.5^4 - 56.5^4) = 3.9807 \times 10^6 \text{ mm}^4$$

$$L_e = L = 2.7 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (120 \times 10^9) (3.9807 \times 10^{-6})}{(2.7)^2} = 646.71 \text{ kN}$$

$$(a) \quad y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] \quad \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{y_{max} + e}{e} \quad \cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e} \right]^2 = \left[\frac{2}{\pi} \arccos \frac{8}{5 + 8} \right]^2 = 0.3341$$

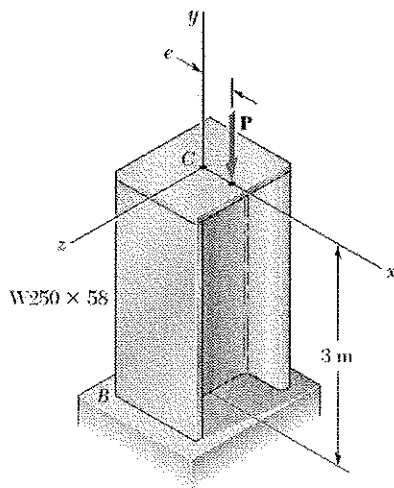
$$P = 0.3341 P_{cr} = 216 \text{ kN}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (216)(5 + 8) = 2808 \text{ kN} \cdot \text{mm}$$

$$A = \pi (c_o^2 - c_i^2) = \pi (62.5^2 - 56.5^2) = 2243 \text{ mm}^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max} c}{I} = \frac{216000}{2243} + \frac{(2808000)(62.5)}{3.9807 \times 10^6} = 140.4 \text{ MPa}$$

Problem 10.37



10.37 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W250 $\times$ 58 rolled-steel column BC . Using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection of the top of the column is 15 mm, (b) the corresponding maximum stress in the column.

$$W250 \times 58 \quad A = 7420 \text{ mm}^2, \quad I_y = 18.8 \times 10^6 \text{ mm}^4$$

$$S_y = 185 \times 10^3 \text{ mm}^3$$

$$L_e = 2L = (2)(3) = 6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (18.8 \times 10^{-6})}{(6)^2}$$

$$= 1030.8 \text{ kN}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos\left(\frac{e}{y_{max} + e}\right) \right]^2 = \left[\frac{2}{\pi} \arccos\left(\frac{12}{15 + 12}\right) \right]^2$$

$$= 0.49957$$

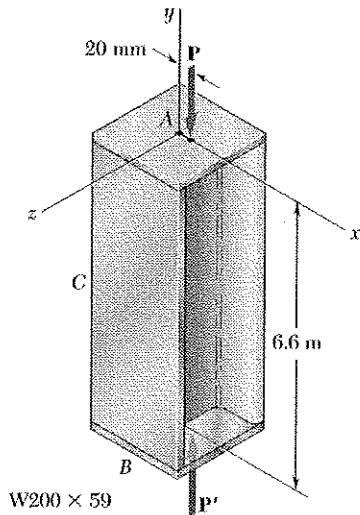
$$(a) \quad P = (0.49957)(1030.8) = 463.8 \text{ kN}$$

$$(b) \quad M_{max} = P(e + y_{max}) = (463.8)(15 + 12) = 12.52 \text{ kNm}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{463800}{7420} + \frac{12520000}{185 \times 10^3} = 130.2 \text{ MPa}$$

Problem 10.38

10.38 The line of action of an axial load P is parallel to the geometric axis of the column AB and intersects the x axis at $x = 20$ mm. Using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection of the midpoint C of the column is 12 mm, (b) the corresponding maximum stress in the column.



$$L_e = L = 6.6 \text{ m}$$

$$e = 20 \text{ mm}$$

$$W200 \times 59 \quad A = 7560 \text{ mm}^2 \quad I_y = 20.4 \times 10^6 \text{ mm}^4 \quad S_y = 199 \times 10^3 \text{ mm}^3$$

$$E = 200 \text{ GPa}$$

$$y_m = 20 \text{ mm}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (20.4 \times 10^{-6})}{(6.6)^2} = 92442.5 \text{ N}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = \frac{y_m + e}{e}$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_m + e} = \frac{20}{12 + 20} = 0.625$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.89567$$

$$\frac{P}{P_{cr}} = \left[\frac{(2)(0.89567)}{\pi} \right]^2 = 0.32513$$

$$(a) \quad P = (0.32513)(92442.5) = 30055.8$$

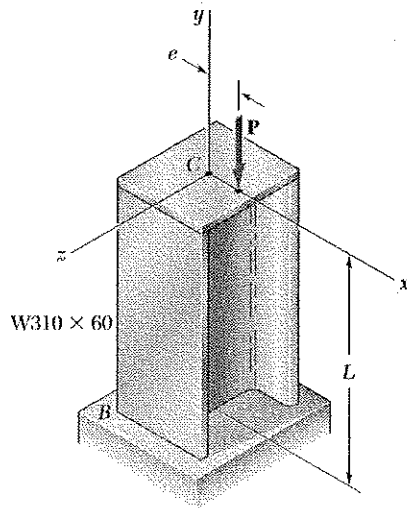
$$P = 300.6 \text{ kN}$$

$$(b) \quad M_{max} = P(e + y_m) = (30055.8)(0.02 + 0.012) = 9617.9 \text{ N}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{S_y} = \frac{30055.8}{7560 \times 10^{-6}} + \frac{9617.9}{199 \times 10^{-6}} \quad \sigma_{max} = 52.3 \text{ MPa}$$

Problem 10.39

10.39 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 $\times$ 60 rolled-steel column BC . Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection at end C is 15 mm, (b) the corresponding maximum stress in the column.



W310 $\times$ 60

$$\begin{aligned} A &= 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2 \\ I_y &= 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4 \\ S_y &= 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3 \end{aligned}$$

$$L = 3.5 \text{ m} \quad L_e = 2L = 7.0 \text{ m}$$

$$\begin{aligned} P_{cr} &= \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (18.3 \times 10^{-6})}{(7.0)^2} \\ &= 737.2 \times 10^3 \text{ N} = 737.2 \text{ kN} \end{aligned}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e} \quad \cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos\left(\frac{e}{y_{max} + e}\right) \right]^2 = \left[\frac{2}{\pi} \arccos\left(\frac{12}{15 + 12}\right) \right]^2 = 0.49957$$

$$(a) \quad P = 0.49957 P_{cr} = 368.28 \text{ kN} \quad P = 368 \text{ kN} \quad \blacktriangleleft$$

$$M_{max} = P(e + y_{max}) = (368.28 \times 10^3)(12 + 15)(10^{-3}) = 9944 \text{ N}\cdot\text{m}$$

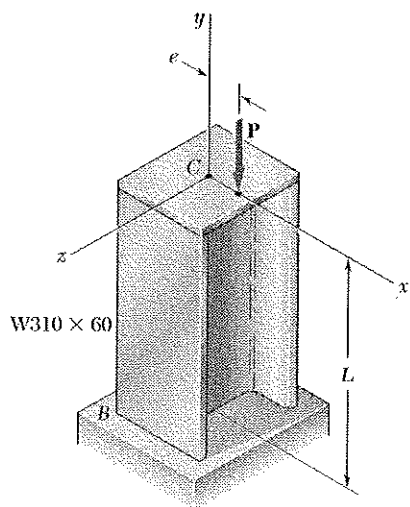
$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{368.28 \times 10^3}{7590 \times 10^{-6}} + \frac{9944}{180 \times 10^{-6}} = 103.8 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 103.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 10.40

10.40 Solve Prob. 10.39, assuming that L is 4.5 m.

10.39 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 $\times$ 60 rolled-steel column BC . Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection at end C is 15 mm, (b) the corresponding maximum stress in the column.



W 310 $\times$ 60

$$A = 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2$$

$$I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4$$

$$S_y = 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3$$

$$L = 4.5 \text{ m}$$

$$L_e = 2L = 9.0 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (18.3 \times 10^{-6})}{(9.0)^2}$$

$$= 445.96 \times 10^3 \text{ N} = 445.96 \text{ kN}$$

$$y_{max} = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right] \quad \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e} \quad \cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos\left(\frac{e}{y_{max} + e}\right) \right]^2 = \left[\frac{2}{\pi} \arccos\left(\frac{12}{15 + 12}\right) \right]^2 = 0.49957$$

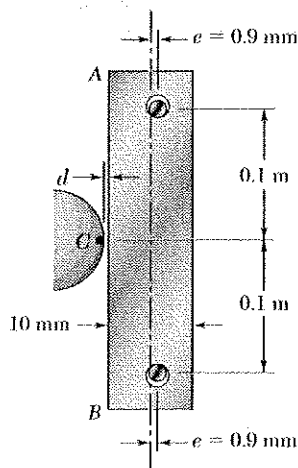
$$(a) \quad P = 0.49957 P_{cr} = 222.79 \text{ kN} \quad P = 223 \text{ kN} \quad \blacktriangleleft$$

$$M_{max} = P(e + y_{max}) = (222.79 \times 10^3)(12 + 15)(10^{-3}) = 6015 \text{ N}\cdot\text{m}$$

$$(b) \quad \sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{222.79 \times 10^3}{7590 \times 10^{-6}} + \frac{6015}{180 \times 10^{-6}} = 62.8 \times 10^6 \text{ Pa}$$

$$\sigma_{max} = 62.8 \text{ MPa} \quad \blacktriangleleft$$

Problem 10.41



10.41 The steel bar AB has a 10×10 -mm square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.9$ mm from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.3$ mm. Use $E = 200$ GPa and the coefficient of thermal expansion $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$.

$$A = (10)^2 = 100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} (10)^4 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = (200 \times 10^9)(833.33 \times 10^{-12}) = 166.667 \text{ N}\cdot\text{m}^2$$

$$L_e = L = 0.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (166.667)}{(0.2)^2} = 41.1234 \times 10^3 \text{ N}$$

Calculate P using the secant formula.

$$y_{max} = d = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] \quad \text{or} \quad \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = 1 + \frac{d}{e} =$$

$$\cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{e}{e+d} = \frac{0.9}{0.9+0.3} = 0.75$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = \cos^{-1}(0.75) = 0.72273$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.72273) \right]^2 = 0.21170$$

$$P = 0.21170 P_{cr} = 8.7058 \times 10^3 \text{ N}$$

If the effects of eccentricity and the shortening due to bending are neglected, the shortening of the bar, if not constrained by the pins, would be

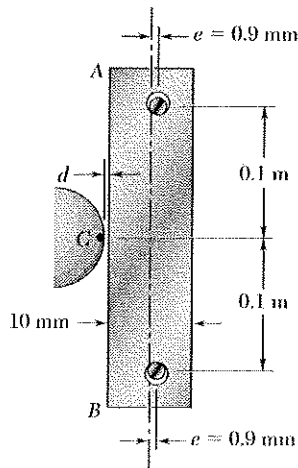
$$\delta = \frac{PL}{EA} = \frac{(8.7058 \times 10^3)(0.2)}{(200 \times 10^9)(100 \times 10^{-6})} = 87.058 \times 10^{-6} \text{ m}$$

This mechanical shortening is compensated by thermal expansion.

$$L\alpha(\Delta T) - \delta = 0$$

$$\Delta T = \frac{\delta}{L\alpha} = \frac{87.058 \times 10^{-6}}{(0.2)(11.7 \times 10^{-6})} = 37.2^\circ\text{C}$$

Problem 10.42



10.42 For the bar of Prob. 10.41, determine the required distance d for which the bar will just make contact with point C when the temperature increases by 60°C .

10.41 The steel bar AB has a $10 \times 10\text{-mm}$ square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.9\text{ mm}$ from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.3\text{ mm}$. Use $E = 200\text{ GPa}$ and the coefficient of thermal expansion $\alpha = 11.7 \times 10^{-6}/^\circ\text{C}$.

$$A = (10)^2 = 100\text{ mm}^2 = 100 \times 10^{-6}\text{ m}^2$$

$$I = \frac{1}{12}(10)^4 = 833.33\text{ mm}^4 = 833.33 \times 10^{-12}\text{ m}^4$$

$$EI = (200 \times 10^9)(833.33 \times 10^{-12}) = 166.667\text{ N}\cdot\text{m}^2$$

$$L_e = L = 0.2\text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (166.667)}{(0.2)^2} = 41.1234 \times 10^3\text{ N}$$

Free thermal expansion of the bar:

$$\delta = L\alpha(\Delta T) = (0.2)(11.7 \times 10^{-6})(60) = 140.4 \times 10^{-6}\text{ m}$$

The free expansion is prevented by constraint of the pins.

If eccentricity and shortening due to bending are neglected, the constraining force P is given by

$$P = \frac{EAS}{L} = \frac{(200 \times 10^9)(100 \times 10^{-6})(140.4 \times 10^{-6})}{0.2} = 14.04 \times 10^3\text{ N}$$

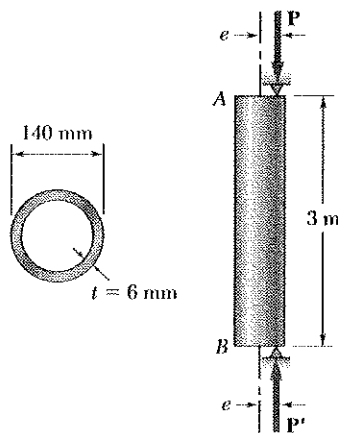
Using the secant formula,

$$y_{max} = d = e \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) - 1 \right]$$

$$= (9 \times 10^{-3}) \left[\sec\left(\frac{\pi}{2} \sqrt{\frac{14.04 \times 10^3}{41.1234 \times 10^3}}\right) - 1 \right] = 5.81 \times 10^{-3}\text{ m}$$

$$d = 5.81\text{ mm} \quad \blacktriangleleft$$

Problem 10.43



10.43 A pipe having the cross section shown is used as a 3-m column. For the grade of steel used $\sigma_Y = 252 \text{ MPa}$ and $E = 200 \text{ GPa}$. Knowing that a factor of safety of 2.8 with respect to permanent deformation is required, determine the allowable load P when the eccentricity e is (a) 8 mm, (b) 4 mm (Hint: Since the factor of safety must be applied to the load P , not to the stress, use Fig. 10.24 to determine P_Y).

$$c = \frac{1}{2}d = \frac{140}{2} = 70 \text{ mm}$$

$$c_1 = c - t = 70 - 6 = 64 \text{ mm}$$

$$A = \pi(c^2 - c_1^2) = 2525.8 \text{ mm}^2$$

$$I = \frac{\pi}{4}(c^4 - c_1^4) = 5686615 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 47.4 \text{ mm}$$

$$L_e = L = 3 \text{ m}$$

$$\frac{L}{r} = \frac{3000}{47.4} = 63.32$$

$$(a) \quad e = 8 \text{ mm} \quad \frac{ec}{r^2} = \frac{(8)(70)}{(47.4)^2} = 0.2492$$

$$\text{Fig 10.24} \quad \frac{P_Y}{A} = 160 \text{ MPa}$$

$$P_Y = (2525.8 \times 10^{-6})(160 \times 10^6) = 404128 \text{ N}$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{404128}{2.8}$$

$$P_{all} = 144.3 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad e = 4 \text{ mm} \quad \frac{ec}{r^2} = \frac{(4)(70)}{(47.4)^2} = 0.1246$$

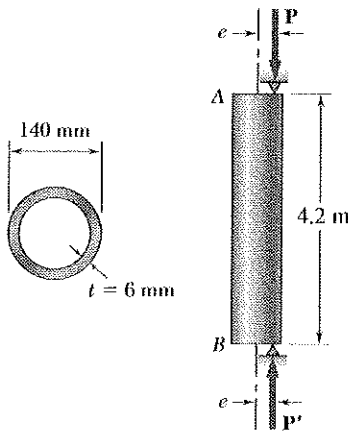
$$\text{Fig 10.24} \quad \frac{P_Y}{A} = 196 \text{ MPa}$$

$$P_Y = (2525.8 \times 10^{-6})(196 \times 10^6) = 495056.8$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{495056.8}{2.8}$$

$$P_{all} = 176.8 \text{ kN} \quad \blacktriangleleft$$

Problem 10.44



10.44 Solve Prob. 10.43, assuming that the length of the column is increased to 4.2 m.

10.43 A pipe having the cross section shown is used as a 4.2 m column. For the grade of steel used $\sigma_Y = 252 \text{ MPa}$ and $E = 200 \text{ GPa}$. Knowing that a factor of safety of 2.8 with respect to permanent deformation is required, determine the allowable load P when the eccentricity e is (a) 8 mm, (b) 4 mm (Hint: Since the factor of safety must be applied to the load P , not to the stress, use Fig. 10.24 to determine P_Y).

$$c = \frac{1}{2}d = \frac{140}{2} = 70 \text{ mm}$$

$$c_1 = c - t = 70 - 6 = 64 \text{ mm}$$

$$A = \pi(c^2 - c_1^2) = 2525.8 \text{ mm}^2$$

$$I = \frac{\pi}{4}(c^4 - c_1^4) = 5680615 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 47.4 \text{ mm}$$

$$L_e = L = 4.2 \text{ m}$$

$$\frac{L_e}{r} = \frac{4200}{47.4} = 88.608$$

$$(a) \quad e = 8 \text{ mm} \quad \frac{ec}{r^2} = \frac{(8)(70)}{(47.4)^2} = 0.2492$$

$$\text{Fig 10.24} \quad \frac{P_Y}{A} = 135 \text{ MPa}$$

$$P_Y = (2525.8 \times 10^{-6})(135 \times 10^6) = 340983 \text{ N}$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{340983}{2.8}$$

$$P_{all} = 121.8 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad e = 4 \text{ mm} \quad \frac{ec}{r^2} = \frac{(4)(70)}{(47.4)^2} = 0.1246$$

$$\text{Fig 10.24} \quad \frac{P_Y}{A} = 165 \text{ MPa}$$

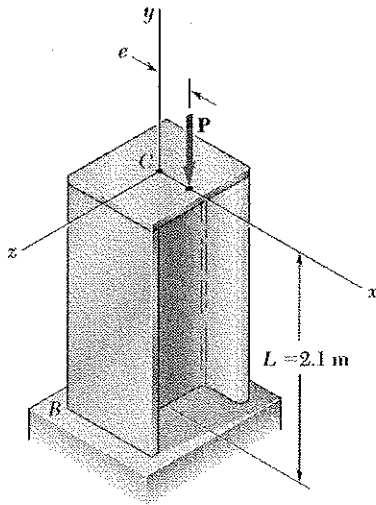
$$P_Y = (2525.8 \times 10^{-6})(165 \times 10^6) = 416757 \text{ N}$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{416757}{2.8}$$

$$P_{all} = 148.8 \text{ kN} \quad \blacktriangleleft$$

Problem 10.45

10.45 An axial load P is applied to the W250 $\times$ 44.8 rolled-steel column BC that is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 12$ mm and that for the grade of steel used $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine (a) the magnitude of P of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (b) the ratio of the load found in part a to the magnitude of the allowable centric load for the column. (See hint of Prob. 10.43).



$$\begin{aligned} \text{W250} \times 44.8 \quad c &= \frac{bf}{2} = 74 \text{ mm} \\ A &= 5720 \text{ mm}^2 = 5.720 \times 10^{-3} \text{ m}^2 \end{aligned}$$

$$I_y = 7.03 \times 10^6 \text{ mm}^4 = 7.03 \times 10^{-6} \text{ m}^4$$

$$r_y = 35.1 \text{ mm} = 35.1 \times 10^{-3} \text{ m} \quad e = 12 \text{ mm}$$

$$L_c = 2L = (2)(2.1) = 4.2 \text{ m}$$

$$\frac{L_c}{r} = \frac{4.2}{35.1 \times 10^{-3}} = 119.658$$

$$\frac{ec}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$$

(a) Fig. 10.24 (b) $\frac{P_Y}{A} = 82.0 \text{ MPa}$

$$P_Y = (5.720 \times 10^{-3})(82.0 \times 10^6) = 469 \times 10^3 \text{ N} = 469 \text{ kN}$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{469}{2.4}$$

$$P_{all} = 195.4 \text{ kN} \quad \blacktriangleleft$$

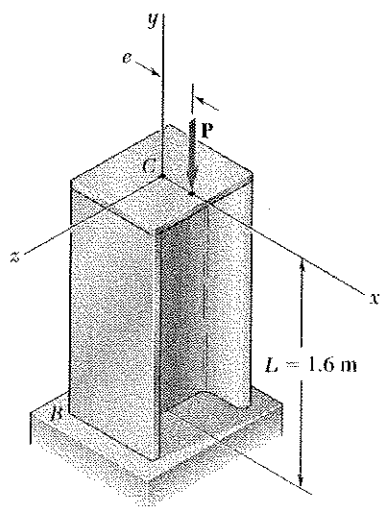
(b) $P_{cr} = \frac{\pi^2 EI}{L_c^2} = \frac{\pi^2 (200 \times 10^9)(7.03 \times 10^{-6})}{(4.2)^2} = 787 \times 10^3 \text{ N} = 787 \text{ kN}$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{787}{2.4} = 328 \text{ kN}$$

$$\text{ratio} = \frac{195.4 \text{ kN}}{328 \text{ kN}}$$

$$\text{ratio} = 0.596 \quad \blacktriangleleft$$

Problem 10.46



10.46 Solve Prob. 10.45, assuming that the length of the column is reduced to 1.6 m.

10.45 An axial load P is applied to the $W250 \times 44.8$ rolled-steel column BC that is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 12$ mm and that for the grade of steel used $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine (a) the magnitude of P of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (b) the ratio of the load found in part a to the magnitude of the allowable centric load for the column. (See hint of Prob. 10.43).

$$W250 \times 44.8 \quad c = \frac{b_F}{2} = 74 \text{ mm}$$

$$A = 5720 \text{ mm}^2 = 5.720 \times 10^{-3} \text{ m}^2$$

$$I_y = 7.03 \times 10^6 \text{ mm}^4 = 7.03 \times 10^{-6} \text{ m}^4$$

$$r_y = 35.1 \text{ mm} = 35.1 \times 10^{-3} \text{ m} \quad e = 12 \text{ mm}$$

$$L_e = 2L = (2)(1.6) = 3.2 \text{ m}$$

$$\frac{L_e}{r} = \frac{3.2}{35.1 \times 10^{-3}} = 91.168$$

$$\frac{ec}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$$

(a) Fig. 10.24(b) $\frac{P_Y}{A} = 103.5 \text{ MPa}$

$$P_Y = (5.72 \times 10^{-3})(103.5 \times 10^6) = 592 \times 10^3 = 592 \text{ kN}$$

$$P_{all} = \frac{P_Y}{F.S.} = \frac{592}{2.4}$$

$$P_{all} = 247 \text{ kN}$$

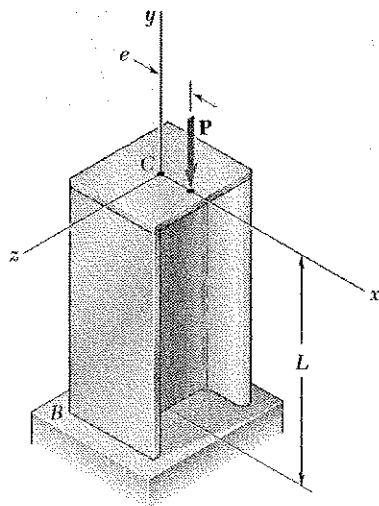
(b) $P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(7.03 \times 10^{-6})}{(3.2)^2} = 1.355 \times 10^6 \text{ N} = 1355 \text{ kN}$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{1355}{2.4} = 565 \text{ kN}$$

$$\text{ratio} = \frac{247 \text{ kN}}{565 \text{ kN}}$$

$$\text{ratio} = 0.437$$

Problem 10.47



10.47 A 100-kN axial load P is applied to the W150 $\times$ 18 rolled-steel column BC that is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 6\text{ mm}$, determine the largest permissible length L if the allowable stress in the column is 80 MPa. Use $E = 200\text{ GPa}$.

$$W150 \times 18 \quad A = 2290\text{ mm}^2 = 2290 \times 10^{-6}\text{ m}^2$$

$$b_p = 102\text{ mm} \quad c = \frac{b_p}{2} = 51\text{ mm}$$

$$I_y = 1.26 \times 10^6\text{ mm}^4 = 1.26 \times 10^{-6}\text{ m}^4 \quad r_y = 23.5\text{ mm}$$

$$\sigma_{\max} = 80 \times 10^6\text{ Pa} \quad P = 100 \times 10^3\text{ N}$$

$$\sigma_{\max} = \frac{P}{A} \left[1 + \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

$$\frac{A\sigma_{\max}}{P} - 1 = \frac{ec}{r_y^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

$$\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = \frac{r_y^2}{ec} \left(\frac{A\sigma_{\max}}{P} - 1 \right) = \frac{(23.5)^2}{(6)(51)} \left[\frac{(2290 \times 10^{-6})(80 \times 10^6)}{100 \times 10^3} - 1 \right]$$

$$= 1.50154$$

$$\cos \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) = 0.66598 \quad \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.84199$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.84199) \right]^2 = 0.28732$$

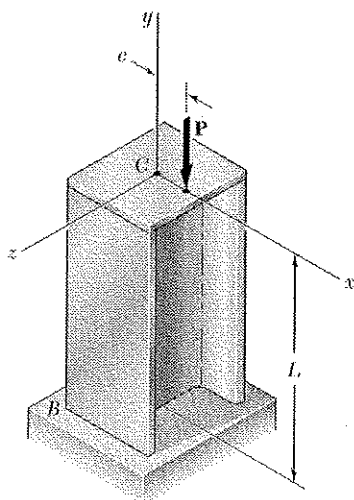
$$P_{cr} = \frac{P}{0.28732} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.28732 \pi^2 EI}{P} = \frac{0.28732 \pi^2 (200 \times 10^9)(1.26 \times 10^{-6})}{100 \times 10^3} = 7.1461\text{ m}^2$$

$$L_e = 2.6732 = 2L$$

$$L = 1.337\text{ m} \quad \blacktriangleleft$$

Problem 10.48



10.48 A 250-kN axial load P is applied to the W200 $\times$ 35.9 rolled-steel column BC which is free at its top C and fixed at its base B . Knowing that the eccentricity of the load is $e = 6$ mm, determine the largest permissible length L if the allowable stress in the column is 80 MPa. Use $E = 200$ GPa.

$$W200 \times 35.9 \quad A = 4580 \text{ mm}^2 = 4580 \times 10^{-6} \text{ m}^2$$

$$b_f = 165 \text{ mm} \quad c = \frac{b_f}{2} = 82.5 \text{ mm}$$

$$I_y = 7.64 \times 10^6 \text{ mm}^4 = 7.64 \times 10^{-6} \text{ m}^4 \quad r_y = 40.8 \text{ mm}$$

$$\sigma_{\max} = 80 \times 10^6 \text{ Pa} \quad P = 250 \times 10^3 \text{ N}$$

$$\sigma_{\max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\frac{A\sigma_{\max}}{P} - 1 = \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec} \left(\frac{A\sigma_{\max}}{P} - 1 \right) = \frac{(40.8)^2}{(6)(82.5)} \left[\frac{(4580 \times 10^{-6})(80 \times 10^6)}{250 \times 10^3} - 1 \right]$$

$$= 1.56657$$

$$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) = 0.63866 \quad \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 0.87804$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} (0.87804) \right]^2 = 0.31245$$

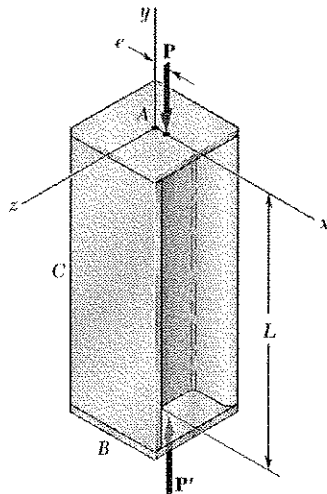
$$P_{cr} = \frac{P}{0.31245} = \frac{\pi^2 EI}{Le^2}$$

$$Le^2 = \frac{0.31245 \pi^2 EI}{P} = \frac{0.31245 \pi^2 (200 \times 10^9)(7.64 \times 10^{-6})}{250 \times 10^3} = 18.848 \text{ m}^2$$

$$Le = 4.34 \text{ m} = 2L$$

$$L = 2.17 \text{ m}$$

Problem 10.49



10.49 Axial loads of magnitude $P = 600 \text{ kN}$ are applied parallel to the geometric axis of the $W250 \times 80$ rolled-steel column AB and intersect the x axis at a distance e from the geometric axis. Knowing that $\sigma_{\text{all}} = 84 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the largest permissible length L when (a) $e = 6 \text{ mm}$, (b) $e = 12 \text{ mm}$.

Data: $P = 600 \text{ kN}$ $E = 200 \text{ GPa}$

$W250 \times 80$ $A = 10200 \text{ mm}^2$ $c = \frac{b_F}{2} = 128 \text{ mm}$

$I_y = 43.1 \times 10^6 \text{ mm}^4$ $r_y = 65 \text{ mm}$

$\sigma_{\text{all}} = \sigma_{\text{max}} = 84 \text{ MPa}$

$$\sigma_{\text{max}} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) \right]$$

$$\frac{A\sigma_{\text{max}}}{P} - 1 = \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right)$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{r^2}{ec} \left[\frac{A\sigma_{\text{max}}}{P} - 1 \right]$$

(a) $e = 6 \text{ mm}$ $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{(65)^2}{(6)(128)} \left[\frac{(10200)(84)}{600000} - 1 \right] = 2.3546$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = 0.4247$ $\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}} = 1.132$

$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi} (1.132) \right]^2 = 0.519$

$P_{\text{cr}} = \frac{P}{0.519} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.519 \pi^2 EI}{P} = \frac{0.519 \pi^2 (200 \times 10^9) (43.1 \times 10^{-6})}{600000} = 73.59 \text{ m}^2$

$L_e = 8.579$ $L = L_e = 8.6 \text{ m}$

(b) $e = 12 \text{ mm}$ $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{(65)^2}{(12)(128)} \left[\frac{(10200)(84)}{600000} - 1 \right] = 1.1773$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = 0.8494$ $\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}} = 0.5559$

$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi} (0.5559) \right]^2 = 0.1252$

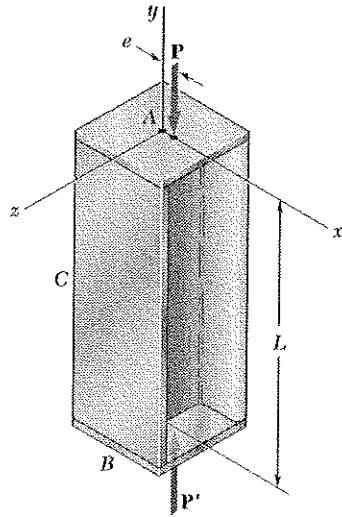
$P_{\text{cr}} = \frac{P}{0.1252} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.1252 \pi^2 EI}{P} = \frac{0.1252 \pi^2 (200 \times 10^9) (43.1 \times 10^{-6})}{600000} = 17.75 \text{ m}^2$

$L_e = 4.21 \text{ m}$ $L = L_e = 4.2 \text{ m}$

Problem 10.50

10.50 Axial loads of magnitude $P = 580 \text{ kN}$ are applied parallel to the geometric axis of the $W250 \times 80$ rolled-steel column AB and intersect the x axis at a distance e from the geometric axis. Knowing that $\sigma_{\text{all}} = 75 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the largest permissible length L when (a) $e = 5 \text{ mm}$ (b) $e = 10 \text{ mm}$.



Data: $P = 580 \times 10^3 \text{ N}$ $E = 200 \times 10^9 \text{ Pa}$

$W 250 \times 80$ $A = 10200 \text{ mm}^2 = 10200 \times 10^{-6} \text{ m}^2$

$b_f = 255 \text{ mm}$ $c = \frac{b_f}{2} = 127.5 \text{ mm}$ $r_y = 65.0 \text{ mm}$

$I_y = 43.1 \times 10^6 \text{ mm}^4 = 43.1 \times 10^{-6} \text{ m}^4$

$\sigma_{\text{all}} = \sigma_{\text{max}} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$

$$\sigma_{\text{max}} = \frac{P}{A} \left[1 + \frac{ec}{r_y^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) \right]$$

$$\frac{A\sigma_{\text{max}}}{P} - 1 = \frac{ec}{r_y^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right)$$

$$\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{r_y^2}{ec} \left(\frac{A\sigma_{\text{max}}}{P} - 1 \right)$$

(a) $e = 5 \text{ mm}$, $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{(65.0)^2}{(5)(127.5)} \left[\frac{(10200 \times 10^{-6})(75 \times 10^6)}{580 \times 10^3} - 1 \right] = 2.1139$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = 0.47305$ $\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}} = 1.07804$

$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi} (1.07804) \right]^2 = 0.47101$

$P_{\text{cr}} = \frac{P}{0.47101} = \frac{\pi^2 EI_y}{L_e^2}$

$L_e^2 = \frac{0.47101 \pi^2 EI_y}{P} = \frac{0.47101 \pi^2 (200 \times 10^9)(43.1 \times 10^{-6})}{580 \times 10^3} = 69.09 \text{ m}^2$

$L_e = 8.31 \text{ m}$

$L = L_e$

$L = 8.31 \text{ m}$ $\blacktriangleleft$

(b) $e = 10 \text{ mm}$, $\sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = \frac{(65)^2}{(10)(127.5)} \left[\frac{(10200 \times 10^{-6})(75 \times 10^6)}{580 \times 10^3} - 1 \right] = 1.05696$

$\cos\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}}\right) = 0.94611$ $\frac{\pi}{2} \sqrt{\frac{P}{P_{\text{cr}}}} = 0.32980$

$\frac{P}{P_{\text{cr}}} = \left[\frac{2}{\pi} (0.32980) \right]^2 = 0.044083$

$P_{\text{cr}} = \frac{P}{0.044083} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.044083 \pi^2 EI}{P} = \frac{0.044083 \pi^2 (200 \times 10^9)(43.1 \times 10^{-6})}{580 \times 10^3} = 6.466 \text{ m}^2$

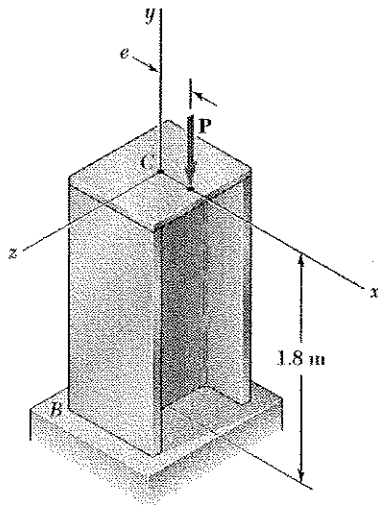
$L_e = 2.54 \text{ m}$

$L = L_e$

$L = 2.54 \text{ m}$ $\blacktriangleleft$

Problem 10.51

10.51 An axial load of magnitude $P = 220 \text{ kN}$ is applied at a point located on the x axis at a distance $e = 6 \text{ mm}$ from the geometric axis of the wide-flange column BC . Knowing that $E = 200 \text{ GPa}$, choose the lightest W200 shape that can be used if $\sigma_{\text{all}} = 120 \text{ MPa}$.



$$P = 220 \times 10^3 \text{ N} \quad L = 1.8 \text{ m} \quad L_e = 2L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{3.6^2} = 152.3 \times 10^9 I_y \text{ N}$$

$$e = 6 \text{ mm} \quad c = \frac{b_f}{2} \quad \frac{ec}{r^2} = \frac{eb_f}{2r_y^2} =$$

$$\sigma_{\max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

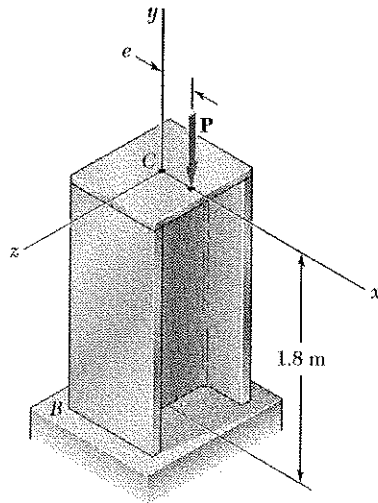
Shape	$A (10^6 \text{ m}^2)$	$b_f (\text{mm})$	$I_y (10^6 \text{ m}^4)$	$r_y (\text{mm})$	$P_{cr} (\text{kN})$	$\frac{ec}{r^2}$	$\sigma_{\max} (\text{MPa})$
W200 $\times$ 41.7	5310	166	9.01	41.2	1372	0.2934	56.5
W200 $\times$ 26.6	3390	133	3.30	31.2	502.6	0.4099	117.4
W200 $\times$ 22.5	2860	102	1.42	22.3	*216.3		

* < P

$$\sigma_{\max} = 117.4 \text{ MPa} < 120 \text{ MPa}$$

Use W200 $\times$ 26.6. $\blacktriangleleft$

Problem 10.52



10.52 Solve Prob. 10.51, assuming that the magnitude of the axial load is $P = 345$ kN.

10.51 An axial load of magnitude $P = 220$ kN is applied at a point located on the x axis at a distance $e = 6$ mm from the geometric axis of the wide-flange column BC . Knowing that $E = 200$ GPa, choose the lightest W200 shape that can be used if $\sigma_{all} = 120$ MPa.

$$P = 345 \times 10^3 \text{ N} \quad L = 1.8 \text{ m} \quad L_e = 2L = 3.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 E I_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{(3.6)^2} = 152.3 \times 10^9 I_y \text{ N}$$

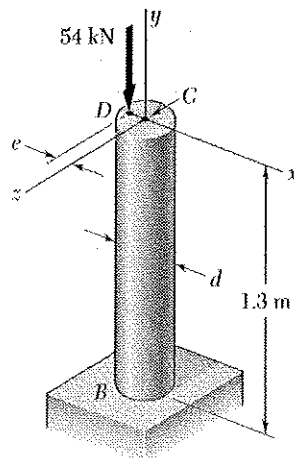
$$e = 6 \text{ mm} \quad c = \frac{b_f}{2} \quad \frac{ec}{r^2} = \frac{e b_f}{2 r_y^2}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

Shape	$A (10^6 \text{ m}^2)$	$b_f (\text{mm})$	$I_y (10^6 \text{ m}^4)$	$r_y (\text{mm})$	$P_{cr} (\text{kN})$	$\frac{ec}{r^2}$	$\sigma_{max} (\text{MPa})$
W 200 $\times$ 41.7	5310	166	9.01	41.2	1372	0.2934	92.0
W 200 $\times$ 26.6	3390	133	3.30	31.2	502.6	0.4099	258
W 200 $\times$ 35.9	4580	165	7.64	40.8	1164	0.2974	109.5
W 200 $\times$ 31.3	4000	134	4.10	32.0	624.4	0.3926	172.6

$$\sigma_{max} = 109.5 \text{ MPa} < 120 \text{ MPa} \quad \text{Use W200} \times 35.9$$

Problem 10.53



10.53 A 54 kN axial load is applied with an eccentricity $e = 10$ mm to the circular steel rod BC that is free at its top C and fixed at its base B . Knowing that the stock of rods available for use have diameters in increments of 4 mm from 44 mm to 72 mm, determine the lightest rod that may be used if $\sigma_{all} = 110$ MPa. Use $E = 200$ GPa.

For a solid circular section $c = \frac{1}{2}d$

$$A = \pi c^2 = \frac{\pi}{4} d^2 \quad I = \frac{\pi}{4} c^4 = \frac{\pi}{64} d^4$$

$$r^2 = \frac{I}{A} = \frac{1}{16} d^2 \quad \frac{ec}{r^2} = \frac{8e}{d}$$

$$P = 54 \times 10^3 \text{ N} \quad e = 10 \text{ mm} = 0.010 \text{ m}$$

$$L_e = 2L = (2)(1.3) = 2.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\frac{P}{P_{cr}} = \frac{PL_e^2}{\pi^2 EI} = \frac{(54 \times 10^3)(2.6)^2(64)}{\pi^2(200 \times 10^9)\pi d^4} = 3.7674 \times 10^{-6} d^{-4}$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 3.0489 \times 10^{-3} d^{-2}$$

$$\frac{P}{A} = \frac{(54 \times 10^3)(4)}{\pi d^2} = 68.755 \times 10^3 d^{-2}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right] \quad \frac{ec}{r^2} = \frac{0.08}{d}$$

$$\sigma_{max} = 68.755 \times 10^3 d^{-2} \left[1 + 0.08 d^{-1} \sec(3.0489 \times 10^{-3} d^{-2}) \right]$$

d (mm)	σ_{max} (MPa)
44	-1.5919 ($P/P_{cr} > 1$)
48	232
52	116.6
56	77.5 ←
60	57.5
64	45.3
68	37.0
72	31.0

The lightest section with $\sigma_{max} < 110$ MPa is $d = 56$ mm.

Checking: $d = 56 \text{ mm} = 0.056 \text{ m} \quad c = 0.028 \text{ m}$

$$A = \frac{\pi}{4} d^2 = 2.4630 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{64} d^4 = 482.75 \times 10^{-9} \text{ m}^4$$

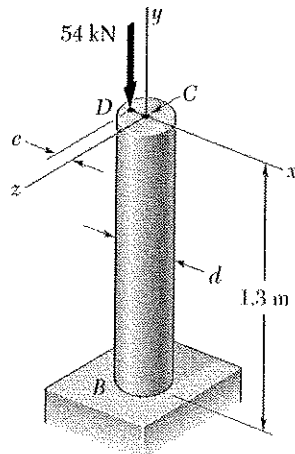
$$r^2 = \frac{I}{A} = 196 \times 10^{-6} \quad \frac{ec}{r^2} = \frac{(0.010)(0.028)}{196 \times 10^{-6}} = 1.4286$$

$$P_{cr} = \frac{\pi^2(200 \times 10^9)(482.75 \times 10^{-9})}{(2.6)^2} = 140.96 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{54 \times 10^3}{140.96 \times 10^3} = 0.38308$$

$$\sigma_{max} = \frac{54 \times 10^3}{2.4630 \times 10^{-3}} \left[1 + 1.4286 \sec\left(\frac{\pi}{2} \sqrt{0.38308}\right) \right] = 77.5 \times 10^6 \text{ Pa}$$

Problem 10.54



10.54 Solve Prob. 10.53, assuming that the 54 kN axial load will be applied to the rod with an eccentricity $e = \frac{1}{2} d$.

10.53 A 54 kN axial load is applied with an eccentricity $e = 10$ mm to the circular steel rod BC that is free at its top C and fixed at its base B. Knowing that the stock of rods available for use have diameters in increments of 4 mm from 44 mm to 72 mm, determine the lightest rod that may be used if $\sigma_{all} = 110$ MPa. Use $E = 200$ GPa.

For a solid circular section $c = \frac{1}{2} d$
 $A = \pi c^2 = \frac{\pi}{4} d^2$ $I = \frac{\pi}{4} c^4 = \frac{\pi}{64} d^4$

$$r^2 = \frac{I}{A} = \frac{1}{16} d^2$$

Use $e = \frac{1}{2} d$ $\frac{ec}{r^2} = (\frac{1}{2})(\frac{1}{2})(16) = 4$

$$P = 54 \times 10^3 \text{ N}$$

$$L_e = 2L = (2)(1.3) = 2.6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$\frac{P}{P_{cr}} = \frac{PL_e^2}{\pi^2 EI} = \frac{(54 \times 10^3)(2.6)^2(64)}{\pi^2(200 \times 10^9) \pi d^4} = 3.7674 \times 10^{-6} d^{-4}$$

$$\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} = 3.0489 \times 10^{-3} d^{-2}$$

$$\frac{P}{A} = \frac{(54 \times 10^3)(4)}{\pi d^2} = 68.755 \times 10^3 d^{-2}$$

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}}\right) \right]$$

$$\sigma_{max} = 68.755 \times 10^3 d^{-2} \left[1 + 4 \sec(3.0489 \times 10^{-3} d^{-2}) \right]$$

$d(\text{mm})$	$\sigma_{max}(\text{MPa})$
44	-35.1
48	517
52	263
56	177.6
60	134.4
64	108.1 ←
68	90.1
72	77.0

The lightest section
 with $\sigma_{max} < 110$ MPa
 is $d = 64$ mm

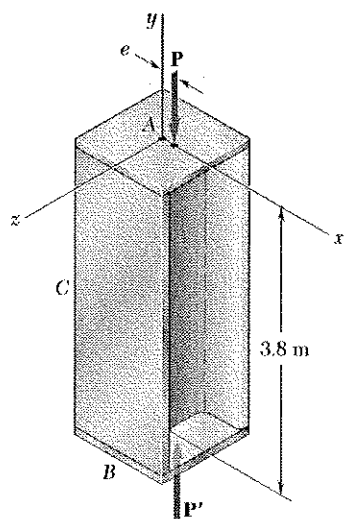
Checking $A = \frac{\pi}{4} (0.064)^2 = 3.217 \times 10^{-3} \text{ m}^2$, $I = \frac{\pi}{64} (0.064)^4 = 823.55 \times 10^{-9} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 (200 \times 10^9) (823.55 \times 10^{-9})}{(2.6)^2} = 240.48 \times 10^3 \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{54 \times 10^3}{240.48 \times 10^3} = 0.22455$$

$$\sigma_{max} = \frac{54 \times 10^3}{3.217 \times 10^{-3}} \left[1 + 4 \sec\left(\frac{\pi}{2} \sqrt{0.22455}\right) \right] = 108.1 \times 10^6 \text{ Pa}$$

Problem 10.55



10.55 Axial loads of magnitude $P = 175$ kN are applied parallel to the geometric axis of a $W250 \times 44.8$ rolled-steel column AB and intersect the axis at a distance $e = 12$ mm from its geometric axis. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (Hint: Since the factor of safety must be applied to the load P , not to the stresses, use Fig. 10.24 to determine P_Y .)

For $W250 \times 44.8$ $A = 5720 \text{ mm}^2$, $r_y = 35.1 \text{ mm}$

$L_e = 3800 \text{ mm}$ $L_e/r = 108.26$

$C = \frac{b_F}{2} = \frac{148}{2} = 74 \text{ mm}$ $e = 12 \text{ mm}$

$\frac{eC}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$

Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{eC}{r^2} = 0.72077$,

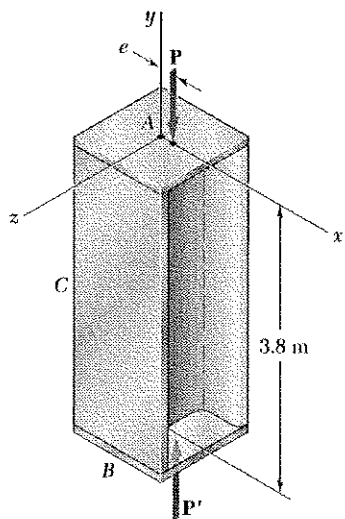
$P_Y/A = 90.37 \text{ MPa} = 90.37 \times 10^6 \text{ N/m}^2$

$P_Y = A(P_Y/A) = (5720 \times 10^{-6})(90.37 \times 10^6) = 517 \times 10^3 \text{ N} = 517 \text{ kN}$

F.S. = $\frac{P_Y}{P} = \frac{517}{175}$

F.S. = 2.95

Problem 10.56



10.56 Solve Prob. 10.55, assuming that $e = 0.16$ mm and $P = 155$ kN.

10.55 Axial loads of magnitude $P = 175$ kN are applied parallel to the geometric axis of a $W250 \times 44.8$ rolled-steel column AB and intersect the axis at a distance $e = 12$ mm from its geometric axis. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (Hint: Since the factor of safety must be applied to the load P , not to the stresses, use Fig. 10.24 to determine P_Y .)

For $W250 \times 44.8$ $A = 5720 \text{ mm}^2$, $r_y = 35.1 \text{ mm}$

$L_e = 3800 \text{ mm}$ $L_e/r = 108.26$

$C = \frac{b_F}{2} = \frac{148}{2} = 74 \text{ mm}$ $e = 16 \text{ mm}$

$\frac{eC}{r^2} = \frac{(16)(74)}{(35.1)^2} = 0.96103$

Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{eC}{r^2} = 0.96103$,

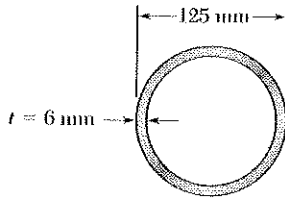
$P_Y/A = 81.17 \text{ MPa} = 81.17 \times 10^6 \text{ N/m}^2$

$P_Y = A(P_Y/A) = (5720 \times 10^{-6})(81.17 \times 10^6) = 464 \times 10^3 \text{ N} = 464 \text{ kN}$

F.S. = $\frac{P_Y}{P} = \frac{464}{155}$

F.S. = 3.00

Problem 10.57



10.57 A steel pipe having the cross section shown is used as a column. Using the AISC allowable stress design formulas, determine the allowable centric load if the effective length of the column is (a) 6 m, (b) 4 m. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$c = \frac{1}{2}d = 62.5 \text{ mm} \quad c_1 = c - t = 56.5 \text{ mm}$$

$$A = \pi(c^2 - c_1^2) = 2.2431 \times 10^3 \text{ mm}^2 = 2.2431 \times 10^{-3} \text{ m}^2$$

$$I = \frac{\pi}{4}(c^4 - c_1^4) = 3.9807 \times 10^6 \text{ mm}^4 = 3.9807 \times 10^{-6} \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = 42.149 \text{ mm} = 42.149 \times 10^{-3} \text{ m}$$

Steel: $\sigma_Y = 250$ MPa $E = 200\,000$ MPa

transition $\frac{L_e}{r}$: $4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200\,000}{250}} = 133.22$

(a) $L_e = 6 \text{ m}$. $\frac{L_e}{r} = \frac{6}{42.149 \times 10^{-3}} = 142.352 > 133.22$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(142.352)^2} = 97.41 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_{cr}}{F.S.} = \frac{0.877 \sigma_e}{1.67} = 51.155 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (51.155 \times 10^6)(2.2431 \times 10^{-3}) \quad P_{all} = 114.7 \text{ kN} \quad \blacktriangleleft$$

(b) $L = 4 \text{ m}$. $\frac{L_e}{r} = \frac{4}{42.149 \times 10^{-3}} = 94.901 < 133.22$

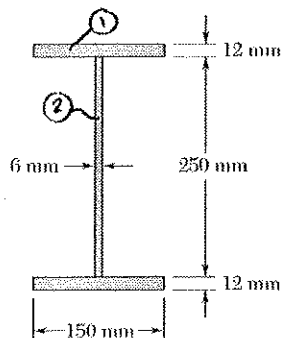
$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(94.901)^2} = 219.17 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{\sigma_Y/\sigma_e}] \sigma_Y = [0.658^{250/219.17}] (250) = 155.09 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_{cr}}{F.S.} = \frac{155.09}{1.67} = 92.871 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (92.871 \times 10^6)(2.2431 \times 10^{-3}) \quad P_{all} = 208 \text{ kN} \quad \blacktriangleleft$$

Problem 10.58



10.58 A column with the cross section shown has a 4-m effective length. Using allowable stress design, determine the largest centric load that can be applied to the column. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$A = 2A_1 + A_2 = (2)(12)(150) + (250)(6) = 5100 \text{ mm}^2$$

$$I_y = 2I_1 + I_2 = (2)\left(\frac{1}{12}\right)(12)(150)^3 + \left(\frac{1}{12}\right)(250)(6)^3 = 6.7545 \times 10^6 \text{ mm}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{6.7545 \times 10^6}{5100}} = 36.4 \text{ mm}$$

$$L_e = 4 \text{ m} \quad \frac{L_e}{r} = 109.9 < C_c$$

$$\text{Steel: } E = 200 \text{ GPa}, \sigma_Y = 250 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$\frac{L_e/r}{C_c} = 0.8743 \quad \text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.8743) - \frac{1}{8}(0.8743)^3 = 1.911$$

$$\sigma_{all} = \frac{\sigma_Y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.911} \left[1 - \frac{1}{2} (0.8743)^2 \right] = 80.82 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (80.82)(5100) = 412.2 \text{ kN}$$

Problem 10.59

10.59 Using allowable stress design, determine the allowable centric load for a column of 6-m effective length that is made from the following rolled-steel shape: (a) W200 × 35.9, (b) W200 × 86. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

Steel: $\sigma_Y = 250$ MPa $E = 200 \times 10^3$ MPa
 transition L/r : $4.71 \sqrt{\frac{E}{\sigma_Y}} = 133.22$

(a) W 200 × 35.9 $A = 4580 \times 10^{-6} \text{ m}^2$ $r_y = 40.8 \times 10^{-3} \text{ m}$

$$\frac{L_e}{r_y} = \frac{6}{40.8 \times 10^{-3}} = 147.06 > 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^3)}{(147.06)^2} = 91.27 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_e}{F.S.} = \frac{(0.877)(91.27)}{1.67} = 47.932 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (47.932 \times 10^6)(4580 \times 10^{-6}) \quad P_{all} = 220 \text{ kN} \quad \blacktriangleleft$$

(b) W 200 × 86 $A = 11000 \times 10^{-6} \text{ m}^2$ $r_y = 53.2 \times 10^{-3} \text{ m}$

$$\frac{L_e}{r_y} = \frac{6}{53.2 \times 10^{-3}} = 112.782 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^3)}{(112.782)^2} = 155.185 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_e}{F.S.} = \frac{1}{1.67} [0.658^{250/155.185}] (250) = 76.276 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (76.276 \times 10^6)(11000 \times 10^{-6}) \quad P_{all} = 839 \text{ kN} \quad \blacktriangleleft$$

Problem 10.60

10.60 A W200 × 46.1 rolled-steel shape is used to form a column of 6-m effective length. Using allowable stress design, determine the allowable centric load if the yield strength of the grade of steel used is (a) $\sigma_Y = 250$ MPa, (b) $\sigma_Y = 345$ MPa. Use $E = 200$ GPa.

Steel: $E = 200 \text{ GPa}$

W200X46.1 $A = 5860 \text{ mm}^2$

$r_{\min} = 51.1 \text{ mm}$

$L_e = 6 \text{ m}$

$L_e/r = 117.4$

$$(a) \sigma_Y = 250 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200 \times 10^3)}{250}} = 125.7$$

$$L_e/r < C_c \quad \frac{L/r}{C_c} = 0.934$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.934) - \frac{1}{8}(0.934)^3 = 1.915$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L/r}{C_c} \right)^2 \right] = \frac{250}{1.915} \left[1 - \frac{1}{2} (0.934)^2 \right] = 73.6 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (73.6)(5860) = 431.3 \text{ kN}$$

$$(b) \sigma_Y = 345 \text{ MPa} \quad C_c = \sqrt{\frac{2\pi^2 (200 \times 10^3)}{345}} = 106.97$$

$$L_e/r > C_c \quad \sigma_{all} = \frac{\pi^2 E}{\frac{23}{12} (L/r)^2} = 74.72 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (74.72)(5860) = 437.9 \text{ kN}$$

Problem 10.61

10.61 A column having a 3.5-m effective length is made of sawn lumber with a 114×140 -mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 7.6$ MPa and the adjusted modulus $E = 2.8$ GPa, determine the maximum allowable centric load for the column.

Sawn lumber: $c = 0.8$ $\sigma_c = 7.6$ MPa $E = 2800$ MPa

$$A = (114)(140) = 15960 \text{ mm}^2 = 15960 \times 10^{-6} \text{ m}^2$$

$$d = 114 \text{ mm} = 114 \times 10^{-3} \text{ m}$$

$$L/d = 3.5 / 114 \times 10^{-3} = 30.70$$

$$\sigma_{ce} = \frac{0.822 E}{(L/d)^2} = \frac{(0.822)(2800)}{(30.70)^2} = 2.442 \quad \frac{\sigma_{ce}}{\sigma_c} = 0.32132$$

$$U = \frac{1 + \sigma_{ce}/\sigma_c}{2c} = \frac{1 + 0.32132}{(2)(0.8)} = 0.82583 \quad v = \frac{\sigma_{ce}/\sigma_c}{c} = 0.40165$$

$$C_p = U - \sqrt{U^2 - v} = 0.29635$$

$$\sigma_{all} = C_p \sigma_c = (0.29635)(7.6) = 2.252 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (2.252 \times 10^6)(15960 \times 10^{-6}) \quad P_{all} = 35.9 \text{ kN}$$

Problem 10.62

10.62 A sawn lumber column with a 190×140 -mm cross section has a 5.5-m effective length. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 8.5$ MPa and that $E = 9$ GPa, determine the maximum allowable centric load for the column.

Sawn lumber: $c = 0.8$, $\sigma_c = 8.5$ MPa $E = 9600$ MPa $K_{ce} = 0.3$

$$A = (190)(140) = 26600 \text{ mm}^2 \quad d = 140 \text{ mm} \quad L = 5.5 \text{ m}$$

$$L/d = 5500 / 140 = 39.286$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{(0.3)(9 \times 10^9)}{(39.286)^2} = 1.75 \text{ MPa} \quad \frac{\sigma_{ce}}{\sigma_c} = 0.206$$

$$U = \frac{1 + \sigma_{ce}/\sigma_c}{2c} = \frac{1.206}{(2)(0.8)} = 0.75375 \quad v = \frac{\sigma_{ce}/\sigma_c}{c} = 0.2575$$

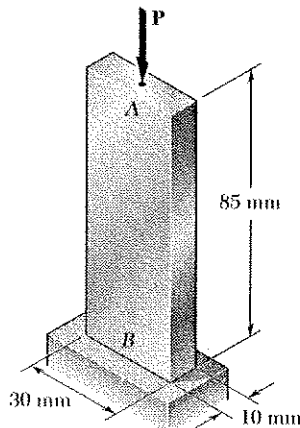
$$C_p = U - \sqrt{U^2 - v} = 0.1964$$

$$\sigma_{all} = \sigma_c C_p = (8.5)(0.1964) = 1.67 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (1.67)(26600) = 44.4 \text{ kN}$$

Problem 10.63

10.63 Bar AB is free at its end A and fixed at its base B. Determine the allowable centric load P if the aluminum alloy is (a) 6061-T6, (b) 2014-T6.



$$A = (30)(10) = 300 \text{ mm}^2 = 300 \times 10^{-6} \text{ m}^2$$

$$I_{\min} = \frac{1}{12}(30)(10)^3 = 2.50 \times 10^3 \text{ mm}^4$$

$$r_{\min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.50 \times 10^3}{300}} = 2.887 \text{ mm}$$

$$L_e = 2L = (2)(85) = 170 \text{ mm} \quad \frac{L_e}{r_{\min}} = 58.88$$

(a) 6061-T6 $L/r < 66$

$$\sigma_{\text{all}} = 139 - 0.868(L/r) = 139 - (0.868)(58.88) = 87.9 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (87.9 \times 10^6)(300 \times 10^{-6}) = 26.4 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 26.4 \text{ kN} \quad \blacktriangleleft$$

(b) 2014-T6 $L/r > 55$

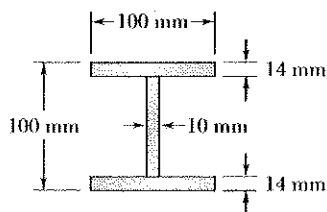
$$\sigma_{\text{all}} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(58.88)^2} = 107.3 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (107.3 \times 10^6)(300 \times 10^{-6}) = 32.2 \times 10^3 \text{ N}$$

$$P_{\text{all}} = 32.2 \text{ kN} \quad \blacktriangleleft$$

Problem 10.64

10.64 A compression member has the cross section shown and an effective length of 1.5 m. Knowing that the aluminum alloy used is 6061-T6, determine the allowable centric load.



$$I_x = \frac{1}{12}(10)(72)^3 + 2\left[\frac{1}{12}(100)(14)^3 + (100)(14)(43)^2\right] = 5533973 \text{ mm}^4$$

$$I_y = \frac{1}{12}(72)(10)^3 + 2\left[\frac{1}{12}(14)(100)^3\right] = 2339333 \text{ mm}^4 = I_{\min}$$

$$A = (10)(72) + (2)(100)(14) = 3520 \text{ mm}^2$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{2339333}{3520}} = 25.78 \text{ mm}$$

$$L = 1.5 \text{ m} \quad \frac{L}{r} = \frac{1500}{25.78} = 58.185$$

6061 T6 aluminum alloy with $L/r < 66$.

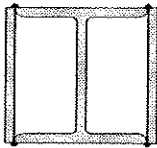
$$\sigma_{\text{all}} = 139 - (0.868)(58.185) = 88.49 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (88.49 \times 10^6)(3520 \times 10^{-6})$$

$$P_{\text{all}} = 311.2 \text{ kN} \quad \blacktriangleleft$$

Problem 10.65

10.65 and 10.66 A compression member of 9-m effective length is obtained by welding two 10-mm-thick steel plates to a W250 × 80 rolled-steel shape as shown. Knowing that $\sigma_Y = 345$ MPa, and $E = 200$ GPa and using allowable stress design, determine the allowable centric load for the compression member.



$$\text{For } W250 \times 80, \quad A = 10200 \text{ mm}^2, \quad d = 256 \text{ mm}, \quad b_F = 255 \text{ mm} \\ I_x = 126 \times 10^6 \text{ mm}^4, \quad I_y = 43.1 \times 10^6 \text{ mm}^4$$

$$\text{For one plate: } A = (256)(10) = 2560 \text{ mm}^2$$

$$I_x = \frac{1}{12}(10)(256)^3 = 13.981 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12}(256)(10)^3 + (2560)\left(\frac{255}{2} + \frac{10}{2}\right)^2 = 44.965 \times 10^6 \text{ mm}^4$$

$$\text{For column: } A = 10200 + (2)(2560) = 15.32 \times 10^3 \text{ mm}^2 = 15.32 \times 10^{-3} \text{ m}^2$$

$$I_x = 126 \times 10^6 + (2)(13.981 \times 10^6) = 153.962 \times 10^6 \text{ mm}^4$$

$$I_y = 43.1 \times 10^6 + (2)(44.965 \times 10^6) = 133.03 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{133.03 \times 10^6}{15.32 \times 10^3}} = 93.185 \text{ mm} = 93.185 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{9}{93.185 \times 10^{-3}} = 96.582$$

$$\text{Steel: transition } L/r = 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40 > 96.582$$

$$\sigma_c = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(96.582)^2} = 211.61 \text{ MPa}$$

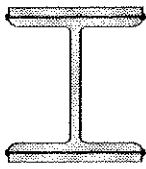
$$\sigma_{\text{all}} = \frac{\sigma_c}{\text{F.S.}} = \frac{1}{1.67} \left[0.658^{345/211.61} \right] (345) = 104.41 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (104.41 \times 10^6)(15.32 \times 10^{-3})$$

$$P_{\text{all}} = 1600 \text{ kN} \quad \blacktriangleleft$$

Problem 10.66

10.65 and 10.66 A compression member of 9-m effective length is obtained by welding two 10-mm-thick steel plates to a W250 × 80 rolled-steel shape as shown. Knowing that $\sigma_Y = 345$ MPa, and $E = 200$ GPa and using allowable stress design, determine the allowable centric load for the compression member.



For W 250 × 80 $A = 10200 \text{ mm}^2$, $d = 256 \text{ mm}$, $b_f = 255 \text{ mm}$
 $I_x = 126 \times 10^6 \text{ mm}^4$, $I_y = 43.1 \times 10^6 \text{ mm}^4$

For one plate: $A = (255)(10) = 2550 \text{ mm}^2$
 $I_x = \frac{1}{12}(255)(10)^3 + (2550)\left(\frac{256}{2} + \frac{10}{2}\right)^2 = 45.128 \times 10^6 \text{ mm}^4$
 $I_y = \frac{1}{12}(10)(255)^3 = 13.818 \times 10^6 \text{ mm}^4$

For column: $A = 10200 + (2)(2550) = 15300 \text{ mm}^2 = 15.3 \times 10^{-3} \text{ m}^2$

$I_x = 126 \times 10^6 + (2)(45.128 \times 10^6) = 216.256 \times 10^6 \text{ mm}^4$

$I_y = 43.1 \times 10^6 + (2)(13.818 \times 10^6) = 70.736 \times 10^6 \text{ mm}^4 = I_{\min}$

$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{70.736 \times 10^6}{15300}} = 67.995 \text{ mm} = 67.995 \times 10^{-3} \text{ m}$

$\frac{L_e}{r} = \frac{9}{67.995 \times 10^{-3}} = 132.363$

Steel: transition $\frac{L}{r}$: $4.71\sqrt{\frac{E}{\sigma_Y}} = 4.71\sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40 < 132.363$

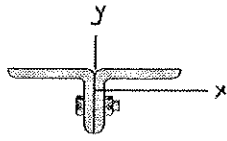
$\sigma_e = \frac{\pi^2 E}{(L/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(132.363)^2} = 112.67 \text{ MPa}$

$\sigma_{\text{all}} = \frac{\sigma_e}{\text{F.S.}} = \frac{0.877 \sigma_e}{1.67} = 59.167 \text{ MPa}$

$P_{\text{all}} = \sigma_{\text{all}} A = (59.167 \times 10^6)(15.3 \times 10^{-3})$

$P_{\text{all}} = 905 \text{ kN}$ ◀

Problem 10.67



10.67 A compression member of 2.3-m effective length is obtained by bolting together two 127 × 76 × 12.7-mm steel angles as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 250$ MPa, and $E = 200$ GPa.

$$4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

L 127 × 76 × 12.7 mm Table gives $A_L = 2420 \text{ mm}^2$, $I_x = 3.93 \times 10^6 \text{ mm}^4$

$$y = 44.4 \text{ mm}, \quad I_y = 1.06 \times 10^6 \text{ mm}^4, \quad x = 19.0 \text{ mm}, \quad r_y =$$

For column, $I_x = 2(I_y)_{\text{angle}} = (2)(1.06 \times 10^6) = 2.12 \times 10^6 \text{ mm}^4$
 $I_y > I_x \quad I_{\min} = I_x = 2.12 \times 10^6 \text{ mm}^4 = 2.12 \times 10^{-6} \text{ m}^4$
 $A = 2A_L = 4840 \text{ mm}^2 = 4840 \times 10^{-6} \text{ m}^2$
 $r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{2.12 \times 10^{-6}}{4840 \times 10^{-6}}} = 20.93 \times 10^{-3} \text{ m}$

$$\frac{L_e}{r} = \frac{2.3}{20.93 \times 10^{-3}} = 109.90 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(109.90)^2} = 163.43 \text{ MPa}$$

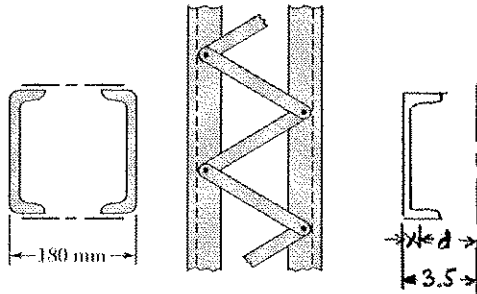
$$\sigma_{\text{all}} = \frac{\sigma_c}{F.S.} = \frac{1}{1.67} \left[0.658^{250/163.43} \right] (250) = 78.916 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (78.916 \times 10^6) (4840 \times 10^{-6})$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (78.916 \times 10^6) (4840 \times 10^{-6})$$

$$P_{\text{all}} = 382 \text{ kN}$$

Problem 10.68



10.68 A column of 6-m effective length is obtained by connecting two C250 × 30 steel channels with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_y = 250$ MPa and $E = 200$ GPa.

C250 × 30 $A = 3780 \text{ mm}^2$ $x = 15.3 \text{ mm}$
 $I_x = 32.6 \times 10^6 \text{ mm}^4$ $I_y = 1.14 \times 10^6 \text{ mm}^4$
 $d = 90 - x = 74.7 \text{ mm}$

For the column: $A = 2(3780) = 7560 \text{ mm}^2$

$I_x = (2)(32.6 \times 10^6) = 65.2 \times 10^6 \text{ mm}^4$

$I_y = 2[1.14 \times 10^6 + (3780)(74.7)^2] = 44.47 \times 10^6 \text{ mm}^4$

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{44.47 \times 10^6}{7560}} = 76.7 \text{ mm}$ $L_e = 6 \text{ m}$

$\frac{L_e}{r} = 78.23$ $C_c = \sqrt{\frac{2\pi^2 E'}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^3)}{250}} = 125.7$

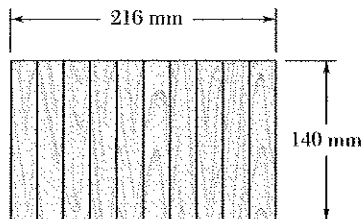
$\frac{L_e}{r} < C_c$ $\frac{L_e/r}{C_c} = 0.6224$

F.S. = $\frac{5}{3} + \frac{3}{8}(0.6224) - \frac{1}{8}(0.6224)^3 = 1.87$

$\sigma_{all} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.87} \left[1 - \frac{1}{2} (0.6224)^2 \right] = 107.8 \text{ MPa}$

$P_{all} = \sigma_{all} A = (107.8)(7560) = 815 \text{ kN}$

Problem 10.69



10.69 A rectangular column with a 4.4-m effective length is made of glued laminated wood. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 8.3$ MPa and the adjusted modulus $E = 4.6$ GPa, determine the maximum allowable centric load for the column.

Glued laminated column: $c = 0.9$ $E = 4600 \text{ MPa}$

$A = (216)(140) = 30240 \text{ mm}^2 = 30240 \times 10^{-6} \text{ m}^2$

$d = 140 \text{ mm} = 140 \times 10^{-3} \text{ m}$ $L = 4.4 \text{ m}$

$\frac{L}{d} = \frac{4.4}{140 \times 10^{-3}} = 31.429$

$\sigma_{ce} = \frac{0.822 E}{(L/d)^2} = \frac{(0.822)(4600)}{(31.429)^2} = 3.8281 \text{ MPa}$

$\frac{\sigma_{ce}}{\sigma_c} = \frac{3.8281}{8.3} = 0.46121$

$U = \frac{1 + \sigma_{ce}/\sigma_c}{2c} = \frac{1.46121}{2(0.9)} = 0.81178$

$v = \frac{\sigma_{ce}/\sigma_c}{c} = 0.51246$

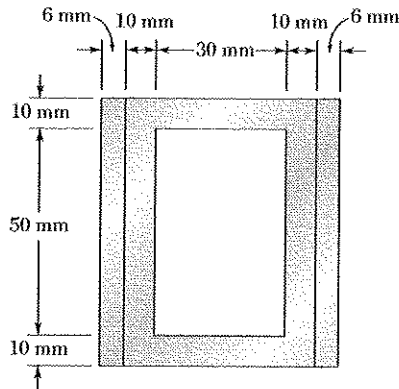
$C_p = U - \sqrt{U^2 - v} = 0.42908$

$\sigma_{all} = C_p \sigma_c = (0.42908)(8.3) = 3.5614 \text{ MPa}$

$P_{all} = \sigma_{all} A = (3.5614 \times 10^6)(30240 \times 10^{-6})$

$P_{all} = 107.7 \text{ kN}$

Problem 10.70



10.70 An aluminum structural tube is reinforced by riveting two plates to it as shown for use as a column of 1.7-m effective length. Knowing that all material is aluminum alloy 2014-T6, determine the maximum allowable centric load.

$$b_o = 2(6) + 2(10) + 30 = 62 \text{ mm}$$

$$b_i = 30 \text{ mm}$$

$$h_o = 2(10) + 50 = 70 \text{ mm}$$

$$h_i = 50 \text{ mm}$$

$$A = b_o h_o - b_i h_i = (62)(70) - (30)(50) = 2840 \text{ mm}^2$$

$$I_x = \frac{1}{12} [b_o h_o^3 - b_i h_i^3] = \frac{1}{12} [(62)(70)^3 - (30)(50)^3] = 1.46 \times 10^6 \text{ mm}^4$$

$$I_y = \frac{1}{12} [h_o b_o^3 - h_i b_i^3] = \frac{1}{12} [(70)(62)^3 - (50)(30)^3] = 1.278 \times 10^6 \text{ mm}^4 = I_{\min}$$

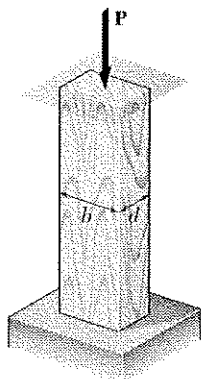
$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{1.278 \times 10^6}{2840}} = 21.2 \text{ mm} \quad L_e = 1700 \text{ mm}$$

$$\frac{L_e}{r} = \frac{1700}{21.2} = 80.2 \geq 55$$

$$\sigma_{\text{all}} = \frac{372000}{(L_e/r)^2} = 57.84 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (57.84)(2840) = 164.3 \text{ kN}$$

Problem 10.71



10.71 An 75-kN centric load is applied to a rectangular sawn lumber column of 6.5-m effective length. Using sawn lumber for which the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 7.3$ MPa and knowing that $E = 10$ GPa, determine the smallest square cross section that may be used. Use $b = 2d$.

Sawn lumber: $c = 0.8$ $K_{CE} = 0.3$

$\sigma_c = 7.3$ MPa $E = 10$ GPa

$A = 2d^2$ $L = 6.5$ m

$L/d = \frac{6000}{d}$

Assumed $C_p = 0.5$

$\sigma_{all} = \sigma_c C_p = (7.3)(0.5) = 3.65$ MPa.

$P_{all} = \sigma_{all} A = 2\sigma_{all} d^2$

$d = \sqrt{\frac{P_{all}}{2\sigma_{all}}} = \sqrt{\frac{75000}{2(3.65)}} = \frac{193.65}{\sqrt{7.3}} = 101.4$ mm

$L/d = 59.17$

$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.3)(10 \times 10^3)}{(L/d)^2} = \frac{3 \times 10^3}{(L/d)^2} = 0.86$ MPa.

$\sigma_{CE} / \sigma_c = 0.1178$

Checked: $C_p = \frac{1 + \sigma_{CE} / \sigma_c}{2C} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2C}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{C}} = 0.1148$

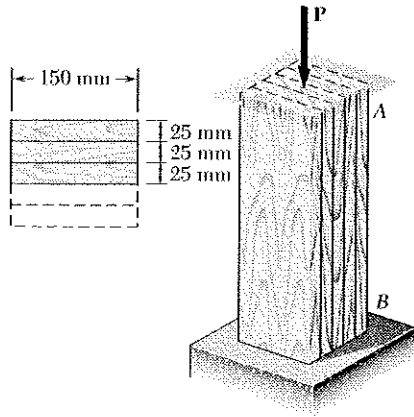
Results of similar trials are summarized in the table below:

Assumed C_p	σ_{all} (MPa)	d (mm)	L/d	σ_{CE} (MPa)	σ_{CE} / σ_c	checked C_p	ΔC_p
0.5	3.65	101.4	59.17	0.86	0.1178	0.1148	-0.3852
0.2983	2.18	131.3	45.70	1.44	0.1975	0.2017	-0.0966
0.2340	1.708	148.2	40.49	1.84	0.2517	0.2538	0.0198
0.25	1.825	143.3	41.87	1.71	0.2342	0.2517	0.0017

$d = 143$ mm.

Problem 10.72

10.72 A column of 2.1-m effective length is to be made by gluing together laminated wood pieces of 25×150 -mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_c = 7.7$ MPa and the adjusted modulus is $E = 5.4$ GPa, determine the number of wood pieces that must be used to support the concentric load shown when (a) $P = 52$ kN, (b) $P = 108$ kN.



Glued laminated column $c = 0.90$

Let n = number of pieces

$$A = (150)(25n) = 3750n \text{ mm}^2 = 3750 \times 10^{-6} n \text{ m}^2$$

$$d = 25 \times 10^{-3} n \text{ m if } n \leq 6$$

$$d = 150 \times 10^{-3} \text{ m if } n > 6$$

$$L_e = 2.1 \text{ m}$$

$$\frac{L_e}{d} = \frac{2.1}{25 \times 10^{-3} n} = \frac{84}{n} \text{ if } n \leq 6$$

$$= \frac{2.1}{0.15} = 14 \text{ if } n \geq 6$$

$$\sigma_c = 7.7 \text{ MPa} \quad E = 5400 \text{ MPa} \quad \sigma_{ce} = \frac{(0.822)(5400)}{(L/d)^2} = \frac{4439}{(L/d)^2} \text{ MPa}$$

$$C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2c} - \left(\frac{1 + \sigma_c/\sigma_{ce}}{2c} \right)^2 - \frac{\sigma_{ce}/\sigma_c}{c}$$

$$\sigma_{all} = C_p \sigma_c$$

$$P_{all} = \sigma_{all} A$$

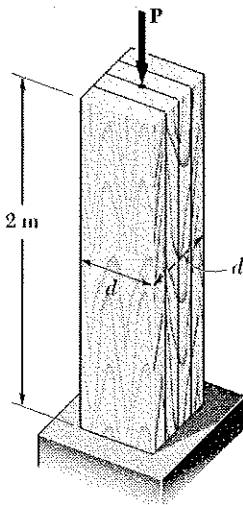
Calculations are carried out in the table below.

n	d (m)	A (10^{-6} m^2)	L/d	σ_{ce} (MPa)	σ_{ce}/σ_c	C_p	σ_{all} (MPa)	P_{all} kN
3	0.075	11250	28	5.662	0.7353	0.6288	4.842	54.5
4	0.100	15000	21	10.066	1.3073	0.8453	6.509	97.6
5	0.125	18750	16.8	15.727	2.0421	0.9237	7.412	133.6

(a) For $P = 52$ kN use $n = 3$ ▶

(b) For $P = 108$ kN use $n = 5$ ▶

Problem 10.73



10.73 The glued laminated column shown is free at its top *A* and fixed at its base *B*. Using wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 9.2$ MPa and an adjusted modulus of elasticity $E = 5.7$ GPa, determine the smallest cross section that can support a centric load of 62 kN.

Glued laminated column $c = 0.9$ $E = 5700$ MPa

$$L_e = 2L = (2)(2) = 4 \text{ m} = 4000 \text{ mm}$$

$$\sigma_c = 9.2 \text{ MPa}$$

$$A = d^2 \quad P_{all} = 62000 \text{ N}$$

$$L_e/d = 4000/d \quad \text{with } d \text{ in mm.}$$

$$\sigma_{all} = \sigma_c C_p$$

$$\text{Assume } C_p, \quad \sigma_{all} = 9.2 C_p \text{ (MPa)}$$

$$d = \sqrt{\frac{P_{all}}{\sigma_{all}}} = \sqrt{\frac{62000}{\sigma_{all}}} = \frac{249}{\sqrt{\sigma_{all}}}$$

$$\sigma_{ce} = \frac{0.822 E}{(L/d)^2} = \frac{(0.822)(5700)}{(L/d)^2} = \frac{4685}{(L/d)^2} \text{ (MPa)}$$

$$\text{Checking: } C_p = \frac{1 + \sigma_{ce}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{ce}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{ce}/\sigma_c}{c}}$$

Calculations are carried out in the following table:

C_p (assumed)	σ_{all} (MPa)	d mm	L/d	σ_{ce} (MPa)	σ_{ce}/σ_c	C_p (calc)	ΔC_p
0.5	4.6	116.1	34.45	3.948	0.4291	0.4021	-0.0979
0.4	3.68	129.8	30.82	4.932	0.5361	0.4892	+0.0892
0.448	4.122	121.7	32.87	4.336	0.4713	0.4373	-0.0107
0.443	4.076	123.3	32.44	4.452	0.4839	0.4476	+0.0046

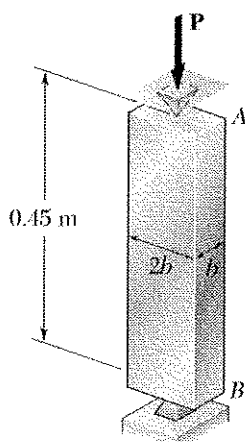
$$C_p = 0.443 + \frac{0.0046}{0.0153}(0.005) = 0.4445$$

$$\sigma_{all} = (0.4445)(9.2) = 4.0894 \text{ MPa}$$

$$d = \frac{249}{\sqrt{4.0894}} = 123.1 \text{ mm}$$

$$d = 123.1 \text{ mm} \quad \blacktriangleleft$$

Problem 10.74



10.74 A 72-kN centric load must be supported by an aluminum column as shown. Using the aluminum alloy 6061-T6, determine the minimum dimension b that can be used.

$$L_e = L = 0.45 \text{ m}, \quad A = 2b^2 \quad I_{min} = \frac{1}{12}(2b)(b)^3 = \frac{1}{6}b^4$$

$$r = \sqrt{\frac{I_{min}}{A}} = \frac{b}{\sqrt{12}} \quad \frac{L_e}{r} = \frac{0.45\sqrt{12}}{b} = \frac{1.55885}{b}$$

6061-T6 aluminum alloy. Assume $\frac{L_e}{r} < 66$.

$$\begin{aligned} \sigma_{all} &= 139 - 0.868(L_e/r) = 139 - \frac{(0.868)(1.55885)}{b} \\ &= 139 - \frac{1.35308}{b} \text{ MPa} \end{aligned}$$

$$P_{all} = \sigma_{all} A = \left(139 - \frac{1.35308}{b}\right)(10^6)(2b^2)$$

$$72 \times 10^3 = 278 \times 10^6 b^2 - 2.70616 \times 10^6 b$$

$$278 b^2 - 2.70616 b - 0.072 = 0$$

$$b = \frac{2.70616 + \sqrt{(2.70616)^2 - (4)(278)(-0.072)}}{(2)(278)} = 21.680 \times 10^{-3} \text{ m}$$

$$r = 6.2586 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{0.45}{6.2586 \times 10^{-3}} = 71.9 \quad \text{Assumption is not verified.}$$

$$\text{Assume } \frac{L_e}{r} > 66. \quad \sigma_{all} = \frac{351 \times 10^3}{(L_e/r)^2} = \frac{351 \times 10^3 b^2}{(1.55885)^2} = 144.444 \times 10^3 b^2 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (144.444 \times 10^3 b^2)(10^6)(2b^2) = 288.89 \times 10^9 b^4 \text{ N}$$

$$72 \times 10^3 = 288.89 \times 10^9 b^4 \quad b^4 = 249.23 \times 10^{-9} \text{ m}^4$$

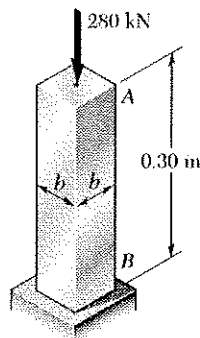
$$b = 22.3 \times 10^{-3} \text{ m} \quad r = 6.45 \times 10^{-3} \text{ m} \quad \frac{L_e}{r} = 69.77 > 66$$

Assumption is verified.

$$b = 22.3 \times 10^{-3} \text{ m} = 22.3 \text{ mm} \quad \blacktriangleleft$$

Problem 10.75

10.75 A 280-kN centric load is applied to the column shown, which is free at its top *A* and fixed at its base *B*. Using aluminum alloy 2014-T6, select the smallest square section that can be used.



$$L_e = 2L = (2)(0.30) = 0.60 \text{ m}$$

$$A = b^2 \quad I = \frac{1}{12} b^4 \quad r = \sqrt{\frac{I}{A}} = \frac{b}{\sqrt{12}}$$

$$\frac{L_e}{r} = \frac{0.60 \sqrt{12}}{b} = \frac{2.0785}{b}$$

2014-T6 aluminum alloy

$$\text{Assume } \frac{L_e}{r} < 55. \quad \sigma_{all} = 212 - 1.585(L/r) = 212 - (1.585)(2.0785/b) \\ = \left(212 - \frac{3.294}{b}\right) \text{ MPa} = \left[212 - \frac{3.294}{b}\right](10^6) \text{ Pa}$$

$$P_{all} = \sigma_{all} A = \left[212 b^2 - 3.294 b\right](10^6) = 280 \times 10^3$$

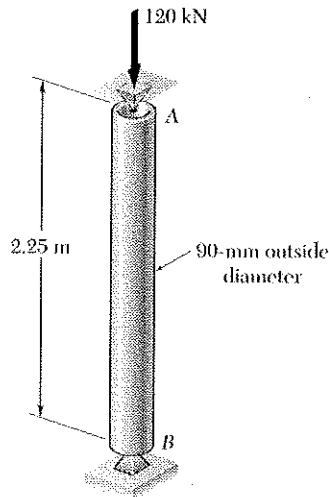
$$212 b^2 - 3.294 b - 280 \times 10^{-3} = 0$$

$$b = \frac{3.294 + \sqrt{(3.294)^2 + (4)(212)(280 \times 10^{-3})}}{(2)(212)} = 44.9 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.0785}{b} = \frac{2.0785}{44.9 \times 10^{-3}} = 46.26 < 55 \quad \text{Assumption is verified.}$$

$$b = 44.9 \text{ mm} \quad \blacktriangleleft$$

Problem 10.76



10.76 An aluminum tube of 90-mm outer diameter is to carry a centric load of 120 kN. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and with wall thicknesses in increments of 3 mm from 6 mm to 15 mm, determine the lightest tube that can be used.

$$L = 2250 \text{ mm}, P = 120 \times 10^3 \text{ N} \quad r_o = 45 \text{ mm}$$

$$r_i = r_o - t \quad A = \pi(r_o^2 - r_i^2) \quad I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

$$r = \sqrt{I/A}$$

For 2014-T6 aluminum alloy,

$$\sigma_{all} = 212 - 1.585(L/r) \text{ MPa} \quad \text{if } L/r < 55$$

$$\sigma_{all} = \frac{372 \times 10^3}{(L/r)^2} \text{ MPa} \quad \text{if } L/r > 55$$

$$P_{all} = \sigma_{all} A$$

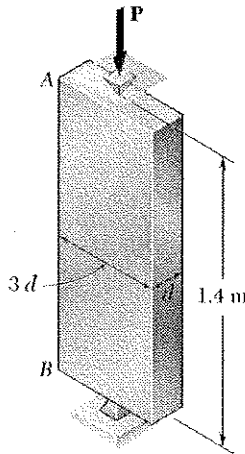
Calculate P_{all} for each thickness.

t mm	r_i mm	A mm^2	I 10^6 mm^4	r mm	L/r	σ_{all} MPa	P_{all} kN
6	39	1583	1.404	29.78	75.56	65.16	103.1
9	36	2290	1.901	28.82	78.08	61.01	139.7
12	33	2941	2.289	27.90	80.65	57.20	168.2
15	30	3534	2.584	27.04	83.20	53.74	189.9

Since P_{all} must be greater than 120 kN, use $t = 9 \text{ mm}$.

Problem 10.77

10.77 A centric load P must be supported by the steel bar AB . Using allowable stress design, determine the smallest dimension d of the cross section that can be used when (a) $P = 108 \text{ kN}$, (b) $P = 166 \text{ kN}$. Use $\sigma_y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$.



$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250}} = 125.66$$

$$L_e = L = 1.4 \text{ m}$$

$$A = (3d)(d) = 3d^2$$

$$I = \frac{1}{12}(3d)(d)^3 = \frac{1}{4}d^4$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} = 0.288675 d$$

(a) $P = 108 \times 10^3 \text{ N}$. Assume $\frac{L_e}{r} > C_c$.

$$P_{all} = \frac{\pi^2 EI}{1.92 L_e^2} \quad I = \frac{(1.92)(P_{all} L_e^2)}{\pi^2 E} = \frac{1}{4}d^4$$

$$d^4 = \frac{(4)(1.92) P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(108 \times 10^3)(1.4)^2}{\pi^2 (200 \times 10^9)} = 823.59 \times 10^{-9} \text{ m}^4$$

$$d = 30.125 \times 10^{-3} \text{ m} \quad r = 8.696 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{8.696 \times 10^{-3}} = 160.99 > 125.66 \quad d = 30.1 \text{ mm}$$

(b) $P = 166 \times 10^3 \text{ N}$. Assume $\frac{L_e}{r} > C_c$

$$d^4 = \frac{(4)(1.92) P L_e^2}{\pi^2 E} = \frac{(4)(1.92)(166 \times 10^3)(1.4)^2}{\pi^2 (200 \times 10^9)} = 1.26588 \times 10^{-6} \text{ m}^4$$

$$d = 33.543 \times 10^{-3} \text{ m} \quad r = 9.68295 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{1.4}{9.68295 \times 10^{-3}} = 144.58 > 125.66 \quad d = 33.5 \text{ mm}$$

Problem 10.78

10.78 A column of 4.5-m effective length must carry a centric load of 900 kN. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa, use allowable stress design to select the wide-flange shape of 250-mm nominal depth that should be used.

$$\text{transition } \frac{L}{r}: \quad 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40$$

$$P < \frac{\sigma_Y A}{1.67} \quad A > \frac{1.67 P}{\sigma_Y} = \frac{(1.67)(900 \times 10^3)}{345 \times 10^6} = 4357 \times 10^{-6} \text{ m}^2 = 4357 \text{ mm}^2$$

$$P < \frac{\sigma_{cr} A}{1.67} = \frac{0.877 \pi^2 E I}{1.67 L_e^2}$$

$$I > \frac{1.67 P L_e^2}{0.877 \pi^2 E} = \frac{(1.67)(900 \times 10^3)(4.5)^2}{0.877 \pi^2 (200 \times 10^9)} = 17.58 \times 10^{-6} \text{ m}^4 = 17.58 \times 10^6 \text{ mm}^4$$

Try W 250 $\times$ 58. $A = 7420 \text{ mm}^2$ $r_y = 50.3 \text{ mm}$

$$\frac{L_e}{r} = \frac{4.5}{50.3 \times 10^{-3}} = 89.46 < 113.40$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(89.46)^2} = 246.64 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_e}) \sigma_Y = [0.658^{345/246.64}] (345) = 192.11 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(192.11 \times 10^6)(7420 \times 10^{-6})}{1.67} = 854 \text{ kN} < 900 \text{ kN}$$

(not acceptable)

Try W 250 $\times$ 67 $A = 8580 \text{ mm}^2$ $r_y = 51.0 \text{ mm}$

$$\frac{L_e}{r} = \frac{4.5}{51.0 \times 10^{-3}} = 88.235 < 113.40$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(88.235)^2} = 253.54 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_e}) \sigma_Y = [0.658^{345/253.54}] (345) = 195.20 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(195.20 \times 10^6)(8580 \times 10^{-6})}{1.67} = 1003 \text{ kN} > 900 \text{ kN}$$

(acceptable)

Use W 250 $\times$ 67. ◀

Problem 10.79

10.79 A column of 6.8-m effective length must carry a centric load of 1200 kN. Using allowable stress design, select the wide-flange shape of 350-mm nominal depth that should be used. Use $\sigma_y = 350$ MPa and $E = 200$ GPa.

$$P < \frac{\sigma_y A}{F.S.} \quad A > \frac{(F.S.)P}{\sigma_y} = \frac{(5/3)(1200 \times 10^3)}{350 \times 10^6} = 5.714 \times 10^{-6} \text{ m}^2$$

$$L_e = 6.8 \text{ m}$$

$$E = 200 \text{ GPa}$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2} \quad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(1.2 \times 10^6)(6.8)^2}{\pi^2 (200 \times 10^9)} = 53.97 \times 10^{-6} \text{ m}^4$$

$$\text{Try } W360 \times 122 \quad A = 15500 \text{ mm}^2, \quad I_{\min} = 61.5 \times 10^6 \text{ mm}^4, \quad r = 63 \text{ mm}$$

$$C_e = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^3)}{350}} = 106.2$$

$$\frac{L_e}{r} = \frac{6800}{63} = 107.9 > 106.2$$

$$\sigma_{\text{all}} = \frac{\pi^2 E}{1.92 (L_e/r)^2} = \frac{\pi^2 (200000)}{(1.92)(107.9)^2} = 88.3 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (88.3)(15500) = 1368.7 \text{ kN} > 1200 \text{ kN}$$

Use $W360 \times 122$

Problem 10.80

10.80 A column of 4.6-m effective length must carry a centric load of 525 kN. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa, use allowable stress design to select the wide-flange shape of 200-mm nominal depth that should be used.

$$\text{transition } \frac{L}{r}: \quad 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40$$

$$P < \frac{\sigma_Y A}{1.67} \quad A > \frac{1.67 P}{\sigma_Y} = \frac{(1.67)(525 \times 10^3)}{345 \times 10^6} = 2.541 \times 10^{-3} \text{ m}^2 = 2541 \text{ mm}^2$$

$$P < \frac{0.877 \pi^2 E I_{\min}}{1.67 L_e^2}$$

$$I_{\min} > \frac{1.67 P L_e^2}{0.877 \pi^2 E} = \frac{(1.67)(525 \times 10^3)(4.6)^2}{0.877 \pi^2 (200 \times 10^9)} = 10.72 \times 10^{-6} \text{ m}^4 = 10.72 \times 10^{-6} \text{ mm}^4$$

$$\text{Try } W 200 \times 46.1 \quad A = 5860 \text{ mm}^2 \quad I_{\min} = 15.3 \times 10^6 \text{ mm}^4 \quad r = 51.1 \text{ mm}$$

$$\frac{L_e}{r} = \frac{4.6}{51.1 \times 10^{-3}} = 90.02 < 113.40$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = 243.59 \text{ MPa}$$

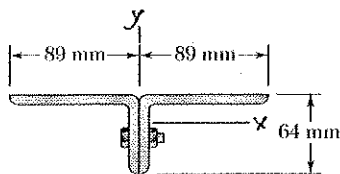
$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_c}) \sigma_Y = [0.658^{345/243.59}] (345) = 190.708 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(190.708 \times 10^6)(5860 \times 10^{-6})}{1.67} = 669 \text{ kN} > 525 \text{ kN}$$

All lighter sections fail the minimum moment of inertia criterion.

Use $W 200 \times 46.1$. 

Problem 10.81



10.81 Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 180 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_Y}}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Use x and y axes as added above.

Try L 89 × 64 × 9.5 $A = (2)(1360) = 2720 \text{ mm}^2 = 2720 \times 10^{-6} \text{ m}^2$

$$r_x = 18.6 \text{ mm} = 18.6 \times 10^{-3} \text{ m} \quad (r_y \text{ in Appendix C})$$

$$L_e/r = 2.4 / 18.6 \times 10^{-3} = 129.032 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(129.032)^2} = 118.559 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_e}) \sigma_Y = [0.658^{250/118.559}] (250) = 103.43 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(103.43 \times 10^6)(2720 \times 10^{-6})}{1.67} = 168.5 \text{ kN} < 180 \text{ kN}$$

Do not use.

Try L 89 × 64 × 12.7 $A = (2)(1780) = 3560 \text{ mm}^2 = 3560 \times 10^{-6} \text{ m}^2$

$$r_x = 18.1 \text{ mm} = 18.1 \times 10^{-3} \text{ m}$$

$$L_e/r = 2.4 / 18.1 \times 10^{-3} = 132.597 < 133.22$$

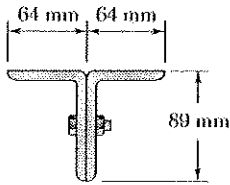
$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(132.597)^2} = 112.27 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_Y/\sigma_e}) \sigma_Y = [0.658^{250/112.27}] (250) = 98.44 \text{ MPa}$$

$$P_{all} = \frac{\sigma_{cr} A}{1.67} = \frac{(98.44 \times 10^6)(3560 \times 10^{-6})}{1.67} = 233 \text{ kN} > 180 \text{ kN}$$

Use L 89 × 64 × 12.7 mm. ◀

Problem 10.82



10.82 Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 325 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_y = 250$ MPa and $E = 200$ GPa.

$$\text{transition } \frac{L_e}{r} : 4.71 \sqrt{\frac{E}{\sigma_y}}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Try L 89 × 64 × 9.5. $A = (2)(1360) = 2720 \text{ mm}^2 = 2720 \times 10^{-6} \text{ m}^2$

$$I_x = (2)(1.07 \times 10^6) = 2.14 \times 10^6 \text{ mm}^4$$

$$I_y = (2)[0.463 \times 10^6 + (1360)(16.9)^2] = 1.70285 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{1.70285 \times 10^6}{2720}} = 25.021 \text{ mm} = 25.021 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.021 \times 10^{-3}} = 95.919 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(95.919)^2} = 214.55 \text{ MPa}$$

$$\sigma_{cr} = (0.658^{\sigma_c/\sigma_y}) \sigma_y = [0.658^{250/214.55}] (250) = 153.51 \text{ MPa}$$

$$P_{\text{all}} = \frac{\sigma_{cr} A}{1.67} = \frac{(153.51 \times 10^6)(2720 \times 10^{-6})}{1.67} = 250 \text{ kN} < 325 \text{ kN}$$

Do not use.

Try L 89 × 64 × 12.7. $A = (2)(1780) = 3560 \text{ mm}^2 = 3560 \times 10^{-6} \text{ m}^2$

$$I_x = (2)(1.36 \times 10^6) = 2.72 \times 10^6 \text{ mm}^4$$

$$I_y = (2)[0.581 \times 10^6 + (1780)(18.1)^2] = 2.3283 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{2.3283 \times 10^6}{3560}} = 25.574 \text{ mm} = 25.574 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.574 \times 10^{-3}} = 93.846 < 133.22$$

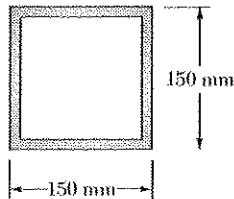
$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(93.846)^2} = 224.13 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{250/224.13}] (250) = 156.74 \text{ MPa}$$

$$P_{\text{all}} = \frac{\sigma_{cr} A}{1.67} = \frac{(156.74 \times 10^6)(3560 \times 10^{-6})}{1.67} = 334 \text{ kN} > 325 \text{ kN}$$

Use L 89 × 64 × 12.7 mm. ◀

Problem 10.83



10.83 A square steel tube having the cross section shown is used as a column of 7.8-m effective length to carry a centric load of 260 kN. Knowing that the tubes available for use are made with wall thicknesses ranging from 6 mm to 18 mm in increments of 1.5 mm, use allowable stress design to determine the lightest tube that can be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$b_o = 150 \text{ mm} \quad b_i = b_o - 2t \quad A = b_o^2 - b_i^2$$

$$I = \frac{1}{12} (b_o^4 - b_i^4)$$

$$L_e = 7.8 \text{ m} \quad P = 260 \text{ kN}$$

$$\sigma_Y = 250 \text{ MPa} \quad E = 200 \text{ GPa}$$

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$\text{Try } t = 12 \text{ mm} \quad b_i = 126 \text{ mm} \quad A = 150^2 - 126^2 = 6624 \text{ mm}^2$$

$$I = \frac{1}{12} [(150)^4 - (126)^4] = 21.18 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 55.6 \text{ mm}$$

$$\frac{L_e}{r} = \frac{7800}{55.6} = 140.3 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L_e/r)^2} = 52.23 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (52.23)(6624) = 346 \text{ kN}$$

P_{all} is approximately proportional to t .

$$\frac{t}{12} = \frac{260}{346} \quad t \approx 9.02 \text{ mm}$$

$$\text{Try } t = 9 \text{ mm} \quad b_i = 132 \text{ mm} \quad A = 5076 \text{ mm}^2$$

$$I = \frac{1}{12} [(150)^4 - (132)^4] = 16.888 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 57.68 \text{ mm}$$

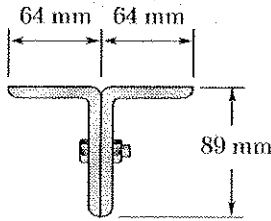
$$\frac{L_e}{r} = \frac{7800}{57.68} = 135.23 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L_e/r)^2} = 56.2 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (56.2)(5076) = 285.3 \text{ kN} > 260 \text{ kN}$$

Use $t = 9 \text{ mm}$

Problem 10.84



10.84 Two 89 × 64-mm angles are bolted together as shown for use as a column of 2.4-m effective length to carry a centric load of 325 kN. Knowing that the angles available have thicknesses of 6.4 mm, 9.5 mm, and 12.7 mm, use allowable stress design to determine the lightest angles that can be used. Use $\sigma_y = 250$ MPa and $E = 200$ GPa.

Steel: $\sigma_y = 250 \times 10^6 \text{ Pa}$ $E = 200 \times 10^9 \text{ Pa}$
 $C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} = \sqrt{\frac{2\pi^2 (200 \times 10^9)}{250 \times 10^6}} = 125.664$

Try L 89 × 64 × 9.5.

$$A = (2)(1360) = 2720 \text{ mm}^2 = 2720 \times 10^{-6} \text{ m}^2$$

$$I_x = (2)(1.07 \times 10^6) = 2.14 \times 10^6 \text{ mm}^4$$

$$I_y = (2)[0.463 \times 10^6 + (1360)(16.9)^2] = 1.70285 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{1.70285 \times 10^6}{2720}} = 25.021 \text{ mm} = 25.021 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.021 \times 10^{-3}} = 95.919 < C_c \quad \frac{L_e/r}{C_c} = 0.76330$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.76330) - \frac{1}{8}(0.76330)^3 = 1.8973$$

$$\sigma_{\text{all}} = \frac{\sigma_y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250 \times 10^6}{1.8973} \left[1 - \frac{1}{2} (0.76330)^2 \right] = 93.38 \times 10^6 \text{ Pa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (93.38 \times 10^6)(2720 \times 10^{-6}) = 254 \times 10^3 \text{ N} = 254 \text{ kN} < 325 \text{ kN}$$

Do not use.

Try L 89 × 64 × 12.7.

$$A = (2)(1780) = 3560 \text{ mm}^2 = 3560 \times 10^{-6} \text{ m}^2$$

$$I_x = (2)(1.36 \times 10^6) = 2.72 \times 10^6 \text{ mm}^4$$

$$I_y = (2)[0.581 \times 10^6 + (1780)(18.1)^2] = 2.3283 \times 10^6 \text{ mm}^4 = I_{\min}$$

$$r = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{2.3283 \times 10^6}{3560}} = 25.574 \text{ mm} = 25.574 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{25.574 \times 10^{-3}} = 93.846 < C_c \quad \frac{L_e/r}{C_c} = 0.7468$$

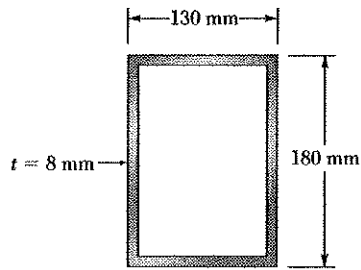
$$F.S. = \frac{5}{3} + \frac{3}{8}(0.7468) - \frac{1}{8}(0.7468)^3 = 1.89465$$

$$\sigma_{\text{all}} = \frac{250 \times 10^6}{1.89465} \left[1 - \left(\frac{1}{2} \right) (0.7468)^2 \right] = 95.155 \times 10^6 \text{ Pa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (95.155 \times 10^6)(3560 \times 10^{-6}) = 339 \times 10^3 \text{ N} = 339 \text{ kN} > 325 \text{ kN}$$

Use L 89 × 64 × 12.7.

Problem 10.85



***10.85** A rectangular tube having the cross section shown is used as a column of 4.4-m effective length. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use load and resistance factor design to determine the largest centric live load that can be applied if the centric dead load is 220 kN. Use a dead load factor $\gamma_D = 1.2$, a live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.

$$L = 4.4 \text{ m}$$

$$b_o = 180 \text{ mm} \quad b_i = 180 - 2(8) = 164 \text{ mm}$$

$$h_o = 130 \text{ mm} \quad h_i = 114 \text{ mm}$$

$$A = (180)(130) - (164)(114) = 4704 \text{ mm}^2$$

$$I = \frac{1}{12} [(180)(130)^3 - (164)(114)^3] = 12707232 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 51.98 \text{ mm}$$

$$\frac{L_e}{r} = 84.65$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_Y}{E}} = \frac{84.65}{\pi} \sqrt{\frac{250}{200000}} = 0.9527 < 1.5$$

$$\lambda_c^2 = 0.9076$$

$$P_v = A (0.658)^{\lambda_c^2} \sigma_Y = (4704) (0.658)^{0.9076} (250) = 804.3 \text{ kN}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_v$$

$$(1.2)(220) + (1.6) P_L = (0.85)(804.3)$$

$$P_L = 262.3 \text{ kN}$$

Problem 10.86

10.86 A column with a 5.8-m effective length supports a centric load, with ratio of dead to live load equal to 1.35. The dead load factor is $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$, and the resistance factor $\phi = 0.90$. Use load and resistance factor design to determine the allowable centric dead and live loads if the column is made of the following rolled-steel shape: (a) W250 $\times$ 67, (b) W 360 $\times$ 101. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.

$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.40$$

(a) W 250 $\times$ 67. $A = 8580 \text{ mm}^2 = 8580 \times 10^{-6} \text{ m}^2$, $r_y = 51.0 \text{ mm} = 51.0 \times 10^{-3} \text{ m}$

$$L_e/r_y = \frac{5.8}{51.0 \times 10^{-3}} = 113.725 > 113.40$$

$$\sigma_{cr} = \frac{0.877 \pi^2 E}{(L_e/r_y)^2} = \frac{0.877 \pi^2 (200 \times 10^9)}{(113.725)^2} = 133.85 \text{ MPa}$$

$$P_u = A \sigma_{cr} = (8580 \times 10^{-6})(133.85 \times 10^6) = 1.1484 \times 10^6 \text{ N}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u \quad \frac{P_D}{P_L} = 1.35 \quad P_L = \frac{P_D}{1.35}$$

$$1.2 P_D + 1.6 \frac{P_D}{1.35} = (0.9)(1.1484 \times 10^6) \quad P_D = 433 \text{ kN} \quad \blacktriangleleft$$

$$P_L = \quad P_L = 321 \text{ kN} \quad \blacktriangleleft$$

(b) W 360 $\times$ 101. $A = 12900 \text{ mm}^2 = 12900 \times 10^{-6} \text{ m}^2$, $r_y = 62.6 \text{ mm} = 62.6 \times 10^{-3} \text{ m}$

$$L_e/r_y = \frac{5.8}{62.6 \times 10^{-3}} = 92.652 < 113.40$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r_y)^2} = \frac{\pi^2 (200 \times 10^9)}{(92.652)^2} = 229.94 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{\sigma_Y/\sigma_c}] \sigma_Y = [0.658^{345/229.94}] (345) = 184.12 \text{ MPa}$$

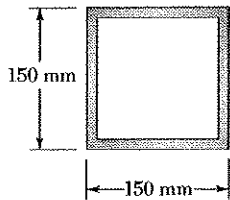
$$P_u = A \sigma_{cr} = (12900 \times 10^{-6})(184.12 \times 10^6) = 2.3751 \times 10^6 \text{ N}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$1.2 P_D + 1.6 \frac{P_D}{1.35} = (0.90)(2.3751 \times 10^6) \quad P_D = 896 \text{ kN} \quad \blacktriangleleft$$

$$P_L = 664 \text{ kN} \quad \blacktriangleleft$$

Problem 10.87



***10.87** The steel tube having the cross section shown is used as a column of 4.5-m effective length to carry a centric dead load of 210 kN and a centric live load of 240 kN. Knowing that the tubes available for use are made with wall thicknesses in increments of 2 mm from 4 mm to 10 mm, use load and resistance factor design to determine the lightest tube that can be used. Use $\sigma_y = 250$ MPa and $E = 200$ GPa. The dead load factor $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$, and the resistance factor $\phi = 0.85$.

$$L_e = 4.5 \text{ m}$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$\text{Required } P_u = \frac{\gamma_D P_D + \gamma_L P_L}{\phi} = \frac{(1.2)(210) + (1.6)(240)}{0.85} = 748.2 \text{ kN}$$

$$\text{Try } t = 6 \text{ mm}$$

$$b_o = 150 \text{ mm} \quad b_i = b_o - 2t = 138 \text{ mm}$$

$$A = b_o^2 - b_i^2 = (150)^2 - (138)^2 = 3456 \text{ mm}^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4) = \frac{1}{12}[(150)^4 - (138)^4] = 11964672 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 58.84 \text{ mm}$$

$$\frac{L_e}{r} = \frac{4500}{58.84} = 76.48$$

$$\lambda_c = \frac{L_e/r}{\pi} \sqrt{\frac{\sigma_y}{E}} = \frac{76.48}{\pi} \sqrt{\frac{250}{200000}} = 0.8607 < 1.5 \quad \lambda_c^2 = 0.7408$$

$$P_u = A (0.658)^{\lambda_c^2} \sigma_y = (3456)(0.658)^{0.7408} (250) = 633.66 < 748.2 \text{ kN}$$

Thickness is too small.

Since P_u is approximately proportional to thickness, the required thickness is approximately

$$\frac{t_{\text{req}}}{6} \approx \frac{P_{u(\text{req})}}{633.66} = \frac{748.2}{633.66} \quad t_{\text{req}} \approx 7.1 \text{ mm}$$

$$\text{Try } t = 8 \text{ mm}$$

$$b_i = 134 \text{ mm}$$

$$A = 4544 \text{ mm}^2, \quad I = 15319338 \text{ mm}^4, \quad r = 58.06 \text{ mm} \quad \frac{L_e}{r} = 77.5$$

$$\lambda_c = \frac{77.5}{\pi} \sqrt{\frac{250}{200000}} = 0.8722 < 1.5 \quad \lambda_c^2 = 0.7607$$

$$P_u = (4544)(0.658)^{0.7607} (250) = 826.2 \text{ kN} > 748.2 \text{ kN}$$

$$\text{Use } t = 8 \text{ mm}$$

Problem 10.88

10.88 A steel column of 5.5-m effective length must carry a centric dead load of 310 kN and a centric live load of 375 kN. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use load and resistance factor design to select the wide-flange shape of 310-mm nominal depth that should be used. The dead load factor $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.90$.

$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$\gamma_D P_D + \gamma_L P_L = \phi P_u$$

$$\text{Required: } P_u = \frac{\gamma_D P_D + \gamma_L P_L}{\phi} = \frac{(1.2)(310) + (1.6)(375)}{0.90} = 1080 \text{ kN}$$

Preliminary calculations.

$$P_u < \sigma_Y A \quad A > \frac{P_u}{\sigma_Y} = \frac{1080 \times 10^3}{250 \times 10^6} = 4.32 \times 10^{-3} \text{ m}^2 = 4320 \text{ mm}^2$$

$$P_u < \frac{0.877 \pi^2 E I_y}{L^2}$$

$$I_y > \frac{P_u L^2}{0.877 \pi^2 E} = \frac{(1080 \times 10^3)(5.5)^2}{0.877 \pi^2 (200 \times 10^9)} = 18.87 \times 10^{-6} \text{ m}^4 = 18.87 \times 10^6 \text{ mm}^4$$

$$\text{Try } W 310 \times 74. \quad A = 9480 \text{ mm}^2 \quad I_y = 23.4 \times 10^6 \text{ mm}^4 \quad r_y = 49.7 \text{ mm}$$

$$\frac{L_e}{r_y} = \frac{5.5}{49.7 \times 10^{-3}} = 110.66 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r_y)^2} = \frac{\pi^2 (200 \times 10^9)}{(110.66)^2} = 161.19 \text{ MPa}$$

$$\sigma_{cr} = [0.658^{\sigma_Y/\sigma_c}] \sigma_Y = [0.658^{250/161.19}] (250) = 130.62 \text{ MPa}$$

$$P_u = A \sigma_{cr} = (9480 \times 10^{-6} \text{ m}^2)(130.62 \times 10^6 \text{ Pa}) = 1238 \text{ kN} > 1080 \text{ kN}$$

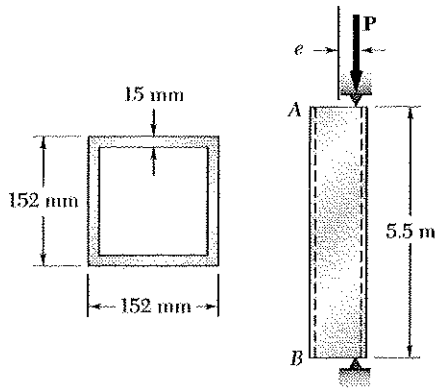
(Acceptable)

The next lighter shape, $W 310 \times 60$, with $I_y = 18.3 \times 10^6 \text{ mm}^4$ fails the moment of inertia criterion given above.

Use $W 310 \times 74$

Problem 10.89

10.89 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load P , knowing that the eccentricity is (a) $e = 0$, (b) $e = 40$ mm.



$$b_o = 152 \text{ mm} \quad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8220 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{5.5}{56.26 \times 10^{-3}} = 97.76 > 55$$

$$\sigma_{all,c} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(97.76)^2} = 38.92 \text{ MPa for centric loading}$$

$$\frac{P}{A \sigma_{all,c}} + \frac{P e c}{I \sigma_{all,b}} = 1$$

$$(a) \quad e = 0 \quad P = A \sigma_{all,c} = (8220 \times 10^{-6})(38.92 \times 10^6) = 320 \times 10^3 \text{ N} = 320 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad e = 40 \times 10^{-3} \text{ m} \quad c = \frac{1}{2}(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$$

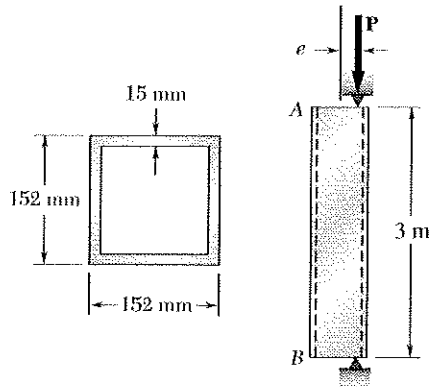
$$\frac{P}{(8220 \times 10^{-6})(38.92 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^6)(220 \times 10^6)} = 3.6568 \times 10^{-6} P = 1$$

$$P = 273 \times 10^3 \text{ N}$$

$$P = 273 \text{ kN} \quad \blacktriangleleft$$

Problem 10.90

10.90 Solve Prob. 10.89, assuming that the effective length of the column is 3.0 m.



10.89 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6, for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load P , knowing that the eccentricity is (a) $e = 0$, (b) $e = 40$ mm.

$$b_o = 152 \text{ mm} \quad b_i = b_o - 2t = 122 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8220 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12} (b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{3.0}{56.26 \times 10^{-3}} = 53.32 < 55$$

$$\sigma_{all,c} = 212 - 1.585 (L/r) = 212 - (1.585)(53.32) = 127.5 \text{ MPa}$$

$$\frac{P}{A \sigma_{all,c}} + \frac{P e c}{I \sigma_{all,b}} = 1$$

$$(a) \quad e = 0 \quad P = A \sigma_{all} = (8220 \times 10^{-6})(127.5 \times 10^6) = 1048 \times 10^3 \text{ N} = 1048 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad e = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} \quad c = \left(\frac{1}{2}\right)(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$$

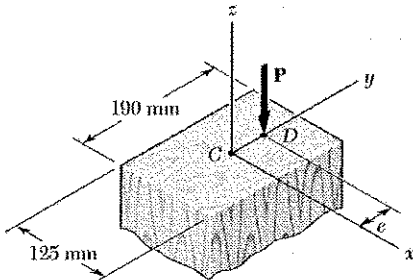
$$\frac{P}{(8220 \times 10^{-6})(127.5 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^6)(220 \times 10^6)} = 1.4852 \times 10^{-6} P = 1$$

$$P = 673 \times 10^3 \text{ N}$$

$$P = 673 \text{ kN} \quad \blacktriangleleft$$

Problem 10.91

10.91 A sawn lumber column of 125×190 -mm cross section has an effective length of 2.5 m. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8$ MPa and a modulus of elasticity $E = 8.3$ GPa. Using the allowable-stress method, determine the largest eccentric load P that can be applied when (a) $e = 12$ mm, (b) $e = 24$ mm.



Sawn lumber: $\sigma_c = 8 \text{ MPa}$ $E = 8.3 \text{ GPa}$
 $C = 0.8$ $K_{CE} = 0.300$

$L_e = 2.5 \text{ m}$

$b = 190 \text{ mm}$, $d = 125 \text{ mm}$, $c = \frac{b}{2} = 95 \text{ mm}$

$A = bd = (190)(125) = 23750 \text{ mm}^2$ $I_x = \frac{1}{12}(125)(190)^3 = 71.448 \times 10^6 \text{ mm}^4$

$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{K_{CE} E d^2}{L^2} = \frac{(0.300)(8.3 \times 10^3)(125)^2}{(2500)^2} = 6.225 \text{ MPa}$

$\sigma_{CE} / \sigma_c = 6.225 / 8 = 0.7781$

$C_P = \frac{1 + \sigma_{CE} / \sigma_c}{2C} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2C}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{C}} = 0.5991$

$\sigma_{all} = \sigma_c C_P = (8)(0.5991) = 4.793 \text{ MPa}$

$\frac{P_{all}}{A} + \frac{P_{all} e c}{I_x} = \sigma_{all}$

$P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{e c}{I_x}}$

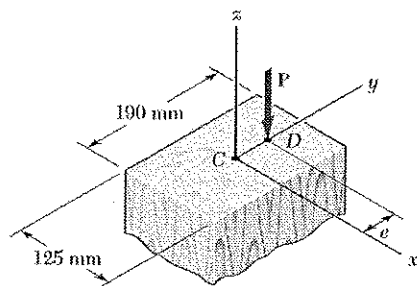
(a) $e = 12 \text{ mm}$

$P_{all} = \frac{4.793}{\frac{1}{23750} + \frac{(12)(95)}{71.448 \times 10^6}} = 82.55 \text{ kN} = 82.6 \text{ kN}$

(b) $e = 24 \text{ mm}$

$P_{all} = \frac{4.793}{\frac{1}{23750} + \frac{(24)(95)}{71.448 \times 10^6}} = 64.76 \text{ kN} = 64.8 \text{ kN}$

Problem 10.92



10.92 Solve Prob. 10.91 using the interaction method and an allowable stress in bending of 9 MPa.

10.91 A sawn lumber column of 125×190 -mm cross section has an effective length of 2.5 m. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8$ MPa and a modulus of elasticity $E = 8.3$ GPa. Using the allowable-stress method, determine the largest eccentric load P that can be applied when (a) $e = 12$ mm, (b) $e = 24$ mm.

$$\begin{aligned} \text{Sawn lumber: } \sigma_c &= 8 \text{ MPa} & E &= 8.3 \text{ MPa} \\ c &= 0.8 & K_{ce} &= 0.300 \\ L_e &= 2.5 \text{ m} \end{aligned}$$

$$b = 190 \text{ mm}, \quad d = 125 \text{ mm}, \quad c = \frac{b}{2} = 95 \text{ mm}$$

$$A = bd = (190)(125) = 23750 \text{ mm}^2 \quad I_x = \frac{1}{12} (125)(190)^3 = 71.448 \times 10^6 \text{ mm}^4$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} = \frac{K_{ce} E d^2}{L^2} = \frac{(0.300)(8.3 \times 10^3)(125)^2}{(2500)^2} = 6.225 \text{ MPa}$$

$$\sigma_{ce} / \sigma_c = 6.225 / 8 = 0.7781$$

$$C_p = \frac{1 + \sigma_{ce} / \sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{ce} / \sigma_c}{2c} \right)^2 - \frac{\sigma_{ce} / \sigma_c}{c}} = 0.5991$$

$$\sigma_{all} = \sigma_c C_p = (8)(0.5991) = 4.793 \text{ MPa}$$

$$\frac{P_{all}}{A \sigma_{all}} + \frac{P e c}{I_x (\sigma_{all})_b} = B P \leq 1 \quad P \leq \frac{1}{B}$$

(a) $e = 12 \text{ mm}$

$$B = \frac{1}{(23750)(4.793)} + \frac{(12)(95)}{(71.448 \times 10^6)(9)} = 10.5576 \times 10^{-6} \text{ N}^{-1}$$

$$P_{all} = \frac{1}{10.5576 \times 10^{-6}} = 94.718 \text{ kN} = 94.7 \text{ kN}$$

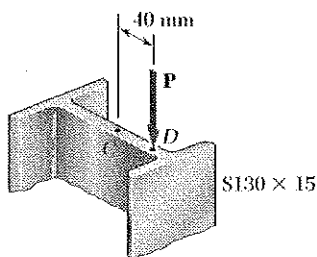
(b) $e = 24 \text{ mm}$

$$B = \frac{1}{(23750)(4.793)} + \frac{(24)(95)}{(71.448 \times 10^6)(9)} = 12.33 \times 10^{-6} \text{ N}^{-1}$$

$$P_{all} = \frac{1}{12.33 \times 10^{-6}} = 81.1 \text{ kN}$$

Problem 10.93

10.93 A steel compression member of 2.75-m effective length supports an eccentric load as shown. Using the allowable-stress method and assuming $e = 40$ mm, determine the maximum allowable load P . Use $\sigma_y = 250$ MPa and $E = 200$ GPa.



$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_y}}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$S130 \times 15: A = 1890 \text{ mm}^2 = 1890 \times 10^{-6} \text{ m}^2$$

$$r_{\min} = r_y = 16.3 \text{ mm} = 16.3 \times 10^{-3} \text{ m}$$

$$S_x = 79.8 \times 10^3 \text{ mm}^3 = 79.8 \times 10^{-6} \text{ m}^3$$

$$e_x = 0$$

$$e_y = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r_{\min}} = \frac{2.75}{16.3 \times 10^{-3}} = 168.712 > 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r_{\min})^2} = \frac{\pi^2 (200 \times 10^9)}{(168.712)^2} = 69.35 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{0.877 \sigma_c}{1.67} = \frac{(0.877)(69.35)}{1.67} = 36.418 \text{ MPa}$$

$$\frac{P}{A} + \left| \frac{M_x}{S_x} \right| + \left| \frac{M_y}{S_y} \right| = \frac{P}{A} + \frac{P e_y}{S_x} + \frac{P e_x}{S_y} = P \left(\frac{1}{A} + \frac{e_y}{S_x} + \frac{e_x}{S_y} \right) = KP$$

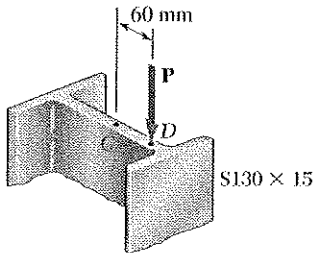
$$KP \leq \sigma_{\text{all}} \quad P \leq \frac{\sigma_{\text{all}}}{K}$$

$$K = \frac{1}{1890 \times 10^{-6}} + \frac{40 \times 10^{-3}}{79.8 \times 10^{-6}} + 0 = 1.03035 \times 10^3 \text{ m}^{-2}$$

$$P \leq \frac{36.418 \times 10^6}{1.03035 \times 10^3} = 35.3 \times 10^3 \text{ N}$$

$$P = 35.3 \text{ kN}$$

Problem 10.94



10.94 Solve Prob. 10.93, using $e = 60$ mm.

10.93 A steel compression member of 2.75-m effective length supports an eccentric load as shown. Using the allowable-stress method and assuming $e = 40$ mm, determine the maximum allowable load P . Use $\sigma_y = 250$ MPa and $E = 200$ GPa.

$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_y}}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$S_{130 \times 15}$$

$$A = 1890 \text{ mm}^2 = 1890 \times 10^{-6} \text{ m}^2$$

$$r_{\min} = r_y = 16.3 \text{ mm} = 16.3 \times 10^{-3} \text{ m}$$

$$S_x = 79.8 \times 10^3 \text{ mm}^3 = 79.8 \times 10^{-6} \text{ m}^3$$

$$e_x = 0$$

$$e_y = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r_{\min}} = \frac{2.75}{16.3 \times 10^{-3}} = 168.712 > 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r_{\min})^2} = \frac{\pi^2 (200 \times 10^9)}{(168.712)^2} = 69.35 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{0.877 \sigma_c}{1.67} = \frac{(0.877)(69.35)}{1.67} = 36.418 \text{ MPa}$$

$$\frac{P}{A} + \left| \frac{M_x}{S_x} \right| + \left| \frac{M_y}{S_y} \right| = \frac{P}{A} + \frac{P e_y}{S_x} + \frac{P e_x}{S_y} = P \left(\frac{1}{A} + \frac{e_y}{S_x} + \frac{e_x}{S_y} \right) = KP$$

$$KP \leq \sigma_{\text{all}}$$

$$P \leq \frac{\sigma_{\text{all}}}{K}$$

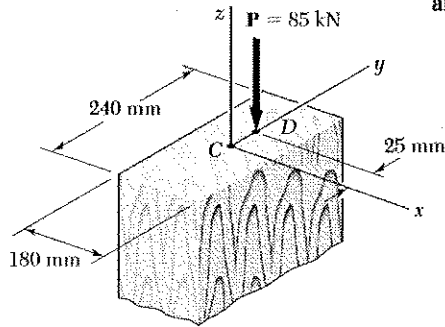
$$K = \frac{1}{1890 \times 10^{-6}} + \frac{60 \times 10^{-3}}{79.8 \times 10^{-6}} + 0 = 1.28098 \times 10^3 \text{ m}^{-2}$$

$$P \leq \frac{36.418 \times 10^6}{1.28098 \times 10^3} = 28.4 \times 10^3 \text{ N}$$

$$P = 28.4 \text{ kN}$$

Problem 10.95

10.95 A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain $F_c = 8.3 \text{ MPa}$ and a modulus of elasticity $E = 11.1 \text{ GPa}$. Using the allowable-stress method, determine the largest allowable effective length L that can be used.



$$d = 180 \text{ mm} = 0.180 \text{ m} \quad b = 240 \text{ mm} = 0.240 \text{ m}$$

$$A = bd = 43.2 \times 10^{-3} \text{ m}^2 \quad E = 11100 \text{ MPa}$$

$$I_x = \frac{1}{12} db^3 = \frac{1}{12} (0.180)(0.240)^3 = 207.36 \times 10^{-6} \text{ m}^4$$

$$e = 25 \text{ mm} = 0.025 \text{ m} \quad c = \frac{b}{2} = 0.120 \text{ m}$$

$$\frac{P}{A} + \frac{Pec}{I} \leq \sigma_{all} \quad \sigma_{all} \geq \frac{85 \times 10^3}{43.2 \times 10^{-3}} + \frac{(85 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6 \text{ Pa}$$

$$= \text{MPa}$$

$$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.9496}{8.3} = 0.38522 = y \quad \text{Let } x = \sigma_{ce}/\sigma_c$$

$$y = \frac{1+x}{2c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \quad \text{where } c = 0.8 \text{ for sawn lumber}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c}\right)^2 - y\left(\frac{1+x}{c}\right) + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$x = y \frac{(1-cy)}{1-y} = (0.38522) \frac{1 - (0.8)(0.38522)}{1 - 0.38522} = 0.43350$$

$$\sigma_{ce} = \sigma_c (0.43350) = (8.3)(0.43350) = 3.598 \text{ MPa}$$

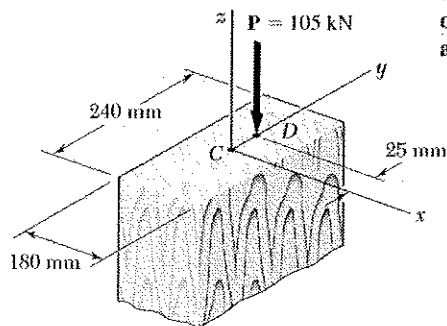
$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} \quad L^2 = \frac{K_{ce} E d^2}{\sigma_{ce}} \quad \text{where } K_{ce} = 0.300$$

$$L = d \sqrt{\frac{K_{ce} E}{\sigma_{ce}}} = (0.180) \sqrt{\frac{(0.300)(11100)}{3.598}} = 5.48 \text{ m}$$

Problem 10.96

10.96 Solve Prob. 10.95, assuming that $P = 105 \text{ kN}$.

10.95 A rectangular column is made of a grade of sawn wood that has an adjusted allowable stress for compression parallel to the grain $F_c = 8.3 \text{ MPa}$ and a modulus of elasticity $E = 11.1 \text{ GPa}$. Using the allowable-stress method, determine the largest allowable effective length L that can be used.



$$d = 180 \text{ mm} = 0.180 \text{ m} \quad b = 240 \text{ mm} = 0.240 \text{ m}$$

$$A = bd = 43.2 \times 10^{-3} \text{ m}^2 \quad E = 11100 \text{ MPa}$$

$$I_x = \frac{1}{12} db^3 = \frac{1}{12} (0.180)(0.240)^3 = 207.36 \times 10^{-6} \text{ m}^4$$

$$e = 25 \text{ mm} = 0.025 \text{ m} \quad c = \frac{b}{2} = 0.120 \text{ m}$$

$$\frac{P}{A} + \frac{Pec}{I_x} \leq \sigma_{all} \quad \sigma_{all} \geq \frac{105 \times 10^3}{43.2 \times 10^{-3}} + \frac{(105 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6 \text{ Pa} = 3.9496 \text{ MPa}$$

$$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.9496}{8.3} = 0.47586 = y \quad \text{Let } x = \sigma_{ce}/\sigma_c$$

$$y = \frac{1+x}{c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \quad \text{where } c = 0.8 \text{ for sawn lumber}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c} - y\right)^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$\left(\frac{1+x}{2c}\right)^2 - \frac{1+x}{c}y + y^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

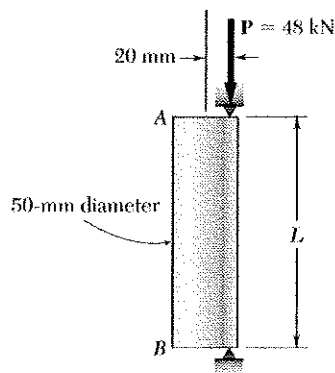
$$x = y \frac{1-cy}{1-y} = (0.47586) \frac{1 - (0.8)(0.47586)}{1 - 0.47586} = 0.56227$$

$$\sigma_{ce} = \sigma_c (0.56227) = (8.3)(0.56227) = 4.6668 \text{ MPa}$$

$$\sigma_{ce} = \frac{K_{ce} E}{(L/d)^2} \quad L^2 = \frac{K_{ce} E d^2}{\sigma_{ce}} \quad \text{where } K_{ce} = 0.300$$

$$L = d \sqrt{\frac{K_{ce} E}{\sigma_{ce}}} = 0.180 \sqrt{\frac{(0.300)(11100)}{4.6668}} = 4.81 \text{ m}$$

Problem 10.97



10.97 An eccentric load $P = 48 \text{ kN}$ is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.

$$c = \frac{1}{2}d = 25 \text{ mm} \quad A = \pi c^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 306.8 \times 10^3 \text{ mm}^4 \quad r = \sqrt{\frac{I}{A}} = 12.5 \text{ mm}$$

$$e = 20 \text{ mm} \quad \sigma_{all,b} = 145 \times 10^6 \text{ Pa}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

$$\frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

$$\frac{1}{\sigma_{all,c}} = \frac{A}{P} \left[1 - \frac{Pec}{I\sigma_{all,b}} \right] = \frac{1.9635 \times 10^{-3}}{48 \times 10^3} \left[1 - \frac{(48 \times 10^3)(20 \times 10^{-3})(25 \times 10^{-3})}{(306.8 \times 10^{-9})(145 \times 10^6)} \right]$$

$$= 18.838 \times 10^{-9} \text{ Pa}^{-1} \quad \sigma_{all,c} = 53.086 \times 10^6 \text{ Pa} = 53.086 \text{ MPa}$$

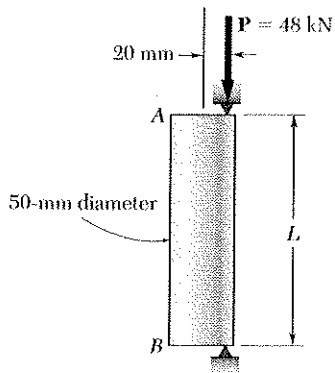
Assume $\frac{L}{r} > 66$. $\sigma_{all,c} = \frac{351 \times 10^3}{(L/r)^2}$

$$\frac{L}{r} = \sqrt{\frac{351 \times 10^3}{\sigma_{all,c}}} = \sqrt{\frac{351 \times 10^3}{53.086}} = 81.314 > 66$$

$$L = 81.314 r = (81.314)(12.5 \times 10^{-3})$$

$$L = 1.016 \text{ m} \quad \blacktriangleleft$$

Problem 10.98



10.98 Solve Prob. 10.97, assuming that the aluminum alloy used is 2014-T6 and that the allowable stress in bending is 180 MPa.

10.97 An eccentric load $P = 48 \text{ kN}$ is applied at a point 20 mm from the geometric axis of a 50-mm-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 145 MPa, determine the largest allowable effective length L that can be used.

$$c = \frac{1}{2}d = 25 \text{ mm} \quad A = \pi c^2 = 1.9635 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4} c^4 = 306.8 \times 10^3 \text{ mm}^4 \quad r = \sqrt{\frac{I}{A}} = 12.5 \text{ mm}$$

$$e = 20 \text{ mm} \quad \sigma_{all,b} = 180 \times 10^6 \text{ Pa}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

$$\frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

$$\frac{1}{\sigma_{all,c}} = \frac{A}{P} \left[1 - \frac{Pec}{I\sigma_{all,b}} \right] = \frac{1.9635 \times 10^{-3}}{48 \times 10^3} \left[1 - \frac{(48 \times 10^3)(20 \times 10^{-3})(25 \times 10^{-3})}{(306.8 \times 10^{-9})(180 \times 10^6)} \right]$$

$$= 23.129 \times 10^{-9} \text{ Pa}^{-1} \quad \sigma_{all,c} = 43.236 \times 10^6 \text{ Pa} = 43.236 \text{ MPa}$$

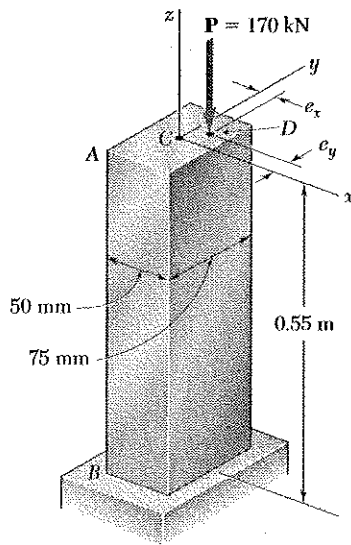
Assume $\frac{L}{r} > 55$. $\sigma_{all,c} = \frac{372 \times 10^3}{(L/r)^2}$

$$\frac{L}{r} = \sqrt{\frac{372 \times 10^3}{\sigma_{all,c}}} = \sqrt{\frac{372 \times 10^3}{43.236}} = 92.757 > 55$$

$$L = 92.757 r = (92.757)(12.5 \times 10^{-3})$$

$$L = 1.159 \text{ m} \quad \blacktriangleleft$$

Problem 10.99



10.99 The compression member AB is made of a steel for which $\sigma_y = 250$ MPa and $E = 200$ GPa. It is free at its top A and fixed at its base B . Using the allowable-stress method, determine the largest allowable eccentricity e_x , knowing that (a) $e_y = 0$, (b) $e_y = 8$ mm.

Steel: $\sigma_y = 250$ MPa $E = 200000$ MPa
 transition $\frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_y}} = 133.22$

$$A = (75 \times 10^{-3})(50 \times 10^{-3}) = 3750 \times 10^{-6} \text{ m}^2$$

$$I_y = \frac{1}{12} (75 \times 10^{-3})(50 \times 10^{-3})^3 = 781.25 \times 10^{-9} \text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = 14.434 \times 10^{-3} \text{ m} = r_{\min}$$

$$I_x = \frac{1}{12} (50 \times 10^{-3})(75 \times 10^{-3})^3 = 1.7578 \times 10^{-6} \text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = 21.651 \times 10^{-3} \text{ m}$$

$$L_e = 2L = (2)(0.55) = 1.10 \text{ m} \quad L_e/r_{\min} = 1.10/14.434 \times 10^{-3} = 76.21 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r_{\min})^2} = \frac{\pi^2 (200 \times 10^9)}{(76.21)^2} = 339.86 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_y}{1.67} = \frac{1}{1.67} [0.658^{250/339.86}] (250) = 110.03 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe_x}{S_y} + \frac{Pe_y}{S_x} = \sigma_{all} \quad \frac{Pe_y}{S_x} = \sigma_{all} - \frac{P}{A} - \frac{Pe_x}{S_y}$$

$$e_x = \frac{S_y}{P} \left[\sigma_{all} - \frac{P}{A} - \frac{Pe_x}{S_x} \right] = S_y \left[\frac{\sigma_{all}}{P} - \frac{1}{A} - \frac{e_y}{S_x} \right]$$

$$S_y = \frac{I_x}{x} = \frac{781.25 \times 10^{-9}}{25 \times 10^{-3}} = 31.25 \times 10^{-6} \text{ m}^3$$

$$S_x = \frac{I_y}{y} = \frac{1.7578 \times 10^{-6}}{37.5 \times 10^{-3}} = 46.875 \times 10^{-6} \text{ m}^3$$

$$P = 170 \times 10^3 \text{ N}$$

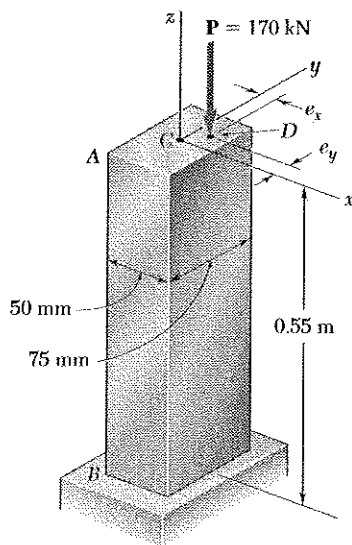
(a) $e_y = 0$
$$e_x = 31.25 \times 10^{-6} \left[\frac{110.03 \times 10^6}{170 \times 10^3} - \frac{1}{3750 \times 10^{-6}} - 0 \right]$$

$$= 11.89 \times 10^{-3} \text{ m} \quad e_x = 11.89 \text{ mm} \quad \blacktriangleleft$$

(b) $e_y = 8 \times 10^{-3} \text{ m}$
$$e_x = 31.25 \times 10^{-6} \left[\frac{110.03 \times 10^6}{170 \times 10^3} - \frac{1}{3750 \times 10^{-6}} - \frac{8 \times 10^{-3}}{46.875 \times 10^{-6}} \right]$$

$$= 6.56 \times 10^{-3} \text{ m} \quad e_x = 6.56 \text{ mm} \quad \blacktriangleleft$$

Problem 10.100



10.100 The compression member AB is made of a steel for which $\sigma_y = 250$ MPa and $E = 200$ GPa. It is free at its top A and fixed at its base B . Using the interaction method with an allowable bending stress equal to 120 MPa and knowing that the eccentricities e_x and e_y are equal, determine their largest allowable common value.

Steel: $\sigma_y = 250$ MPa $E = 200000$ MPa

transition $\frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_y}} = 133.22$

$A = (75 \times 10^{-3})(50 \times 10^{-3}) = 3750 \times 10^{-6} \text{ m}^2$

$I_y = \frac{1}{12}(75 \times 10^{-3})(50 \times 10^{-3})^3 = 781.25 \times 10^{-9} \text{ m}^4$

$r_y = \sqrt{\frac{I_y}{A}} = 14.434 \times 10^{-3} \text{ m}$

$I_x = \frac{1}{12}(50 \times 10^{-3})(75 \times 10^{-3})^3 = 1.7578 \times 10^{-6} \text{ m}^4$

$r_x = \sqrt{\frac{I_x}{A}} = 21.651 \times 10^{-3} \text{ m}$

$L_e = 2L = (2)(0.55) = 1.10 \text{ m}$ $L_e/r_{min} = 1.10 / 14.434 \times 10^{-3} = 76.21 < 133.22$

$\sigma_c = \frac{\pi^2 E}{(L_e/r_{min})^2} = \frac{\pi^2 (200 \times 10^9)}{(76.21)^2} = 339.86 \text{ MPa}$

$\sigma_{all(centric)} = \frac{\sigma_y}{1.67} = \frac{1}{1.67} [0.658^{250/339.86}] (250) = 110.03 \text{ MPa}$

$\sigma_{all(bending)} = 120 \text{ MPa}$

$\frac{P}{A \sigma_{all(centric)}} + \frac{P e_y y}{I_x \sigma_{all(bending)}} + \frac{P e_x x}{I_y \sigma_{all(bending)}} = 1$ with $e_x = e_y$

$\frac{P}{\sigma_{all(bending)}} \left(\frac{y}{I_x} + \frac{x}{I_y} \right) e = 1 - \frac{P}{A \sigma_{all(centric)}}$

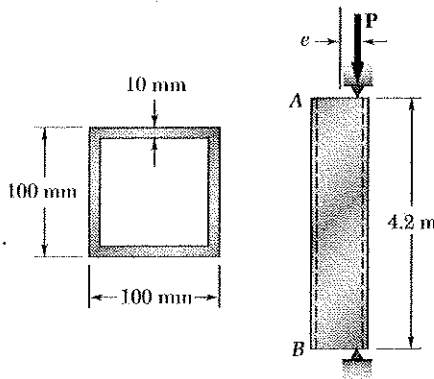
$\frac{170 \times 10^3}{120 \times 10^6} \left(\frac{37.5 \times 10^{-3}}{1.7578 \times 10^{-6}} + \frac{25 \times 10^{-3}}{781.25 \times 10^{-9}} \right) e = 1 - \frac{170 \times 10^3}{(3750 \times 10^{-6})(110.03 \times 10^6)}$

$75.556 e = 1 - 0.41201$

$e = 7.78 \times 10^{-3} \text{ m}$

$e = 7.78 \text{ mm}$

Problem 10.101



10.101 A column of 4.2-m effective length consists of a section of steel having the cross section shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 220$ kN, (b) $P = 140$ kN. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$E = 200 \text{ GPa}$$

$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_Y}} = 133.22$$

$$b_o = 100 \text{ mm} \quad b_i = b_o - 2t = 80 \text{ mm} \quad c = 50 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 3600 \text{ mm}^2 \quad I_y = \frac{1}{12}(b_o^4 - b_i^4) = 4.92 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 36.97 \text{ mm} \quad L_e = 4.2 \text{ m}$$

$$\frac{L_e}{r} = \frac{4200}{36.97} = 113.61 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(113.61)^2} = 125.93 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_Y}{1.67} = \frac{1}{1.67} [0.658^{250/125.93}] (250) = 65.22 \text{ MPa}$$

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I} = \sigma_{all}$$

$$\frac{P_{all} e c}{I} = \sigma_{all} - \frac{P_{all}}{A}$$

$$e = \frac{I}{c P_{all}} \left(\sigma_{all} - \frac{P_{all}}{A} \right)$$

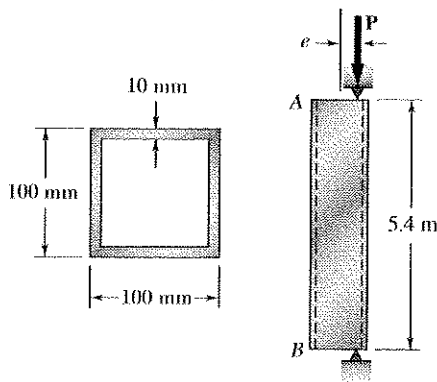
(a) $P_{all} = 220 \text{ kN}$

$$e = \frac{4.92 \times 10^{-6}}{(0.05)(220000)} \left[65.22 \times 10^6 - \frac{220000}{3600 \times 10^{-6}} \right] = 0.001838 \text{ m} \quad e = 1.84 \text{ mm}$$

(b) $P_{all} = 140 \text{ kN}$

$$e = \frac{4.92 \times 10^{-6}}{(0.05)(140000)} \left[65.22 \times 10^6 - \frac{140000}{3600 \times 10^{-6}} \right] = 0.01851 \text{ m} \quad e = 18.5 \text{ mm}$$

Problem 10.102



10.102 Solve Prob. 10.101, assuming that the effective length of the column is increased to 5.4 m and that (a) $P = 110$ kN, (b) $P = 70$ kN.

10.101 A column of 4.2-m effective length consists of a section of steel tubing having the cross section shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (a) $P = 220$ kN, (b) $P = 140$ kN. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$E = 200 \text{ GPa}$$

$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_Y}} = 133.22$$

$$b_o = 100 \text{ mm} \quad b_i = b_o - 2t = 80 \text{ mm} \quad c = 50 \text{ mm}$$

$$A = b_o^2 - b_i^2 = 3600 \text{ mm}^2 \quad I = \frac{1}{12}(b_o^4 - b_i^4) = 4.92 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 36.97$$

$$L_e = 5.4 \text{ m}$$

$$L_e/r = 146.06 > 133.22$$

$$\sigma_{cr} = \frac{0.877 \pi^2 E}{(L_e/r)^2} = 81.146 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_{cr}}{1.67} = 48.59 \text{ MPa}$$

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I} = \sigma_{all}$$

$$\frac{P_{all} e c}{I} = \sigma_{all} - \frac{P_{all}}{A}$$

$$e = \frac{I}{c P_{all}} \left(\sigma_{all} - \frac{P_{all}}{A} \right)$$

(a) $P_{all} = 110 \text{ kN}$

$$e = \frac{4.92 \times 10^{-6}}{(0.05)(110000)} \left[48.59 \times 10^6 - \frac{110000}{3600 \times 10^{-6}} \right] = 0.01613 \text{ m}$$

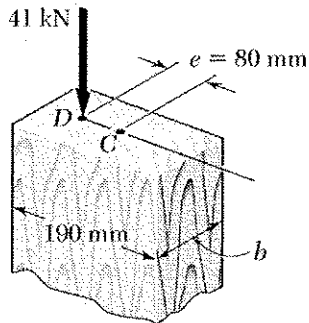
$$e = 16.1 \text{ mm}$$

(b) $P_{all} = 70 \text{ kN}$

$$e = \frac{4.92 \times 10^{-6}}{(0.05)(70000)} \left[48.59 \times 10^6 - \frac{70000}{3600 \times 10^{-6}} \right] = 0.04097 \text{ m}$$

$$e = 41 \text{ mm}$$

Problem 10.103



10.103 A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41 kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8.1$ MPa and $E = 8.3$ GPa. Use the allowable-stress method to determine the lightest section that can be used.

Sawn lumber: $\sigma_c = 8.1$ MPa $E = 8.3$ GPa
 $c = 0.8$ $K_{CE} = 0.300$
 $L_e = 2.2$ m

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I_x} = \sigma_{all}$$

$$e = 80 \times 10^{-3} \text{ m}, \quad c = \frac{1}{2}(190) = 95 \text{ mm} = 95 \times 10^{-3} \text{ m}$$

$$A = 0.190 b \text{ m}^2 \quad I_x = \frac{1}{12} b (0.190)^3 = 571.58 \times 10^{-6} b \text{ m}^4$$

$$\frac{P_{all}}{0.190 b} + \frac{P_{all}(80 \times 10^{-3})(95 \times 10^{-3})}{571.58 \times 10^{-6} b} = \frac{18.56 P_{all}}{b} = \sigma_{all}$$

$$P_{all} = 0.05388 \sigma_{all} b$$

$d = 0.190$ m or b , whichever is smaller.

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5 d^2 \text{ MPa}$$

$$\sigma_{CE} / \sigma_c = \frac{514.5 d^2}{8.1} = 63.51 d^2$$

$$C_P = \frac{1 + \sigma_{CE} / \sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{2c}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{c}}$$

$$= \frac{1 + \sigma_{CE} / \sigma_c}{1.6} - \sqrt{\left(\frac{1 + \sigma_{CE} / \sigma_c}{1.6}\right)^2 - \frac{\sigma_{CE} / \sigma_c}{0.8}}$$

$$\sigma_{all} = \sigma_c C_P = (8.1 \times 10^6) C_P$$

$$P_{all} = (0.05388 b)(8.1 \times 10^6) C_P = 472.47 \times 10^3 b C_P$$

Calculate P_{all} for all four values of b . See table below.

b (m)	d (m)	σ_{CE} / σ_c	C_P	P_{all} (kN)
0.090	0.090	0.51443	0.44367	18.87
0.140	0.140	1.24480	0.76081	50.3
0.190	0.190	2.2927	0.8878	79.7
0.240	0.190	2.2927	0.8878	100.7

→

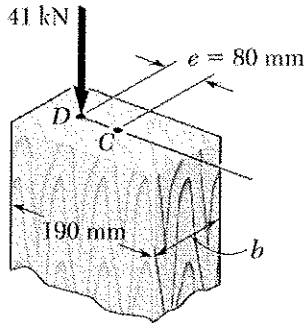
← $P = 41$ kN

Use $b = 0.140$ m

= 140 mm

Problem 10.104

10.104 Solve Prob. 10.103, assuming that $e = 40$ mm.



10.103 A sawn lumber column of rectangular cross section has a 2.2-m effective length and supports a 41 kN load as shown. The sizes available for use have b equal to 90 mm, 140 mm, 190 mm and 240 mm. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_c = 8.1$ MPa and $E = 8.3$ GPa. Use the allowable-stress method to determine the lightest section that can be used.

Sawn lumber: $\sigma_c = 8.1$ MPa $E = 8.3$ GPa
 $c = 0.8$ $K_{CE} = 0.300$
 $L_e = 2.2$ m

$$\frac{P_{all}}{A} + \frac{P_{all} e c}{I_x} = \sigma_{all}$$

$e = 40 \times 10^{-3}$ m, $c = \frac{1}{2}(190) = 95$ mm $= 95 \times 10^{-3}$ m
 $A = 0.190$ b m² $I_x = \frac{1}{12} b (0.190)^3 = 571.58 \times 10^{-6}$ b m⁴

$$\frac{P_{all}}{0.190 b} + \frac{P_{all} (40 \times 10^{-3}) (95 \times 10^{-3})}{571.58 \times 10^{-6} b} = \frac{11.911 P_{all}}{b} = \sigma_{all}$$

$$P_{all} = 0.083953 \sigma_{all} b$$

$d = 0.190$ m or b , whichever is smaller.

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.300)(8300)}{(2.2/d)^2} = 514.5 d^2 \text{ MPa}$$

$$\sigma_{CE}/\sigma_c = \frac{514.5 d^2}{8.1} = 63.51 d^2$$

$$C_p = \frac{1 + \sigma_{CE}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_c}{c}}$$

$$= \frac{1 + \sigma_{CE}/\sigma_c}{1.6} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_c}{1.6}\right)^2 - \frac{\sigma_{CE}/\sigma_c}{0.8}}$$

$$\sigma_{all} = \sigma_c C_p = (8.1 \times 10^6) C_p$$

$$P_{all} = (0.083953 k)(8.1 \times 10^6) C_p = 680.02 \times 10^3 b C_p$$

Calculate P_{all} for all four values of b . See table below.

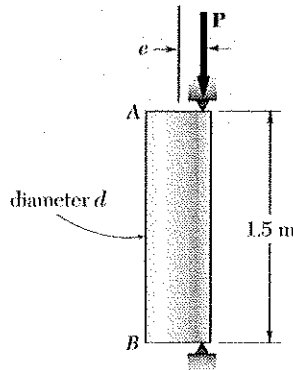
b (m)	d (m)	σ_{CE}/σ_c	C_p	P_{all} (kN)
0.090	0.090	0.51443	0.44367	27.2
0.140	0.140	1.24480	0.76081	72.4
0.190	0.190	2.2927	0.8878	114.7
0.240	0.190	2.2927	0.8878	144.9

→ $P = 41$ kN

Use $b = 0.140$ m
 $= 140$ mm

Problem 10.105

10.105 The eccentric load P has a magnitude of 85 kN and is applied at a point located at a distance $e = 30$ mm from the geometric axis of a rod made of the aluminum alloy 6016-T6. Using the interaction method with a 140-MPa allowable stress in bending, determine the smallest diameter d that can be used.



$$\text{Assume } L/r > 66, \quad \sigma_{all} = \frac{B}{(L/r)^2} \quad B = 351 \times 10^9 \text{ Pa}$$

$$c = \frac{d}{2} \quad A = \pi c^2 = \frac{\pi}{4} d^2 \quad I = \frac{\pi}{4} c^4 = \frac{\pi}{64} d^4$$

$$r = \sqrt{\frac{I}{A}} = \frac{1}{4} d$$

$$\frac{P}{A \sigma_{all, \text{centric}}} + \frac{P e c}{I \sigma_{all, \text{bending}}} = 1$$

$$\frac{P L^2}{A B r^2} + \frac{P e d}{2 I \sigma_{all, \text{bending}}} = 1$$

$$\frac{64 P L^2}{\pi B d^4} + \frac{32 P e d}{\pi \sigma_{all, \text{bending}}} = 1$$

$$\frac{(64)(85 \times 10^3)(1.5)^2}{\pi (351 \times 10^9) d^4} + \frac{(32)(85 \times 10^3)(30 \times 10^{-3})}{\pi (140 \times 10^6) d^3} = 1 \quad \text{let } x = \frac{1}{d}$$

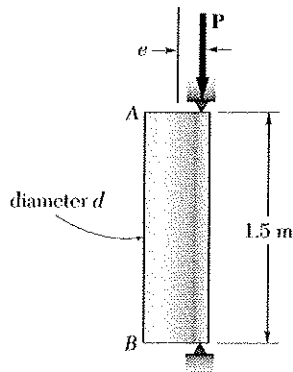
$$11.1 \times 10^{-6} x^4 + 185.53 \times 10^{-6} x^3 = 1 \quad \text{Solving, } x = 14.2725 \text{ m}^{-1}$$

$$d = \frac{1}{x} = 70.0 \times 10^{-3} \text{ m} \quad d = 70.0 \text{ mm} \quad \blacktriangleleft$$

$$r = \frac{d}{4} = 17.50 \times 10^{-3} \text{ m} \quad \frac{L}{r} = \frac{1.5}{17.5 \times 10^{-3}} = 85.7 > 66$$

Assumption is verified.

Problem 10.106



10.106 Solve Prob. 10.105, using the allowable-stress method and assuming that the aluminum alloy used is 2014-T6.

10.105 The eccentric load P has a magnitude of 85 kN and is applied at a point located at a distance $e = 30$ mm from the geometric axis of a rod made of the aluminum alloy 6016-T6. Using the interaction method with a 140-MPa allowable stress in bending, determine the smallest diameter d that can be used.

Assume $L/r > 55$. $\sigma_{all} = \frac{B}{(L/r)^2}$ $B = 372 \times 10^9 \text{ Pa}$

$c = \frac{d}{2}$ $A = \pi c^2 = \frac{\pi}{4} d^2$ $I = \frac{\pi}{4} c^4 = \frac{\pi}{64} d^4$

$r = \sqrt{\frac{I}{A}} = \frac{1}{4} d$

$\frac{P}{A} + \frac{Pec}{I} = \sigma_{all} = \frac{Br^2}{L^2}$

$\frac{PL^2}{ABr^2} + \frac{PL^2 e \frac{1}{2} d}{IBr^2} = 1$ $\frac{64 PL^2}{\pi d^4 B} + \frac{32 PL^2}{\pi d^3 B} = 1$ Let $x = \frac{1}{d}$

$\frac{64 PL^2}{\pi B} x^4 + \frac{(16)(64) PL^2 e}{2\pi B} x^5 = 1$

$\frac{(64)(85 \times 10^3)(1.5)^2}{\pi (372 \times 10^9)} x^4 + \frac{(16)(64)(85 \times 10^3)(1.5)^2 (30 \times 10^{-3})}{2\pi (372 \times 10^9)} x^5 = 1$

$10.473 \times 10^{-6} x^4 + 2.5136 \times 10^{-6} x^5 = 1$ $x = 12.441 \text{ m}^{-1}$

$d = \frac{1}{x} = 80.4 \times 10^{-3} \text{ m}$

$d = 80.4 \text{ mm}$

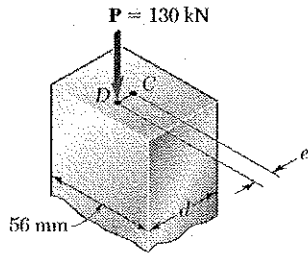
$r = \frac{d}{4} = 20.1 \times 10^{-3} \text{ m}$

$\frac{L}{r} = \frac{1.5}{20.1 \times 10^{-3}} = 74.5 > 55$

Assumption is verified.

Problem 10.107

10.107 A compression member of rectangular cross section has an effective length of 0.9 m and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 165 MPa. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 10$ mm.



$$A = 56d \quad c = \frac{1}{2}d \quad e = 10 \text{ mm} \quad L_e = 900 \text{ mm}$$

$$\sigma_{all,b} = 165 \text{ MPa} \quad P = 130 \text{ kN}$$

$$I_x = \frac{1}{12}(56)d^3 \quad r_x = \frac{d}{\sqrt{12}}$$

Assume $r_x = r_{min}$, i.e. $d < 56 \text{ mm}$

$$L_e/r_{min} = \sqrt{12} L_e/d$$

Assume $L_e/r_{min} > 55$. $\sigma_{all,c} = \frac{372 \times 10^3}{(L_e/r_x)^2} = \frac{372000 d^2}{12 L_e^2} = \frac{372000}{(12)(900)^2} d^2 = 0.03827 d^2$

$$\frac{P}{A \sigma_{all,c}} + \frac{Pec}{I \sigma_{all,b}} = \frac{130000}{(56d)(0.03827 d^2)} + \frac{(12)(130000)(10)(\frac{1}{2}d)}{(56 d^3)(165)} = 1$$

$$\frac{60659}{d^3} + \frac{844.2}{d^2} = 1 \quad \text{Let } x = \frac{1}{d} \quad 60659x^3 + 844.2x^2 = 1$$

Solving for x , $x = 0.02107$, $d = \frac{1}{x} = 47.46 < 56 \text{ mm}$

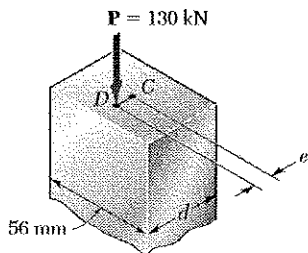
$$L/r_x = (\sqrt{12})(900)/(47.46) = 65.7 > 55$$

$$d = 47.5 \text{ mm} \quad \blacktriangleleft$$

Problem 10.108

10.108 Solve Prob. 10.107, assuming that $e = 5$ mm.

10.107 A compression member of rectangular cross section has an effective length of 0.9 m and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 165 MPa. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 10$ mm.



$$A = 56d \quad c = \frac{1}{2}d \quad e = 5 \text{ mm} \quad L_e = 900 \text{ mm}$$

$$\sigma_{all,b} = 165 \text{ MPa} \quad P = 130 \text{ kN}$$

$$I_x = \frac{1}{12}(56)d^3 \quad r_x = \frac{d}{\sqrt{12}}$$

Assume $r_x = r_{min}$, i.e. $d < 56 \text{ mm}$

$$L_e/r_{min} = \sqrt{12} L_e/d$$

Assume $L_e/r_{min} > 55$. $\sigma_{all,c} = \frac{372000}{(L_e/r_x)^2} = \frac{372000 d^2}{12 L_e^2} = \frac{372000}{(12)(900)^2} d^2 = 0.03827 d^2$

$$\frac{P}{A \sigma_{all,c}} + \frac{Pec}{I \sigma_{all,b}} = \frac{130000}{(56d)(0.03827 d^2)} + \frac{(12)(130000)(5)(\frac{1}{2}d)}{(56 d^3)(165)} = 1$$

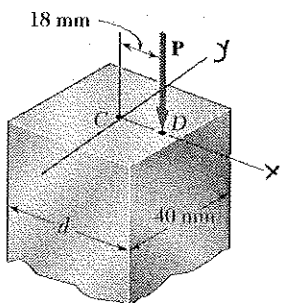
$$\frac{60659}{d^3} + \frac{422.1}{d^2} = 1 \quad \text{Let } x = \frac{1}{d} \quad 60659x^3 + 422.1x^2 = 1$$

Solving for x , $x = 0.02267 \text{ m}^{-1}$ $d = \frac{1}{x} = 44.1 \text{ mm} < 56 \text{ mm}$

$$L_e/r_x = (\sqrt{12})(900)/44.1 = 70.7 > 55$$

$$d = 44.1 \text{ mm} \quad \blacktriangleleft$$

Problem 10.109



10.109 A compression member made of steel has a 720-mm effective length and must support the 198-kN load P as shown. For the material used $\sigma_y = 250$ MPa and $E = 200$ GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

Using dimensions in meters,

$$A = 40 \times 10^{-3} d \quad L_e = 720 \text{ mm} = 0.720 \text{ m}$$

$$I_x = \frac{1}{12} (40 \times 10^{-3})^3 d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12} (40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-3} d^3$$

$$|x| = \frac{d}{2}, \quad |y| = 20 \text{ mm} = 0.020 \text{ m} \quad |e_x| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

$$\text{transition } \frac{L_e}{r}: \quad 4.71 \sqrt{\frac{E}{\sigma_y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Assume $d > 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$. Then $I_{\min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{40 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m}, \quad \frac{L_e}{r} = 62.35 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(62.35)^2} = 507.76 \text{ MPa}$$

$$\sigma_{\text{all, centric}} = \frac{\sigma_c}{1.67} = \frac{1}{1.67} (0.658^{250/507.76}) (250) = 121.82 \text{ MPa}$$

$$\sigma_{\text{all, bending}} = 150 \text{ MPa}$$

$$\frac{P}{A \sigma_{\text{all, centric}}} + \frac{P e_x x}{I_y \sigma_{\text{all, bending}}} = 1$$

$$\frac{198 \times 10^3}{(40 \times 10^{-3} d)(121.82 \times 10^6)} + \frac{(198 \times 10^3)(18 \times 10^{-3})(\frac{1}{2} d)}{(3.3333 \times 10^{-3} d^3)(150 \times 10^6)} = 1$$

$$\frac{40.634 \times 10^{-3}}{d} + \frac{3.5640 \times 10^{-3}}{d^2} = 1$$

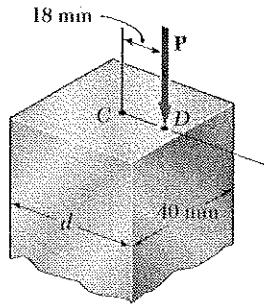
$$d^2 - 40.634 \times 10^{-3} d - 3.5640 \times 10^{-3} = 0$$

$$d = \frac{1}{2} \left\{ 40.634 \times 10^{-3} + \sqrt{(40.634 \times 10^{-3})^2 + 4(3.5640 \times 10^{-3})} \right\}$$

$$d = 83.4 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m}$$

$$d = 83.4 \text{ mm} \quad \blacktriangleleft$$

Problem 10.110



10.110 Solve Prob. 10.109, assuming that the effective length is 1.62 m and that the magnitude P of the eccentric load is 128 kN.

10.109 A compression member made of steel has a 720-mm effective length and must support the 198-kN load P as shown. For the material used $\sigma_y = 250$ MPa and $E = 200$ GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

Using dimensions in meters,

$$A = 40 \times 10^{-3} d \quad L_e = 1.62 \text{ m}$$

$$I_x = \frac{1}{12} (40 \times 10^{-3})^3 d = 5.3333 \times 10^{-6} d$$

$$I_y = \frac{1}{12} (40 \times 10^{-3}) d^3 = 3.3333 \times 10^{-5} d^3$$

$$|x| = \frac{1}{2} d, \quad |y| = 20 \text{ mm} = 20 \times 10^{-3} \text{ m} \quad |e_x| = 18 \text{ mm} = 18 \times 10^{-3} \text{ m}$$

$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

Assume $d > 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$. Then $I_{\min} = I_x$.

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333 \times 10^{-6} d}{3.3333 \times 10^{-3} d}} = 11.547 \times 10^{-3} \text{ m} \quad \frac{L_e}{r} = 140.29 > 133.22$$

$$\sigma_{cr} = \frac{0.877 \pi^2 E}{(L_e/r)^2} = \frac{0.877 \pi^2 (200 \times 10^9)}{(140.29)^2} = 87.958 \text{ MPa}$$

$$\sigma_{all, \text{centric}} = \frac{\sigma_{cr}}{1.67} = 52.67 \text{ MPa}$$

$$\sigma_{all, \text{bending}} = 150 \text{ MPa}$$

$$\frac{P}{A \sigma_{all, \text{centric}}} + \frac{P e_x}{I_y \sigma_{all, \text{bending}}} = 1$$

$$\frac{128 \times 10^3}{(40 \times 10^{-3} d)(52.67 \times 10^6)} + \frac{(128 \times 10^3)(18 \times 10^{-3})(\frac{1}{2} d)}{(3.3333 \times 10^{-5} d^3)(150 \times 10^6)} = 1$$

$$\frac{60.756 \times 10^{-3}}{d} + \frac{2.304 \times 10^{-3}}{d^2} = 1$$

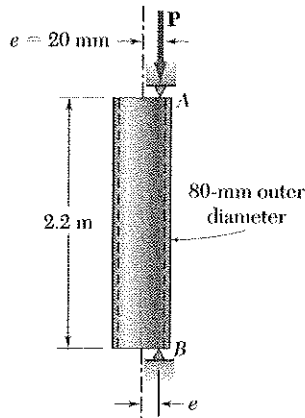
$$d^2 - 60.756 \times 10^{-3} d - 2.304 \times 10^{-3} = 0$$

$$d = \frac{1}{2} \left\{ 60.756 \times 10^{-3} + \sqrt{(60.756 \times 10^{-3})^2 + (4)(2.304 \times 10^{-3})} \right\}$$

$$d = 87.2 \times 10^{-3} \text{ m} > 40 \times 10^{-3} \text{ m}$$

$$d = 87.2 \text{ mm} \quad \blacktriangleleft$$

Problem 10.111



10.111 A steel tube of 80-mm outer diameter is to carry a 93-kN load P with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable-stress method, determine the lightest tube that can be used. Assume $E = 200$ GPa and $\sigma_Y = 250$ MPa.

$$r_o = \frac{1}{2} d_o = 40 \text{ mm}, \quad r_i = r_o - t$$

$$A = \pi (r_o^2 - r_i^2), \quad I = \frac{\pi}{4} (r_o^4 - r_i^4) \quad r = \sqrt{\frac{I}{A}}$$

t (mm)	r_i (mm)	A (mm ²)	I (10 ⁶ mm ⁴)	r (mm)
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$$L_e = 2.2 \text{ m}$$

$$P = 93 \times 10^3 \text{ N}$$

$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$\text{Try } t = 9 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < 133.22$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(86.92)^2} = 261.27 \text{ MPa}$$

$$\sigma_{all} = \frac{\sigma_e}{1.67} = \frac{1}{1.67} [0.658^{250/261.27}] (250) = 100.30 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2007 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.285 \times 10^{-6}} = 104.2 \text{ MPa} > 100.3 \text{ MPa}$$

(not allowed)

$$\text{Approximate required area: } \left(\frac{104.2}{100.3} \right) (2007) = 2085 \text{ mm}^2$$

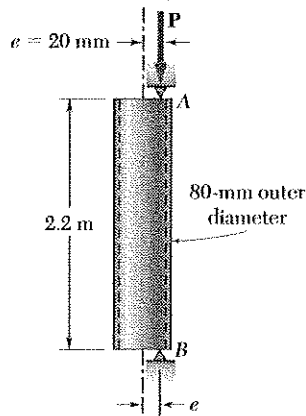
$$\text{Try } t = 12 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < 133.22$$

$$\sigma_e = 243.05 \text{ MPa} \quad \sigma_{all} = 97.33 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2564 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.528 \times 10^{-6}} = 85.0 \text{ MPa} < 97.3 \text{ MPa}$$

$$\text{Use } t = 12 \text{ mm.} \quad \blacktriangleleft$$

Problem 10.112



10.112 Solve Prob. 10.111, using the interaction method with $P = 165 \text{ kN}$, $e = 15 \text{ mm}$, and an allowable stress in bending of 150 MPa .

10.111 A steel tube of 80-mm outer diameter is to carry a 93-kN load P with an eccentricity of 20 mm . The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm . Using the allowable-stress method, determine the lightest tube that can be used. Assume $E = 200 \text{ GPa}$ and $\sigma_Y = 250 \text{ MPa}$.

$$r_o = \frac{1}{2} d_o = 40 \text{ mm} \quad r_i = r_o - t$$

$$A = \pi (r_o^2 - r_i^2) \quad I = \frac{\pi}{4} (r_o^4 - r_i^4) \quad r = \sqrt{\frac{I}{A}}$$

t (mm)	r_i (mm)	A (mm ²)	I (10 ⁶ mm ⁴)	r (mm)
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$$L_e = 2.2 \text{ m}$$

$$P = 165 \times 10^3 \text{ N}$$

$$\sigma_{\text{all, bending}} = 150 \text{ MPa}$$

$$\text{transition } \frac{L_e}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$\text{Try } t = 9 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < 133.22$$

$$\sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(86.92)^2} = 261.27 \text{ MPa}$$

$$\sigma_{\text{all, centric}} = \frac{\sigma_Y}{1.67} = \frac{1}{1.67} [0.658^{250/261.27}] (250) = 100.30 \text{ MPa}$$

$$\frac{P}{A \sigma_{\text{all, centric}}} + \frac{P e c}{I \sigma_{\text{all, bending}}} = \frac{165 \times 10^3}{(2007 \times 10^{-6})(100.3 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.285 \times 10^{-6})(150 \times 10^6)}$$

$$= 0.820 + 0.514 = 1.334 > 1 \quad (\text{not allowed})$$

$$\text{Approximate required area: } A = (1.334)(2007) = 2677 \text{ mm}^2$$

$$\text{Try } t = 12 \text{ mm. } \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < 133.22$$

$$\sigma_c = 243.05 \text{ MPa}$$

$$\sigma_{\text{all, centric}} = 97.33 \text{ MPa}$$

$$\frac{P}{A \sigma_{\text{all, centric}}} + \frac{P e c}{I \sigma_{\text{all, bending}}} = \frac{165 \times 10^3}{(2564 \times 10^{-6})(97.33 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.528 \times 10^{-6})(150 \times 10^6)}$$

$$0.661 + 0.432 = 1.093 > 1 \quad (\text{not allowed})$$

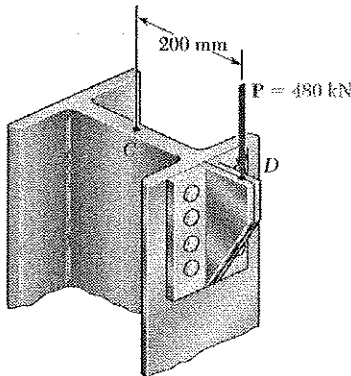
$$\text{Try } t = 15 \text{ mm. } \frac{L_e}{r} = 93.26 \quad \sigma_c = 226.95 \text{ MPa} \quad \sigma_{\text{all, centric}} = 94.40 \text{ MPa}$$

$$\frac{165 \times 10^3}{(3063 \times 10^{-6})(94.40 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.704 \times 10^{-6})(150 \times 10^6)} = 0.958 < 1$$

$$\text{Use } t = 15 \text{ mm.}$$

Problem 10.113

10.113 A steel column having a 7.2-m effective length is loaded eccentrically as shown. Using the allowable stress method, select the wide-flange shape of 350-mm nominal depth that should be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.



$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$L = 7.2 \text{ m}$$

If column is short and loading is centric

$$F.S. = \frac{5}{3} \quad \sigma_{all} = \frac{\sigma_Y}{F.S.} = \frac{250}{5/3} = 150 \text{ MPa}$$

$$A > \frac{P}{\sigma_{all}} = \frac{480000}{150} = 3200 \text{ mm}^2$$

If column is long and loading is centric, $F.S. = 1.92$

$$P < \frac{P_{cr}}{1.92} = \frac{\pi^2 E I_y}{1.92 L^2}, \quad I_y > \frac{1.92 P L^2}{\pi^2 E} = \frac{(1.92)(480000)(7200)^2}{\pi^2 (200000)} = 24.2 \times 10^6 \text{ mm}^4$$

$$\frac{P}{A} + \frac{P e c}{I_x} = \frac{P}{A} + \frac{P e}{S_x} < \sigma_{all}$$

Try W 360 X 101 $A = 12900 \text{ mm}^2$, $I_y = 50.6 \times 10^6 \text{ mm}^4$, $r_y = 62.6 \text{ mm}$, $S_x = 1690000 \text{ mm}^3$

$$\frac{L}{r_y} = \frac{7200}{62.6} = 115 < C_c \quad \frac{L/r_y}{C_c} = 0.9149$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.9149) - \frac{1}{8}(0.9149)^2 = 1.9051$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L/r_y}{C_c} \right)^2 \right] = \frac{250}{1.9051} \left[1 - \frac{1}{2} (0.9149)^2 \right] = 76.3 \text{ MPa}$$

$$\frac{P}{A} + \frac{P e}{S_x} = \frac{480000}{12900} + \frac{(480000)(200)}{1690000} = 90 \text{ MPa} > \sigma_{all} \quad \text{unsafe}$$

Assuming that σ_{max} is approximately proportional to $\frac{1}{A}$, the required area is

$$A = (12900) \frac{90}{76.3} = 15216.3 \text{ mm}^2$$

Try W 360 X 216 $A = 27600 \text{ mm}^2$, $S_x = 3860000 \text{ mm}^3$, $r_y = 101 \text{ mm}$

$$\frac{L}{r_y} = 71.29 \quad \frac{L/r_y}{C_c} = 0.5671 \quad F.S. = 1.8391 \quad \sigma_{all} = 114.08 \text{ MPa}$$

$$\frac{P}{A} + \frac{P e}{S_x} = 56.8 \text{ MPa} < 114.08 \text{ MPa} \quad \text{safe}$$

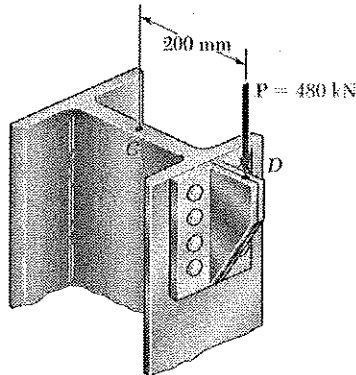
Try W 360 X 122 $A = 15500 \text{ mm}^2$, $S_x = 2010000 \text{ mm}^3$, $r_y = 63 \text{ mm}$

$$\frac{L}{r_y} = 114.3 \quad \frac{L/r_y}{C_c} = 0.9093 \quad F.S. = 1.9043 \quad \sigma_{all} = 77 \text{ MPa}$$

$$\frac{P}{A} + \frac{P e}{S_x} = 78.7 \text{ MPa} > 77 \text{ MPa} \quad \text{unsafe}$$

Use W 360 X 216

Problem 10.114



10.114 Solve Prob. 10.113 using the interaction method, assuming that $\sigma_Y = 350$ MPa and an allowable stress in bending of 200 MPa.

10.113 A steel column having a 7.2-m effective length is loaded eccentrically as shown. Using the allowable stress method, select the wide-flange shape of 350-mm nominal depth that should be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$\text{Steel: } C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{350}} = 106.2$$

$$L_e = 7.2 \text{ m}$$

$$\text{Assume } \frac{L}{r_y} > 106.2$$

$$r_y < \frac{L}{106.2} = \frac{7200}{106.2} = 67.8$$

$$\sigma_{all,c} = \frac{\pi^2 E}{1.92 (L/r_y)^2} = \frac{\pi^2 E r_y^2}{1.92 L^2}$$

$$\text{Interaction formula: } \frac{P}{A \sigma_{all,c}} + \frac{P e c}{I_x \sigma_{all,b}} < 1 \quad (1)$$

The first term of inequality (1) becomes

$$\frac{1.92 P L^2}{\pi^2 E A r_y^2} = \frac{1.92 P L^2}{\pi^2 E I_y} = \frac{(1.92)(480000)(7200)^2}{\pi^2 (200000) I_y} = \frac{24.2 \times 10^6}{I_y}$$

The second term of inequality (1) becomes

$$\frac{P e}{S_x \sigma_{all,b}} = \frac{(480000)(200)}{(S_x)(200)} = \frac{0.457 \times 10^6}{S_x}$$

$$\text{Then inequality (1) becomes } \frac{24.2 \times 10^6}{I_y} + \frac{0.457 \times 10^6}{S_x} = \beta < 1$$

$$\text{Obviously } I_y > 24.2 \times 10^6 \text{ mm}^4 \text{ and } S_x > 457 \times 10^3 \text{ mm}^3$$

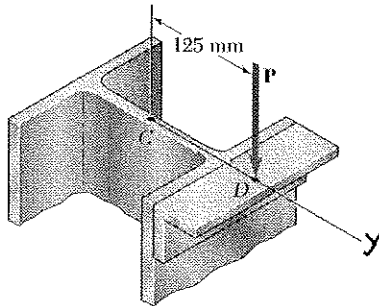
Shape	I_y (mm ⁴)	S_x (x10 ³ mm ³)	β
W200 x 71	25.4 x 10 ⁶	709	> 1
W360 x 101	50.6 x 10 ⁶	1690	0.75

W360 x 101 is the lightest shape with $\beta < 1$

Use W360 x 101

Problem 10.115

10.115 A steel compression member of 5.8-m effective length is to support a 296-kN eccentric load P . Using the interaction method, select the wide-flange shape of 200-mm nominal depth that should be used. Use $E = 200$ GPa, $\sigma_Y = 250$ MPa, and $\sigma_{all} = 150$ MPa in bending.



$$\text{transition } \frac{L}{r} : 4.71 \sqrt{\frac{E}{\sigma_Y}} \quad L_e = 5.8 \text{ m}$$

$$4.71 \sqrt{\frac{200 \times 10^9}{250 \times 10^6}} = 133.22$$

$$\text{At transition: } \frac{5.8}{r_y} = 133.22$$

$$r_y = 45.4 \times 10^{-3} \text{ m} = 45.4 \text{ mm}$$

For 200 mm nominal depth wide flange sections,

$$c = \frac{1}{2}d \approx 104 \text{ mm} \quad r_x \approx 88 \text{ mm} \quad \frac{ec}{r_x^2} \approx \frac{(125)(104)}{(88)^2} = 1.68$$

$$\text{For } r_y = 45.4 \text{ mm, } \sigma_{all, \text{centric}} = \frac{0.877 \pi^2 E}{(1.67)(133.22)^2} = 58.4 \times 10^6 \text{ Pa}$$

$$\text{Interaction formula: } \frac{P}{A \sigma_{all, \text{centric}}} + \frac{Pec}{A r_x^2 \sigma_{all, \text{bending}}} < 1$$

$$A > P \left[\frac{1}{\sigma_{all, \text{centric}}} + \frac{ec/r_x^2}{\sigma_{all, \text{bending}}} \right] = 296 \times 10^3 \left(\frac{1}{58.4 \times 10^6} + \frac{1.68}{150 \times 10^6} \right) \\ = 8.38 \times 10^{-3} \text{ m}^2 = 8380 \text{ mm}^2$$

$$\text{Try W200} \times 59. \quad A = 7560 \text{ mm}^2, \quad S_x = 582 \times 10^3 \text{ mm}^3 \quad r_y = 51.9 \text{ mm}$$

$$\frac{L_e}{r_y} = \frac{5.8}{51.9 \times 10^{-3}} = 111.75 < 133.22 \quad \sigma_c = \frac{\pi^2 E}{(L_e/r)^2} = 158.06 \text{ MPa}$$

$$\sigma_{all, \text{centric}} = \frac{\sigma_c}{1.67} = \frac{1}{1.67} [0.658^{250/158.06}] (250) = 77.217 \text{ MPa}$$

$$\frac{P}{A \sigma_{all, \text{centric}}} + \frac{Pe}{S_x \sigma_{all, \text{bending}}} = \frac{296 \times 10^3}{(7560 \times 10^{-6})(77.217 \times 10^6)} + \frac{(296 \times 10^3)(125 \times 10^{-3})}{(582 \times 10^{-6})(150 \times 10^6)} \\ = 0.5075 + 0.4238 = 0.9313 < 1 \quad (\text{allowed})$$

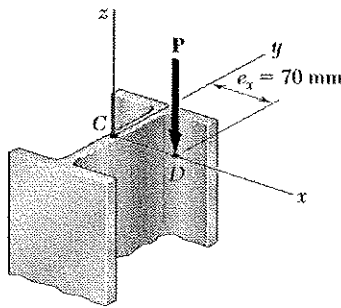
$$\text{Try W200} \times 52. \quad A = 6660 \text{ mm}^2 \quad S_x = 512 \times 10^3 \text{ mm}^3 \quad r_y = 51.7$$

$$\frac{L_e}{r_y} = 112.186 \quad \sigma_c = 156.84 \text{ MPa} \quad \sigma_{all, \text{centric}} = 76.821 \text{ MPa}$$

$$\frac{296 \times 10^3}{(6660 \times 10^{-6})(76.821 \times 10^6)} + \frac{(296 \times 10^3)(125 \times 10^{-3})}{(512 \times 10^{-6})(150 \times 10^6)} = 1.060 > 1 \quad (\text{not allowed})$$

Use W200 $\times$ 59

Problem 10.116



10.116 A steel column of 7.2-m effective length is to support an 83-kN eccentric load P at a point D , located on the x axis as shown. Using the allowable-stress method, select the wide-flange shape of 250-mm nominal depth that should be used. Use $E = 200$ GPa and $\sigma_y = 250$ MPa.

$$\text{transition } \frac{L}{r} = 4.71 \sqrt{\frac{E}{\sigma_y}} = 133.22$$

$$\text{At the transition: } \frac{7.2}{r_y} = 133.22$$

$$r_y = 54.05 \times 10^{-3} \text{ m} = 54.05 \text{ mm}$$

$$\text{All sections meet } \frac{L}{r_y} > 133.22.$$

$$\sigma_{all} = \frac{\Phi_{cr}}{1.67} = \frac{0.877 \pi^2 E}{1.67 (L/r_y)^2} = \frac{0.877 \pi^2 (200 \times 10^9)}{(1.67)(7.2/r_y)^2} = 19.9962 \times 10^9 r_y^2$$

$$\text{Try } W 250 \times 49.1 \quad A = 6250 \text{ mm}^2, \quad I_y = 15.1 \times 10^6 \text{ mm}^4$$

$$S_y = 150 \times 10^3 \text{ mm}^3 \quad r_y = 49.2 \text{ mm}$$

$$\sigma_{all} = (19.9962 \times 10^9)(49.2 \times 10^{-3})^2 = 48.404 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe}{S_y} = \frac{83 \times 10^3}{6250 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})}{150 \times 10^{-6}} = 52.013 \text{ MPa} > \sigma_{all}$$

(not allowed)

$$\text{Approximate required area: } A \approx \frac{52.013}{48.404} (6250) = 6716 \text{ mm}^2$$

$$\text{Try } W 250 \times 58 \quad A = 7420 \text{ mm}^2 \quad S_y = 185 \times 10^3 \text{ mm}^3 \quad r_y = 50.3 \text{ mm}$$

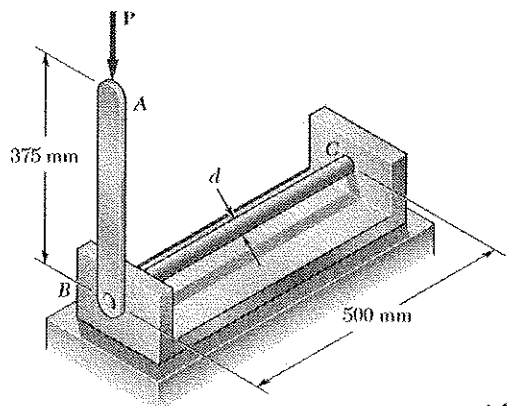
$$\sigma_{all} = (19.9962 \times 10^9)(50.3 \times 10^{-3})^2 = 50.592 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe}{S_y} = \frac{83 \times 10^3}{7420 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})}{185 \times 10^{-6}} = 42.591 \text{ MPa} < \sigma_{all}$$

Use $W 250 \times 58$ ▶

Problem 10.117

10.117 The steel rod BC is attached to the rigid bar AB and to the fixed support at C . Knowing that $G = 77 \text{ GPa}$, determine the critical load P_{cr} of the system when $d = 12 \text{ mm}$.



Look at torsion spring BC .

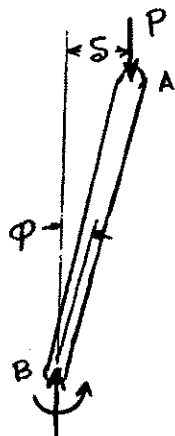
$$\phi = \frac{TL}{GJ} \quad T = \frac{GJ}{L} \phi = K\phi$$

$$G = 77 \text{ GPa}$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi}{2} (6)^4 = 2035.7 \text{ mm}^4$$

$$L = 0.5 \text{ m}$$

$$K = \frac{GJ}{L} = \frac{(77 \times 10^9)(2035.7 \times 10^{-12})}{0.5} = 313.5 \text{ Nm/rad.}$$



$$s = l \sin \phi = 0.375 \sin \phi$$

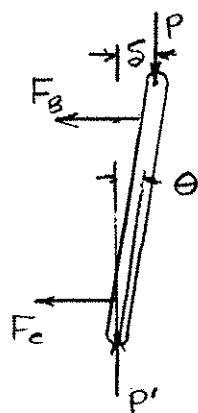
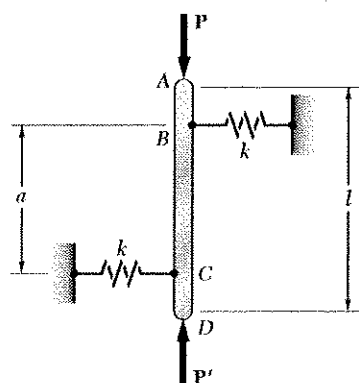
$$+\circlearrowleft \sum M_B = 0: -Ps + T = 0$$

$$P = \frac{T}{s} = \frac{K\phi}{l \sin \phi} = \frac{313.5 \phi}{0.375 \sin \phi} = 836 \frac{\phi}{\sin \phi} \text{ N}$$

$$\text{Let } \phi \rightarrow 0.$$

$$P_{cr} = 836 \text{ N} \quad \blacktriangleleft$$

Problem 10.118



10.118 The rigid bar AD is attached to two springs of constant k and is in equilibrium in the position shown. Knowing that the equal and opposite loads P and P' remain vertical, determine the magnitude P_{cr} of the critical load for the system. Each spring can act in either tension or compression.

Let x_B and x_C be the deflections of points B and C , positive $\rightarrow$.

Then, $F_B = -kx_B$ and $F_C = -kx_C$

$$\pm \sum F_x = 0: -F_B - F_C = 0 \quad F_C = -F_B$$

$$x_C = -x_B \quad F_B \text{ and } F_C \text{ form a couple } \curvearrowright.$$

Let θ be the angle change. $y_B = -y_C = \frac{1}{2}a \sin \theta$

$$s = l \sin \theta$$

P and P' form a couple Ps

$$+\circlearrowleft \sum M = 0: k\left(\frac{1}{2}a \sin \theta\right)a \cos \theta - Pl \sin \theta = 0$$

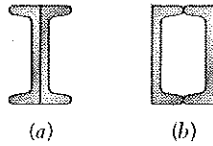
$$P = \frac{ka^2 \cos \theta}{2l}$$

Let $\theta \rightarrow 0$

$$P_{cr} = \frac{ka^2}{2l} \quad \blacktriangle$$

Problem 10.119

10.119 A column of 3-m effective length is to be made by welding together two C130 × 13 rolled-steel channels. Using $E = 200$ GPa, determine for each arrangement shown the allowable centric load if a factor of safety of 2.4 is required.



For channel C 130 × 13, $A = 1710 \text{ mm}^2$ $b_x = 48 \text{ mm}$
 $I_x = 3.70 \times 10^6 \text{ mm}^4$ $I_y = 0.264 \times 10^6 \text{ mm}^4$ $\bar{x} = 12.2 \text{ mm}$

Arrangement (a). $I_x = (2)(3.70 \times 10^6) = 7.40 \times 10^6 \text{ mm}^4$

$$I_y = 2[0.264 \times 10^6 + (1710)(12.2)^2] = 1.0370 \times 10^6 \text{ mm}^4$$

$$I_{\min} = I_y = 1.0370 \times 10^6 \text{ mm}^4 = 1.0370 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI_{\min}}{L_c^2} = \frac{\pi^2 (200 \times 10^9)(1.0370 \times 10^{-6})}{(3.0)^2} = 227 \times 10^3 \text{ N} = 227 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{227}{2.4}$$

$$P_{all} = 94.8 \text{ kN} \quad \blacktriangleleft$$

Arrangement (b). $I_x = (2)(3.70 \times 10^6) \text{ mm}^4$

$$I_y = 2[0.264 \times 10^6 + (1710)(48 - 12.2)^2] = 4.911 \times 10^6 \text{ mm}^4$$

$$I_{\min} = I_y = 4.911 \times 10^6 \text{ mm}^4 = 4.911 \times 10^{-6} \text{ m}^4$$

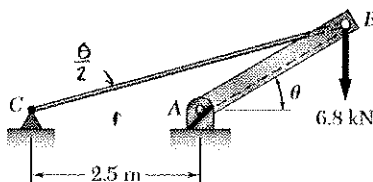
$$P_{cr} = \frac{\pi^2 EI_{\min}}{L_c^2} = \frac{\pi^2 (200 \times 10^9)(4.911 \times 10^{-6})}{(3.0)^2} = 1077 \times 10^3 \text{ N} = 1077 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{1077}{2.4}$$

$$P_{all} = 449 \text{ kN} \quad \blacktriangleleft$$

Problem 10.120

10.120 Member AB consists of a single C130 × 10.4 steel channel of length 2.5 m. Knowing that the pins A and B pass through the centroid of the cross section of the channel, determine the factor of safety for the load shown with respect to buckling in the plane of the figure when $\theta = 30^\circ$. Use $E = 200$ GPa.



Since $AB = 2.5 \text{ m}$, triangle ABC is isosceles.

$$+\uparrow \sum F_y = 0:$$

$$F_{AB} \sin 30^\circ - F_{AC} \sin 15^\circ - 6.8 = 0$$

$$F_{AB} \left(\sin 30^\circ - \frac{\sin 15^\circ \cos 30^\circ}{\cos 15^\circ} \right) = 0.26795 F_{AB} = 6.8$$

$$F_{AB} = 25.378 \text{ kN}$$

$$C 130 \times 10.4$$

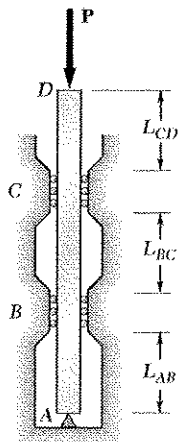
$$I_{\min} = 0.229 \times 10^6 \text{ mm}^4 = 0.229 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI_{\min}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9)(0.229 \times 10^{-6})}{(2.5)^2} = 72.324 \times 10^3 \text{ N} = 72.324 \text{ kN}$$

$$F.S. = \frac{P_{cr}}{F_{AB}} = \frac{72.324}{25.378}$$

$$F.S. = 2.85 \quad \blacktriangleleft$$

Problem 10.121



10.121 A 25-mm-square aluminum strut is maintained in the position shown by a pin support at *A* and by sets of rollers at *B* and *C* that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 0.9$ m, $L_{BC} = 1.2$ m, and $L_{CD} = 0.3$ m, determine the allowable load **P** using a factor of safety with respect to buckling of 3.2. Consider only buckling in the plane of the figure and use $E = 72$ GPa.

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(25)^3 = 32552 \text{ mm}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \quad P_{all} = \frac{(P_{cr})_{min}}{F.S.} = \frac{\pi^2 EI}{(F.S.)(L_e)_{max}^2}$$

$$\text{Portion AB: } L_e = 0.7L_{AB} = (0.7)(0.9) = 0.63 \text{ m}$$

$$\text{Portion BC: } L_e = 0.5L_{BC} = (0.5)(1.2) = 0.6 \text{ m}$$

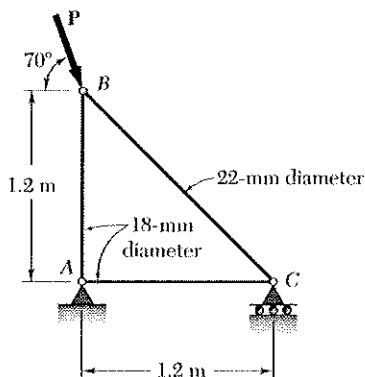
$$\text{Portion CD } L_e = 2L_{CD} = (2)(0.3) = 0.6 \text{ m}$$

$$(L_e)_{max} = 0.63 \text{ m}$$

$$P_{all} = \frac{\pi^2 (72 \times 10^9) (32552 \times 10^{-12})}{(3.2)(0.63)^2} = 18212.9 \text{ N} = 18.2 \text{ kN}$$

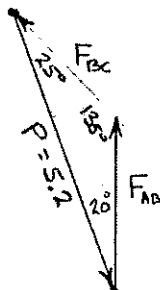
Problem 10.122

10.122 Knowing that $P = 5.2$ kN, determine the factor of safety for the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



Joint B :

From force triangle,



$$\frac{F_{AB}}{\sin 25^\circ} = \frac{F_{BC}}{\sin 20^\circ} = \frac{5.2}{\sin 135^\circ}$$

$$F_{AB} = 3.1079 \text{ kN (comp)}$$

$$F_{BC} = 2.5152 \text{ kN (comp)}$$

$$\text{Member AB: } I_{AB} = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} \left(\frac{18}{2} \right)^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$$

$$F_{AB,cr} = \frac{\pi^2 E I_{AB}}{L_{AB}^2} = \frac{\pi^2 (200 \times 10^9) (5.153 \times 10^{-9})}{(1.2)^2} = 7.0636 \times 10^3 \text{ N} = 7.0636 \text{ kN}$$

$$F.S. = \frac{F_{AB,cr}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

$$\text{Member BC: } I_{BC} = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} \left(\frac{22}{2} \right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$$

$$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88 \text{ m}^2$$

$$F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9) (11.499 \times 10^{-9})}{2.88} = 7.8813 \times 10^3 \text{ N} = 7.8813 \text{ kN}$$

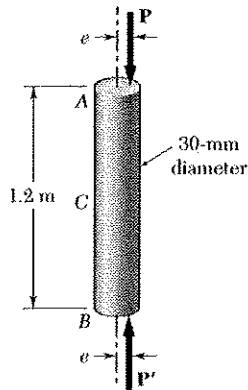
$$F.S. = \frac{F_{BC,cr}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest F.S. governs.

$$F.S. = 2.27$$

Problem 10.123

10.123 An axial load P is applied to the 30-mm-diameter steel rod AB as shown. For $P = 35$ kN and $e = 1.5$ mm, determine (a) the deflection at the midpoint C of the rod, (b) the maximum stress in the rod. Use $E = 200$ GPa.



$$c = \frac{1}{2}d = \frac{30}{2} = 15 \text{ mm}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(15)^4 = 39761 \text{ mm}^4$$

$$L_e = L = 1.2 \text{ m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9) (39761 \times 10^{-12})}{(1.2)^2} = 54.5 \text{ kN}$$

$$\frac{P}{P_{cr}} = \frac{35}{54.5} = 0.6422$$

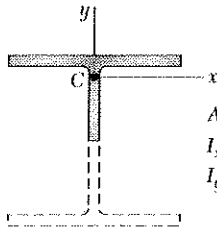
$$(a) \ y_{max} = e \left[\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = 1.5 [2.2577] = 3.4 \text{ mm}$$

$$M_{max} = P(e + y_{max}) = (35)(1.5 + 3.4) = 171.5 \text{ N}\cdot\text{m}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(30)^2 = 706.9 \text{ mm}^2$$

$$(b) \ \sigma_{max} = \frac{P}{A} + \frac{M_c}{I} = \frac{35000}{706.9} + \frac{(171.5 \times 10^3)(15)}{39761} = 114.21 \text{ MPa} = 114.2 \text{ MPa}$$

Problem 10.124



$$\begin{aligned} A &= 13.8 \times 10^3 \text{ mm}^2 \\ I_x &= 26.0 \times 10^6 \text{ mm}^4 \\ I_y &= 142.0 \times 10^6 \text{ mm}^4 \end{aligned}$$

10.124 A column is made from half of a W360 x 216 rolled-steel shape, with the geometric properties as shown. Using allowable stress design, determine the allowable centric load if the effective length of the column is (a) 4.0 m, (b) 6.5 m. Use $\sigma_Y = 345 \text{ MPa}$ and $E = 200 \text{ GPa}$.

$$\begin{aligned} r &= \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{26.0 \times 10^6}{13.8 \times 10^3}} = 43.406 \text{ mm} \\ &= 43.406 \times 10^{-3} \text{ m} \\ A &= 13.8 \times 10^{-3} \text{ m}^2 \end{aligned}$$

$$\text{transition } L/r: 4.71 \sqrt{\frac{E}{\sigma_Y}} = 4.71 \sqrt{\frac{200 \times 10^9}{345 \times 10^6}} = 113.4$$

$$(a) \quad L_e = 4.0 \text{ m}, \quad \frac{L_e}{r} = \frac{4.0}{43.406 \times 10^{-3}} = 92.153 < 113.4$$

$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(92.153)^2} = 232.44 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{1}{F.S.} \sigma_{cr} = \frac{1}{1.67} \left[0.658^{345/232.44} \right] (345) = 111.0 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (111.0 \times 10^6) (13.8 \times 10^{-3}) \quad P_{\text{all}} = 1532 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad L_e = 6.5 \text{ m}, \quad \frac{L_e}{r} = \frac{6.5}{43.4 \times 10^{-3}} = 149.77 > 113.4$$

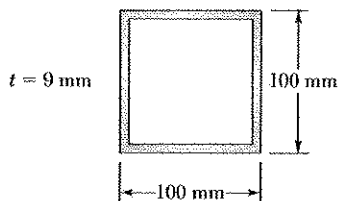
$$\sigma_e = \frac{\pi^2 E}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^9)}{(149.77)^2} = 88.0 \text{ MPa}$$

$$\sigma_{\text{all}} = \frac{1}{F.S.} \sigma_{cr} = \frac{1}{1.67} \left[(0.877)(88.0) \right] = 46.21 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (46.21 \times 10^6) (13.8 \times 10^{-3}) \quad P_{\text{all}} = 638 \text{ kN} \quad \blacktriangleleft$$

Problem 10.125

10.125 A compression member has the cross section shown and an effective length of 1.5 m. Knowing that the aluminum alloy used is 2014-T6, determine the allowable centric load.



$$b_o = 100 \text{ mm} \quad b_i = b_o - 2t = 82 \text{ mm}$$

$$A = (100)^2 - (82)^2 = 3276 \text{ mm}^2$$

$$I = \frac{1}{12} [(100)^4 - (82)^4] = 4565652 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 37.3 \text{ mm}$$

$$L_e = 1.5 \text{ m}$$

$$\frac{L}{r} = \frac{1500}{37.3} = 40.21 < 55 \quad \text{for 2014-T6 aluminum alloy.}$$

$$\sigma_{\text{all}} = 212 - 1.585(L/r) = 212 - (1.585)(40.21) = 148.3 \text{ MPa}$$

$$P_{\text{all}} = \sigma_{\text{all}} A = (148.3) (3276) = 485.8 \text{ kN} \quad \blacktriangleleft$$

Problem 10.126

10.126 A column of 5-m effective length must carry a centric load of 950 kN. Using allowable stress design, select the wide-flange shape of 250-mm nominal depth that should be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

$$P < \frac{\sigma_Y A}{F.S.} \quad A > \frac{(F.S.)P}{\sigma_Y} = \frac{(5/3)(950000)}{250 \times 10^6} = 6.33 \times 10^{-3} \text{ m}^2$$

$$L_e = 5 \text{ m} \quad E = 200 \text{ GPa}$$

$$P < \frac{\pi^2 EI}{1.92 L^2} \quad I > \frac{1.92 P L_e^3}{\pi^2 E} = \frac{(1.92)(950 \times 10^3)(5)^3}{\pi^2 (200 \times 10^9)} = 23.1 \times 10^{-6} \text{ m}^4$$

Try W250x80. $A = 10200 \text{ mm}^2$ $I_y = 43.1 \times 10^6 \text{ mm}^4$ $r = 65 \text{ mm}$.

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$\frac{L_e}{r} = \frac{5000}{65} = 76.92 < C_c \quad \frac{L_e/r}{C_c} = \frac{76.92}{125.7} = 0.6119$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.6119) - \frac{1}{8}(0.6119)^3 = 1.8675$$

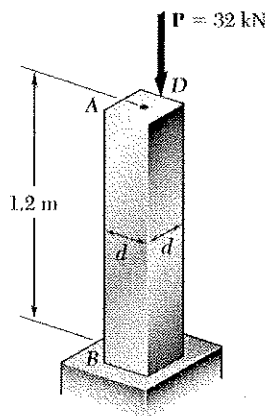
$$\sigma_{all} = \frac{\sigma_Y}{F.S.} \left[1 - \frac{1}{2} \left(\frac{L_e/r}{C_c} \right)^2 \right] = \frac{250}{1.8675} \left[1 - \frac{1}{2} (0.6119)^2 \right] = 108.8 \text{ MPa}$$

$$P_{all} = \sigma_{all} A = (108.8)(10200) = 1109.7 \text{ kN} > 950 \text{ kN}$$

Use W250x80

Problem 10.127

10.127 A 32-kN vertical load P is applied at the midpoint of one edge of the square cross section of the aluminum compression member AB that is free at its top A and fixed at its base B . Knowing that the alloy used is 6061-T6 and using the allowable-stress method, determine the smallest allowable dimension d .



$$L_e = 2L = 2.4 \text{ m}$$

$$A = d^2 \quad I = \frac{1}{12} d^4 \quad r = \sqrt{\frac{I}{A}} = \frac{1}{\sqrt{12}} d \quad c = \frac{1}{2} d \quad e = \frac{1}{2} d$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{P}{d^2} + \frac{P(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{1}{12}d^4} = \frac{4P}{d^2} = \sigma_{all}$$

Assume $L/r > 66$. $\sigma_{all} = \frac{B}{(L/r)^2}$ $B = 351 \times 10^9 \text{ Pa}$

$$\frac{Br^2}{L^2} = \frac{Bd^2}{12L^2} = \frac{4P}{d^2} \quad d^4 = \frac{48PL^2}{B}$$

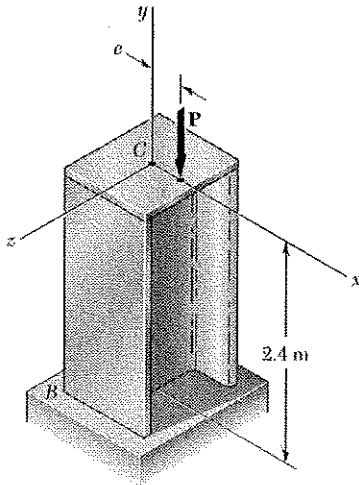
$$d = \sqrt[4]{\frac{48PL^2}{B}} = \sqrt[4]{\frac{(48)(32 \times 10^3)(2.4)^2}{351 \times 10^9}} = 70.9 \times 10^{-3} \text{ m}$$

$$r = \frac{70.9 \times 10^{-3}}{\sqrt{12}} = 20.45 \times 10^{-3} \text{ m}$$

$$\frac{L_e}{r} = \frac{2.4}{20.45 \times 10^{-3}} = 117.3 > 66$$

$$d = 70.9 \text{ mm}$$

Problem 10.128



10.128 A 180-kN axial load P is applied to the rolled-steel column BC at a point on the x axis at a distance $e = 62$ mm from the geometric axis of the column. Using the allowable-stress method, select the wide-flange shape of 200-mm nominal depth that should be used. Use $E = 200$ GPa and $\sigma_Y = 250$ MPa.

Steel: $E = 200 \text{ GPa}$ $\sigma_Y = 250 \text{ MPa}$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2 (200000)}{250}} = 125.7$$

$$L = 2.4 \text{ m}$$

$$L_e = 2L = 4.8 \text{ m}$$

Try W200x46.1 $r_y = 51.1 \text{ mm}$, $\frac{L_e}{r_y} = 93.93 < C_c$

$$\frac{L_e/r_y}{C_c} = 0.747$$

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}(0.747) - \frac{1}{8}(0.747)^3 = 1.894$$

$$\sigma_{\text{all}} = \frac{\sigma_Y}{\text{F.S.}} \left[1 - \frac{1}{2} \left(\frac{L_e/r_y}{C_c} \right)^2 \right] = \frac{250}{1.894} \left[1 - \frac{1}{2} (0.747)^2 \right] = 95.1 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{180000}{5860} + \frac{(180000)(62)(204/2)}{15.3 \times 10^6} = 104.75 \text{ MPa} > 95.1 \text{ MPa}$$

(not allowed)

Approximate required area = $\left(\frac{104.75}{95.1} \right) (5860) = 6454.6 \text{ mm}^2$

Try W200x52. $r_y = 51.7 \text{ mm}$ $\frac{L_e}{r_y} = 92.84 < C_c$ $\frac{L_e/r_y}{C_c} = 0.7386$

$$\text{F.S.} = 1.893 \quad \sigma_{\text{all}} = 96 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{180000}{6660} + \frac{(180000)(62)(204/2)}{17.8 \times 10^6} = 90.97 < 96 \text{ MPa} \quad (\text{allowed})$$

Use W200x52

PROBLEM 10.C1

10.C1 A solid steel rod having an effective length of 500 mm is to be used as a compression strut to carry a centric load P . For the grade of steel used $E = 200$ GPa and $\sigma_y = 245$ MPa. Knowing that a factor of safety of 2.8 is required and using Euler's formula, write a computer program and use it to calculate the allowable centric load P_{all} for values of the radius of the rod from 6 mm to 24 mm, using 2-mm increments.

SOLUTION

ENTER RADIUS RAD, EFFECTIVE LENGTH L_e
AND FACTOR OF SAFETY FS

COMPUTE RADIUS OF GYRATION

$$A = \pi \text{ RAD}^2$$

$$I = \frac{1}{4} \pi \text{ RAD}^4$$

$$r = \sqrt{\frac{I}{A}}$$

DETERMINE ALLOWABLE CENTRIC LOAD

CRITICAL STRESS:

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$$

LET σ EQUAL SMALLER OF σ_{cr} AND σ_y

$$P_{all} = \frac{\sigma A}{FS}$$

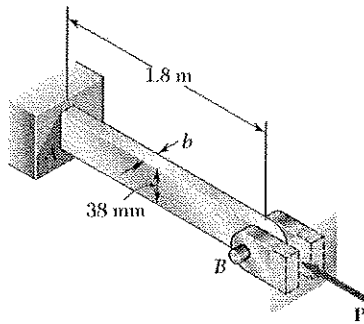
PROGRAM OUTPUT

Radius of rod m	Critical stress MPa	Allowable load kN
.006	71.1	2.87
.008	126.3	9.07
.010	197.4	22.15

.012	284.2	39.58
.014	386.9	53.88
.016	505.3	70.37
.018	639.6	89.06
.020	789.6	109.96
.022	955.4	133.05
.024	1137.0	158.34

Below the dashed line we have:
critical stress > yield strength

PROBLEM 10.C2



10.C2 An aluminum bar is fixed at end A and supported at end B so that it is free to rotate about a horizontal axis through the pin. Rotation about a vertical axis at end B is prevented by the brackets. Knowing that $E = 70$ GPa, use Euler's formula with a factor of safety of 2.5 to determine the allowable centric load P for values of b from 20 mm to 38 mm, using 3-mm increments.

SOLUTION

ENTER E , LENGTH L AND FACTOR OF SAFETY FS
FOR $b = 20$ mm TO 38 mm WITH 3 mm INCREMENTS

COMPUTE RADII OF GYRATION

$$A = 38 \ b$$

$$I_x = \frac{1}{12} b \ 38^3 \quad r_x = \sqrt{\frac{I_x}{A}}$$

$$I_y = \frac{19}{6} b^3 \quad r_y = \sqrt{\frac{I_y}{A}}$$

COMPUTE CRITICAL STRESSES

$$(\sigma_{cr})_x = \frac{\pi^2 E}{(0.7L/r_x)^2}$$

$$(\sigma_{cr})_y = \frac{\pi^2 E}{(0.5L/r_y)^2}$$

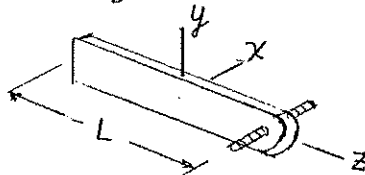
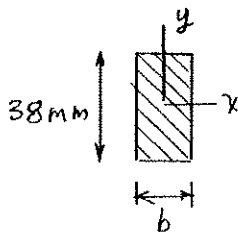
LET σ_{cr} EQUAL SMALLER STRESS

COMPUTE ALLOWABLE CENTRIC LOAD

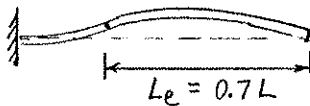
$$P_{all} = \frac{\sigma_{cr} A}{FS}$$

PROGRAM OUTPUT

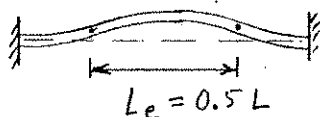
b mm	Critical stress x axis MPa	Critical stress y axis MPa	Allowable load kN
20	52.365	28.431	8.643
23	52.365	37.600	13.145
26	52.365	48.048	18.989
29	52.365	59.776	23.082
32	52.365	72.783	25.470
35	52.365	87.070	27.858
38	52.365	102.636	30.246



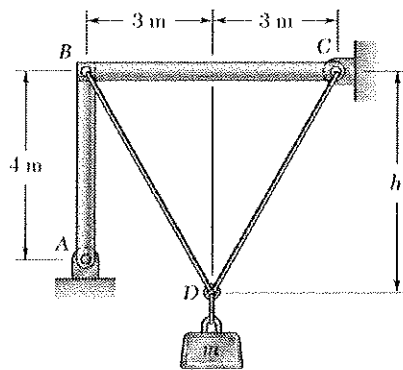
BUCKLING IN yz PLANE



BUCKLING IN xz PLANE



PROBLEM 10.C3



JOINT D:

$$\begin{aligned} & \begin{array}{c} h \Delta \\ 3 \end{array} \quad \begin{array}{c} \Delta h \\ 3 \end{array} \\ & \begin{array}{c} T \\ I \end{array} \quad \begin{array}{c} T \\ I \end{array} \\ & \begin{array}{c} W \\ \downarrow \end{array} \\ & \sum F_y = 0 \text{ YIELDS} \\ & T_y = \frac{1}{2} W \\ & \frac{T_x}{T_y} = \frac{3}{h} \text{ YIELDS} \\ & T_x = \frac{1.5 W}{h} \end{aligned}$$

JOINT B:

$$\begin{aligned} & \begin{array}{c} F_{BC} = \frac{1.5 W}{h} \\ \leftarrow \end{array} \\ & \begin{array}{c} F_{AB} = \frac{1}{2} W \\ \uparrow \end{array} \quad \begin{array}{c} T_x = \frac{1.5 W}{h} \\ \rightarrow \end{array} \\ & \begin{array}{c} T_y = \frac{1}{2} W \\ \downarrow \end{array} \end{aligned}$$

10.C3 The pin-ended members AB and BC consist of sections of aluminum pipe of 120-mm outer diameter and 10-mm wall thickness. Knowing that a factor of safety of 3.5 is required, determine the mass m of the largest block that can be supported by the cable arrangement shown for values of h from 4 m to 8 m, using 0.25-m increments. Use $E = 70$ GPa and consider only buckling in the plane of the structure.

SOLUTION

COMPUTE MOMENT OF INERTIA

$$I = \frac{\pi}{4} (0.06^4 - 0.05^4)$$

FOR $h = 4$ TO 8 USING 0.25 INCREMENTS

COMPUTE ALLOWABLE LOADS FOR MEMBERS

$$(F_{AB})_{cr} = \frac{\pi^2 EI}{3.5(4)^2}; (F_{BC})_{cr} = \frac{\pi^2 EI}{3.5(6)^2}$$

DETERMINE ALLOWABLE W

$$(W_{all})_1 = 2(F_{AB})_{cr}; (W_{all})_2 = \frac{h}{1.5}(F_{BC})_{cr}$$

W_{all} EQUALS SMALLER VALUE

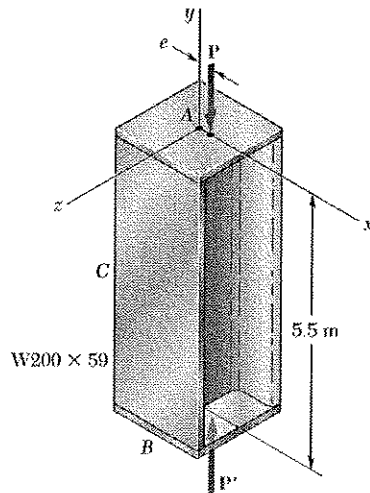
COMPUTE MASS m

$$m = \frac{W_{all}}{9.81}$$

PROGRAM OUTPUT

h	Weight critical stress AB	Weight critical stress BC	mass
m	kN	kN	kg
4.00	455.11	269.7	7854.88
4.25	455.11	286.6	8345.80
4.50	455.11	303.4	8836.74
4.75	455.11	320.3	9327.66
5.00	455.11	337.1	9818.59
5.25	455.11	354.0	10309.52
5.50	455.11	370.8	10800.45
5.75	455.11	387.7	11291.38
6.00	455.11	404.5	11782.31
6.25	455.11	421.4	12273.24
6.50	455.11	438.3	12764.17
6.75	455.11	455.1	13255.10
7.00	455.11	472.0	13255.10
7.25	455.11	488.8	13255.10
7.50	455.11	505.7	13255.10
7.75	455.11	522.5	13255.10
8.00	455.11	539.4	13255.10

PROBLEM 10.C4



10.C4 An axial load P is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W200 $\times$ 59 rolled-steel column AB . Using $E = 200$ GPa, write a computer program and use it to calculate for values of P from 100 to 300 kN, using 20-kN increments, (a) the horizontal deflection at the midpoint C , (b) the maximum stress in the column.

SOLUTION

ENTER LENGTH L , ECCENTRICITY e

ENTER PROPERTIES A, I_y, r_y, b_f

COMPUTE CRITICAL LOAD

$$P_{cr} = \frac{\pi^2 E I_y}{L^2}$$

FOR $P = 100$ kN TO 300 kN IN INCREMENTS OF 20 kN

COMPUTE HORIZONTAL DEFLECTION AT C

$$y_c = e \left(\sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1.0 \right)$$

COMPUTE MAXIMUM STRESS

$$\sigma_{max} = \frac{P}{A} \left(1 + \frac{e b_f}{2 r_y^2} \sec \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right)$$

PROGRAM OUTPUT

Load kN	maximum deflection mm	maximum stress MPa
100	1.205	19.874
160	2.042	32.680
200	2.655	41.560
240	3.329	50.923
300	4.473	65.723

PROBLEM 10.C5

10.C5 A column of effective length L is made from a rolled-steel shape and carries a centric axial load P . The yield strength for the grade of steel used is denoted by σ_y , the modulus of elasticity by E , the cross-sectional area of the selected shape by A , and its smallest radius of gyration by r . Using the AISC design formulas for allowable stress design, write a computer program to determine the allowable load P . Use this program to solve (a) Prob. 10.59, (b) Prob. 10.60, (c) Prob. 10.124.

SOLUTION

ENTER L, E, σ_y

ENTER PROPERTIES A, r_y

DETERMINE ALLOWABLE STRESS

$$C = 4.71 \sqrt{\frac{E}{\sigma_y}}$$

$$\text{IF } L/r_y \geq C$$

$$\sigma_{all} = \frac{0.877 \pi^2 E}{1.67 (L/r_y)^2}$$

$$\text{IF } L/r_y < C$$

$$\sigma_e = \frac{\pi^2 E}{(L/r_y)^2}$$

$$\sigma_{all} = \frac{1}{1.67} \left[0.658^{\sigma_y / \sigma_e} \right] \sigma_y$$

CALCULATE ALLOWABLE LOAD:

$$P_{all} = \sigma_{all} A$$

CONTINUED

PROBLEM 10.C5 CONTINUED

PROGRAM OUTPUT

Problem 10.60 a

Effective Length = 6m
A = 5860 mm²
ry = 51.5 mm
Yield strength = 250 MPa
E = 200000 MPa

Allowable centroid load: P = 431.296 kN

Problem 10.60 b

Effective Length = 6m
A = 5860 mm²
ry = 51.5 mm
Yield strength = 345 MPa
E = 200000 MPa

Allowable centroid load: P = 437.859 kN

Problem 10.60 a

Effective Length = 6.00 m
A = 4580.0 mm²
ry = 40.8 mm
Yield strength = 250.0 MPa
E = 200 GPa

Allowable centroid load: P = 217.727 kN

Problem 10.60 b

Effective Length = 6.00 m
A = 11000.0 mm²
ry = 53.2 mm
Yield strength = 250.0 MPa
E = 200 GPa

Allowable centroid load: P = 858.637 kN

Problem 10.124 a

Effective Length = 4.00 m
A = 13.8 mm²
ry = 43.4 mm
Yield strength = 345.0 MPa
E = 200 GPa

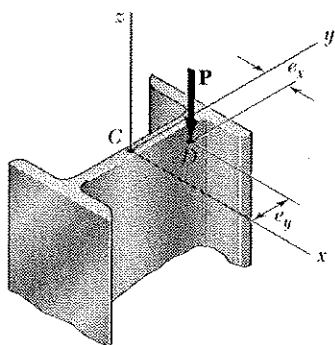
Allowable centroid load: P = 1532 kN

Problem 10.124 b

Effective Length = 6.50 m
A = 13800.0 mm²
ry = 43.4 mm
Yield strength = 345.0 MPa
E = 200 GPa

Allowable centroid load: P = 638.0 kN

PROBLEM 10.C6



10.C6 A column of effective length L is made from a rolled-steel shape and is loaded eccentrically as shown. The yield strength of the grade of steel used is denoted by σ_Y , the allowable stress in bending by σ_{all} , the modulus of elasticity by E , the cross-sectional area of the selected shape by A , and its smallest radius of gyration by r . Write a computer program to determine the allowable load P , using either the allowable-stress method or the interaction method. Use this program to check the given answer for (a) Prob. 10.113, (b) Prob. 10.114.

SOLUTION

ENTER $L, E, \sigma_Y, (\sigma_{all})_{bending}, e_x, e_y$

ENTER PROPERTIES A, s_x, s_y, r_y

DETERMINE ALLOWABLE STRESS

$$C = 4.71 \sqrt{\frac{E}{\sigma_Y}}$$

$$\text{IF } L/r_y \geq C$$

$$\sigma_{all} = \frac{0.877 \pi^2 E}{1.67 (L/r_y)^2}$$

$$\text{IF } L/r_y < C$$

$$\sigma_e = \frac{\pi^2 E}{(L/r_y)^2}$$

$$\sigma_{all} = \frac{1}{1.67} \left[0.658^{\sigma_Y/\sigma_e} \right] \sigma_Y$$

FOR ALLOWABLE-STRESS METHOD

$$COEF = \frac{1}{A} + \frac{e_x}{s_x} + \frac{e_y}{s_y}$$

$$P_{all} = \frac{\sigma_{all}}{COEF}$$

FOR INTERACTION METHOD

$$COEF = \frac{1}{A \sigma_{all}} + \frac{(e_x/s_x) + (e_y/s_y)}{(\sigma_{all})_{bending}}$$

$$P_{all} = \frac{1.0}{COEF}$$

CONTINUED

PROBLEM 10.C6 CONTINUED

PROGRAM OUTPUT

Problem 10.113

Effective Length = 7.2 m
A = 27600 mm²
R_Y = 101 mm
S_X = 3.8 x 10⁶ mm³
Yield strength = 250 MPa
E = 200000 MPa

Using Allowable-Stress Method
Allowable load: P = 1567.68 kN

Problem 10.114

Effective Length = 7.2 m
A = 12900 mm²
R_Y = 62.6 mm
S_X = 1690 x 10³ mm³
Yield strength = 350 MPa
E = 200000 MPa

Using Interaction Method
Allowable load: P = 1212.781 kN

Chapter 11

Problem 11.1**11.1** Determine the modulus of resilience for each of the following aluminum alloys:

- | | | |
|---------------|----------------------|------------------------------|
| (a) 1100-H14: | $E = 70 \text{ GPa}$ | $\sigma_Y = 55 \text{ MPa}$ |
| (b) 2014-T6 | $E = 72 \text{ GPa}$ | $\sigma_Y = 220 \text{ MPa}$ |
| (c) 6061-T6 | $E = 69 \text{ GPa}$ | $\sigma_Y = 150 \text{ MPa}$ |

Aluminum alloys

$$(a) \quad E = 70 \times 10^9 \text{ Pa} \quad \sigma_Y = 55 \times 10^6 \text{ Pa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(55 \times 10^6)^2}{(2)(70 \times 10^9)} = 21.6 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 21.6 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(b) \quad E = 72 \times 10^9 \text{ Pa} \quad \sigma_Y = 220 \times 10^6 \text{ Pa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(220 \times 10^6)^2}{(2)(72 \times 10^9)} = 336 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 336 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(c) \quad E = 69 \times 10^9 \text{ Pa} \quad \sigma_Y = 150 \times 10^6 \text{ Pa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(150 \times 10^6)^2}{(2)(69 \times 10^9)} = 163.0 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 163.0 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

Problem 11.2**11.2** Determine the modulus of resilience for each of the following grades of structural steel:

- | | |
|--------------------------|------------------------------|
| (a) ASTM A709 Grade 50: | $\sigma_Y = 350 \text{ MPa}$ |
| (b) ASTM A913 Grade 65: | $\sigma_Y = 450 \text{ MPa}$ |
| (c) ASTM A709 Grade 100: | $\sigma_Y = 700 \text{ MPa}$ |

Structural steel $E = 200 \text{ GPa}$ for all three steels given.

$$(a) \quad \sigma_Y = 350 \text{ MPa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(350 \times 10^6)^2}{(2)(200 \times 10^9)} = 306.25 \text{ N}\cdot\text{m}/\text{m}^3 = 306.3 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(b) \quad \sigma_Y = 450 \text{ MPa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(450 \times 10^6)^2}{(2)(200 \times 10^9)} = 506.25 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 506.3 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$(c) \quad \sigma_Y = 700 \text{ MPa}$$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(700 \times 10^6)^2}{(2)(200 \times 10^9)} = 1225 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 1225 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$= 1.225 \text{ MJ}/\text{m}^3$$

Problem 11.3

11.3 Determine the modulus of resilience for each of the following alloys:

- | | |
|-----------------------------|--|
| (a) Titanium: | $E = 115 \text{ GPa}$, $\sigma_Y = 875 \text{ MPa}$ |
| (b) Magnesium: | $E = 45 \text{ GPa}$, $\sigma_Y = 200 \text{ MPa}$ |
| (c) Cupronickel (annealed): | $E = 140 \text{ GPa}$, $\sigma_Y = 125 \text{ MPa}$ |

(a) $E = 115 \text{ GPa}$, $\sigma_Y = 875 \text{ MPa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(875 \times 10^6)^2}{(2)(115 \times 10^9)} = 3328.8 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 3328.8 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

$$= 3.33 \text{ MJ}/\text{m}^3$$

(b) $E = 45 \text{ GPa}$, $\sigma_Y = 200 \text{ MPa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(200 \times 10^6)^2}{(2)(45 \times 10^9)} = 444.4 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 444.4 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

(c) $E = 140 \text{ GPa}$, $\sigma_Y = 125 \text{ MPa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(125 \times 10^6)^2}{(2)(140 \times 10^9)} = 55.8 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 55.8 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

Problem 11.4

11.4 Determine the modulus of resilience for each of the following metals:

- | | | |
|-----------------------------|-----------------------|------------------------------|
| (a) Stainless steel | | |
| AISI 302 (annealed): | $E = 190 \text{ GPa}$ | $\sigma_Y = 260 \text{ MPa}$ |
| (b) Stainless steel 2014-T6 | | |
| AISI 302 (cold-rolled): | $E = 190 \text{ GPa}$ | $\sigma_Y = 520 \text{ MPa}$ |
| (c) Malleable cast iron: | $E = 165 \text{ GPa}$ | $\sigma_Y = 230 \text{ MPa}$ |

(a) $E = 190 \times 10^9 \text{ Pa}$, $\sigma_Y = 260 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(260 \times 10^6)^2}{(2)(190 \times 10^9)} = 177.9 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 177.9 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

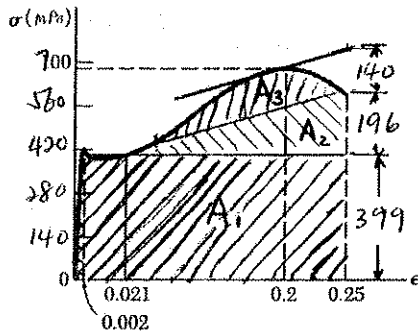
(b) $E = 190 \times 10^9 \text{ Pa}$, $\sigma_Y = 520 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(520 \times 10^6)^2}{(2)(190 \times 10^9)} = 712 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 712 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

(c) $E = 165 \times 10^9 \text{ Pa}$, $\sigma_Y = 230 \times 10^6 \text{ Pa}$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(230 \times 10^6)^2}{(2)(165 \times 10^9)} = 160.3 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 160.3 \text{ kJ}/\text{m}^3 \quad \blacktriangleleft$$

Problem 11.5



(a) $\sigma_Y = E \epsilon_Y$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2} E \epsilon_Y^2 = \frac{1}{2} (200 \times 10^9) (0.002)^2$$

$$= 400 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 400 \text{ kJ}/\text{m}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

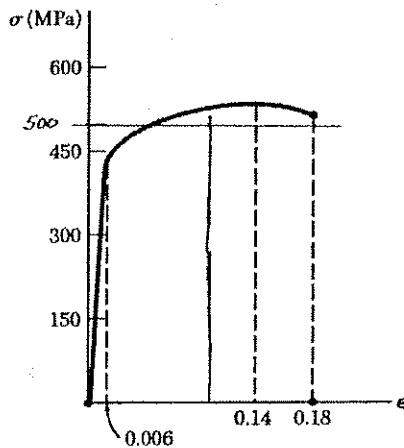
$$A_1 = (399) (0.25 - 0.002) = 98.95 \times 10^6 \text{ N}/\text{m}^2$$

$$A_2 = \frac{1}{2} (196) (0.25 - 0.021) = 22.44 \times 10^6 \text{ N}/\text{m}^2$$

$$A_3 = \frac{2}{3} (140) (0.25 - 0.075) = 16.33 \times 10^6 \text{ N}/\text{m}^2$$

$$\text{modulus of toughness} = U_Y + A_1 + A_2 + A_3 \approx 137.7 \text{ MJ}/\text{m}^3$$

Problem 11.6



(a) $\sigma_Y = E \epsilon_Y$

$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{1}{2} E \epsilon_Y^2 = \frac{1}{2} (72 \times 10^9) (0.006)^2$$

$$= 1296 \times 10^3 \text{ N}\cdot\text{m}/\text{m}^3 = 1296 \text{ kJ}/\text{m}^3$$

(b) Modulus of toughness = total area under the stress-strain curve

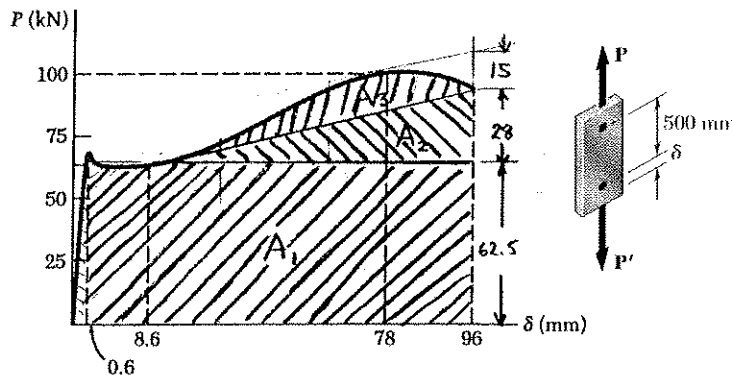
The average ordinate of the stress-strain curve is $500 \text{ MPa} = 500 \times 10^6 \text{ N}/\text{m}^2$

$$\text{The area under the curve is } A = (500 \times 10^6) (0.18) = 90 \times 10^6 \text{ N}/\text{m}^2$$

$$\text{modulus of toughness} = 90 \times 10^6 \text{ J}/\text{m}^3 = 90 \text{ MJ}/\text{m}^3$$

Problem 11.7

11.7 The load-deformation diagram shown has been drawn from data obtained during a tensile test of structural steel. Knowing that the cross-sectional area of the specimen is 250 mm^2 and that the deformation was measured using a 500-mm gage length, determine (a) the modulus of resilience of the steel, (b) the modulus of toughness of the steel.



Assuming that yielding occurs at $P = 62.5 \text{ kN}$ and $S = 0.6 \text{ mm}$

$$\begin{aligned} U_Y &= \frac{1}{2} (62.5 \times 10^3) (0.6 \times 10^{-3}) \\ &= 18.75 \text{ N}\cdot\text{m} \\ &= 18.75 \text{ J} \end{aligned}$$

Volume of stressed material: $V = AL = (250)(500) = 125 \times 10^3 \text{ mm}^3$
 $= 125 \times 10^{-6} \text{ m}^3$

$$U_Y = \frac{U_Y}{V} = \frac{18.75}{125 \times 10^{-6}} = 150 \times 10^3 = 150 \text{ kJ/m}^3$$

$$A_1 = (62.5 \times 10^3) (96 \times 10^{-3}) = 6 \times 10^3 \text{ N}\cdot\text{m} = 6 \times 10^3 \text{ J}$$

$$A_2 = \frac{1}{2} (28 \times 10^3) (96 - 8.6) \times 10^{-3} = 1.22 \times 10^3 \text{ N}\cdot\text{m} = 1.22 \times 10^3 \text{ J}$$

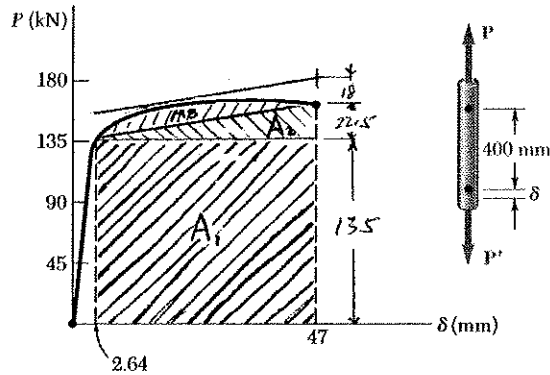
$$A_3 = \frac{2}{3} (15 \times 10^3) (61 \times 10^{-3}) = 0.61 \times 10^3 \text{ N}\cdot\text{m} = 0.61 \times 10^3 \text{ J}$$

Total energy: $U = U_Y + A_1 + A_2 + A_3 = 7.85 \times 10^3 \text{ J}$

$$\begin{aligned} \text{modulus of toughness} &= \frac{U}{V} = \frac{7.85 \times 10^3}{125 \times 10^{-6}} = 63 \times 10^6 \text{ J/m}^3 \\ &= 63 \text{ MJ/m}^3 \end{aligned}$$

Problem 11.8

11.8 The load-deformation diagram shown has been drawn from data obtained during a tensile test of a 18-mm-diameter rod of an aluminum alloy. Knowing that the deformation was measured using a 400-mm gage length, determine (a) the modulus of resilience of the alloy, (b) the modulus of toughness of the alloy.



Volume of stressed material involved in the measurement:

$$V = \frac{\pi}{4} d^2 L = \frac{\pi}{4} (0.018)^2 (0.4) = 101.79 \times 10^{-6} \text{ m}^3$$

(a) Modulus of resilience.

$$P_Y = 135 \text{ kN}, \quad \delta_Y = 0.00264 \text{ m}$$

$$U_Y = \frac{1}{2} P_Y \delta_Y = \frac{1}{2} (135000)(0.00264) = 178.2 \text{ N m}$$

$$\text{modulus of resilience} \quad u_Y = \frac{U_Y}{V} = \frac{178.2}{101.79} = 1.75 \text{ N m/m}^3 = 1.75 \text{ J/m}^3$$

(b) Modulus of toughness.

$$A_1 = (135000)(0.047 - 0.00264) = 5.9886 \text{ kN m} = 5.9886 \text{ kJ}$$

$$A_2 = \frac{1}{2} (22500)(0.047 - 0.00264) = 0.9981 \text{ kN m} = 0.9981 \text{ kJ}$$

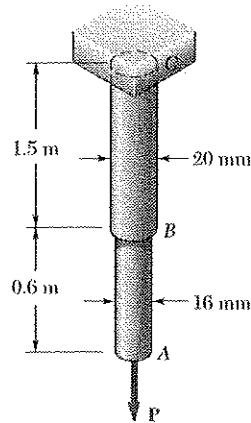
$$A_3 = \frac{2}{3} (18000)(0.047 - 0.00264) = 0.79848 \text{ kN m} = 0.79848 \text{ kJ}$$

$$U = U_Y + A_1 + A_2 + A_3 = 7.963 \text{ kJ}$$

$$\text{modulus of toughness} \quad \frac{U}{V} = \frac{7.963}{101.79 \times 10^{-6}} = 78.2 \text{ MJ/m}^3$$

Problem 11.9

11.9 Using $E = 200 \text{ GPa}$, determine (a) the strain energy of the steel rod ABC when $P = 35 \text{ kN}$, (b) the corresponding strain energy density in portions AB and BC of the rod.



$$P = 35 \text{ kN}, \quad E = 200 \text{ GPa}$$

$$A = \frac{\pi}{4} d^2, \quad V = AL, \quad \sigma = \frac{P}{A}, \quad u = \frac{\sigma^2}{2E}$$

$$U = uV$$

Portion	d m	L m	A m^2	V m^3	σ MPa	u kJ/m^3	U J
AB	0.020	1.5	201×10^{-6}	120.6×10^{-6}	174.1	75.777	9.093
BC	0.016	0.6	314.2×10^{-6}	471.3×10^{-6}	111.4	31.025	14.622
Σ							23.715

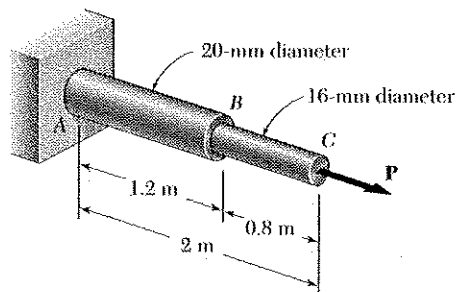
(a) $U = 23.7 \text{ J}$

(b) In AB, $u = 75.8 \text{ kJ/m}^3$

In BC, $u = 31.0 \text{ kJ/m}^3$

Problem 11.10

11.10 Using $E = 200 \text{ GPa}$, determine (a) the strain energy of the steel rod ABC when $P = 25 \text{ kN}$, (b) the corresponding strain-energy density in portions AB and BC of the rod.



$$A_{AB} = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2 = 201.06 \times 10^{-6} \text{ m}^2$$

$$P = 25 \times 10^3 \text{ N}$$

$$U = \sum \frac{P^2 L}{2EA} = \frac{(25 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(314.16 \times 10^{-6})} + \frac{(25 \times 10^3)^2 (0.8)}{(2)(200 \times 10^9)(201.06 \times 10^{-6})}$$

$$(a) \quad U = 5.968 + 6.213 = 12.18 \text{ N}\cdot\text{m} = 12.18 \text{ J}$$

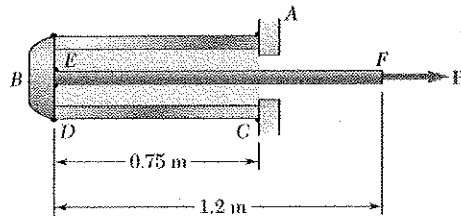
$$(b) \quad \sigma_{AB} = \frac{P}{A_{AB}} = \frac{25 \times 10^3}{314.16 \times 10^{-6}} = 79.58 \times 10^6 \text{ Pa}$$

$$U_{AB} = \frac{\sigma_{AB}^2}{2E} = \frac{(79.58 \times 10^6)^2}{(2)(200 \times 10^9)} = 15.83 \times 10^3 = 15.83 \text{ kJ/m}^3$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{25 \times 10^3}{201.06 \times 10^{-6}} = 124.28 \times 10^6 \text{ Pa}$$

$$U_{BC} = \frac{\sigma_{BC}^2}{2E} = \frac{(124.28 \times 10^6)^2}{(2)(200 \times 10^9)} = 38.6 \times 10^3 = 38.6 \text{ kJ/m}^3$$

Problem 11.11



11.11 A 0.75-m length of aluminum pipe of cross-sectional area 1190 mm² is welded to a fixed support A and to a rigid cap B. The steel rod EF, of 18-mm diameter, is welded to cap B. Knowing that the modulus of elasticity is 200 GPa for the steel and 73 GPa for the aluminum, determine (a) the total strain of the system when $P = 40$ kN, (b) the corresponding strain-energy density of the pipe CD and in the rod EF.

For EF: $A = \frac{\pi}{4} d^2 = 254.47 \text{ mm}^2$

$$\text{CD: } U_{CD} = \frac{P^2 L}{2EA} = \frac{(40 \times 10^3)^2 (0.75)}{2(73 \times 10^9)(1190 \times 10^{-6})} = 6.907 \text{ J}$$

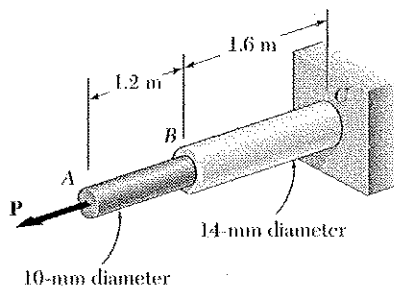
$$\text{EF: } U_{EF} = \frac{P^2 L}{2EA} = \frac{(40 \times 10^3)^2 (1.2)}{2(200 \times 10^9)(254.47 \times 10^{-6})} = 18.863 \text{ J}$$

(a) Total: $U = U_{CD} + U_{EF} = 25.77 \text{ J}$

(b) CD: $\epsilon = -\frac{40000}{1190} = -33.61 \text{ MPa}$, $U = \frac{\epsilon^2}{2E} = \frac{(-33.61)^2}{(2)(73 \times 10^3)} = 0.0077 \text{ J/m}^3$

EF: $\epsilon = \frac{40000}{254.47} = 157.19 \text{ MPa}$, $U = \frac{\epsilon^2}{2E} = \frac{157.19^2}{(2)(200 \times 10^3)} = 0.062 \text{ J/m}^3$

Problem 11.12



11.12 Rod AB is made of a steel for which the yield strength is $\sigma_Y = 450$ MPa and $E = 200$ GPa; rod BC is made of an aluminum alloy for which $\sigma_Y = 280$ MPa and $E = 73$ GPa. Determine the maximum strain energy that can be acquired by the composite rod ABC without causing any permanent deformations.

$$A_{AB} = \frac{\pi}{4} (10)^2 = 78.54 \text{ mm}^2 = 78.54 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} (14)^2 = 153.94 \text{ mm}^2 = 153.94 \times 10^{-6} \text{ m}^2$$

$$P_{all} = \sigma_Y A \text{ for each portion.}$$

$$\text{AB: } P_{all} = (450 \times 10^6)(78.54 \times 10^{-6}) = 35.343 \times 10^3 \text{ N}$$

$$\text{BC: } P_{all} = (280 \times 10^6)(153.94 \times 10^{-6}) = 43.103 \times 10^3 \text{ N}$$

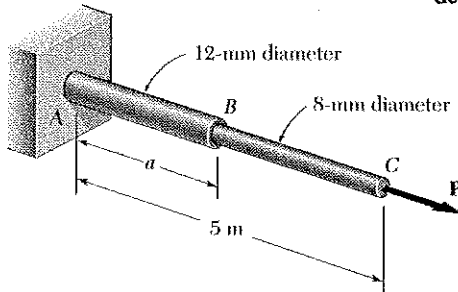
Use the smaller value. $P = 35.343 \times 10^3 \text{ N}$

$$U = \frac{P^2 L_{AB}}{2E_{AB} A_{AB}} + \frac{P^2 L_{BC}}{2E_{BC} A_{BC}} = \frac{(35.343 \times 10^3)^2 (1.2)}{(2)(200 \times 10^9)(78.54 \times 10^{-6})} + \frac{(35.343 \times 10^3)^2 (1.6)}{(2)(73 \times 10^9)(153.94 \times 10^{-6})}$$

$$U = 136.6 \text{ J}$$

Problem 11.13

11.13 Rods AB and BC are made of a steel for which the yield strength is $\sigma_Y = 300$ MPa and the modulus of elasticity is $E = 200$ GPa. Determine the maximum strain energy that can be acquired by the assembly without causing permanent deformation when the length a of rod AB is (a) 2 m, (b) 4 m.



$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(8)^2 = 50.265 \text{ mm}^2 = 50.265 \times 10^{-6} \text{ m}^2$$

$$P = \sigma_Y A_{\min} = (300 \times 10^6)(50.265 \times 10^{-6}) \\ = 15.08 \times 10^3 \text{ N}$$

$$U = \sum \frac{P^2 l}{2EA} = \frac{P^2}{2E} \sum \frac{l}{A}$$

(a) $a = 2 \text{ m}$, $L - a = 5 - 2 = 3 \text{ m}$

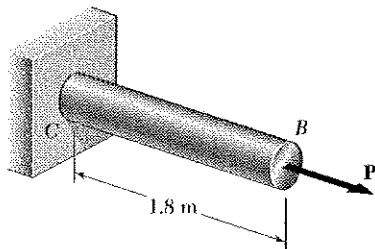
$$U = \frac{(15.08 \times 10^3)^2}{(2)(200 \times 10^9)} \left[\frac{2}{113.097 \times 10^{-6}} + \frac{3}{50.265 \times 10^{-6}} \right] = 44.0 \text{ N}\cdot\text{m} = 44.0 \text{ J}$$

(b) $a = 4 \text{ m}$, $L - a = 1 \text{ m}$

$$U = \frac{(15.08 \times 10^3)^2}{(2)(200 \times 10^9)} \left[\frac{4}{113.097 \times 10^{-6}} + \frac{1}{50.265 \times 10^{-6}} \right] = 31.4 \text{ N}\cdot\text{m} = 31.4 \text{ J}$$

Problem 11.14

11.14 Rod BC is made of a steel for which the yield strength is $\sigma_Y = 300$ MPa and the modulus of elasticity is $E = 200$ GPa. Knowing that a strain energy of 10 J must be acquired by the rod when the axial load P is applied, determine the diameter of the rod for which the factor of safety with respect to permanent deformation is six.



For factor of safety of six on the energy,

$$U_Y = (6)(10) = 60 \text{ J}$$

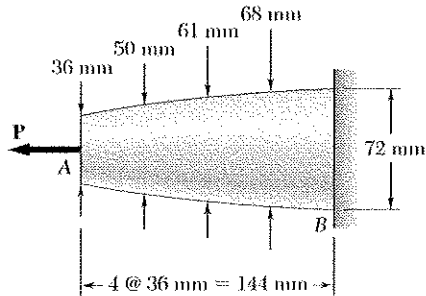
$$U_Y = \frac{\sigma_Y^2}{2E} = \frac{(300 \times 10^6)^2}{(2)(200 \times 10^9)} = 225 \times 10^3 \text{ J/m}^3$$

$$A = \frac{U_Y}{L U_Y} = \frac{60}{(1.8)(225 \times 10^3)} = 148.148 \times 10^{-6} \text{ m}^2$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(148.148 \times 10^{-6})}{\pi}} = 13.73 \times 10^{-3} \text{ m} \\ d = 13.73 \text{ mm}$$

Problem 11.15

11.15 Using $E = 74 \text{ GPa}$, determine by approximate means the maximum strain energy that can be acquired by the aluminum rod shown if the allowable normal stress is $\sigma_{all} = 154 \text{ MPa}$.



$$A_{min} = \frac{\pi}{4} (36)^2 = 1017.9 \text{ mm}^2$$

$$\sigma_{all} = 154 \text{ MPa}$$

$$P_{all} = \sigma_{all} A_{min} = 156756.6 \text{ N}$$

$$U = \int \frac{P^2 dx}{2EA} = \frac{P^2}{2E} \int \frac{dx}{\frac{\pi}{4} d^2} = \frac{2P^2}{\pi E} \int \frac{dx}{d^2}$$

Use Simpson's rule to compute the integral,

$$h = 36 \text{ mm}$$

Section	$d(\text{mm})$	$1/d^2 (\text{mm}^{-2})$	multiplier	$m(1/d^2) (\text{mm}^{-2})$
1	36	771.6	1	771.6
2	50	400	4	1600
3	61	268.7	2	537.4
4	68	216.3	4	865.2
5	72	192.9	1	192.9
Σ				3967.1

$$\int_A^B \frac{dx}{d^2} = \frac{h}{3} \Sigma m\left(\frac{1}{d^2}\right) = \frac{0.036}{3} (3967.1) = 47.6052 \text{ m}^{-1}$$

$$U = \frac{(2)(156756.6)^2 (47.6052)}{\pi (74 \times 10^9)}$$

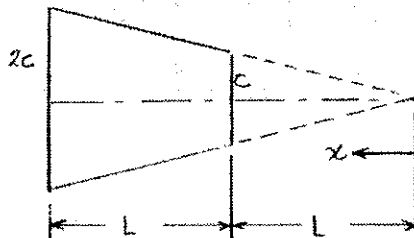
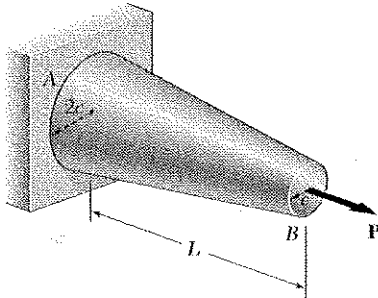
$$U = 10.06 \text{ J}$$

Problem 11.16

11.16 Show by integration that the strain energy of the tapered rod AB is

$$U = \frac{1}{4} \frac{P^2 L}{EA_{min}}$$

where A_{min} is the cross-sectional area at end B.



$$\text{radius: } r = \frac{cx}{L}$$

$$A_{min} = \pi c^2$$

$$A = \pi r^2 = \frac{\pi c^2}{L^2} x^2$$

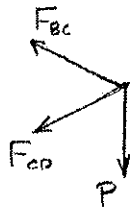
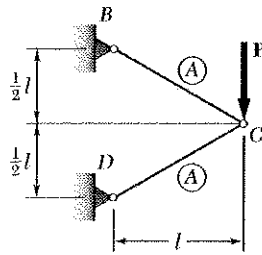
$$U = \int_L^{2L} \frac{P^2 dx}{2EA} = \frac{P^2}{2E} \int_L^{2L} \frac{L^2}{\pi c^2} \frac{dx}{x^2}$$

$$= \frac{P^2 L^2}{2E \pi c^2} \left(-\frac{1}{x} \right) \Big|_L^{2L}$$

$$= \frac{P^2 L^2}{2EA_{min}} \left(-\frac{1}{2L} + \frac{1}{L} \right)$$

$$U = \frac{1}{4} \frac{P^2 L}{EA_{min}}$$

Problem 11.17



11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.

$$L_{BC} = L_{CD} = \sqrt{l^2 + (\frac{1}{2}l)^2} = \frac{\sqrt{5}}{2}l$$

Joint C

$$+\rightarrow \Sigma F_x = 0: -\frac{2}{\sqrt{5}}F_{BC} - \frac{2}{\sqrt{5}}F_{CD} = 0$$

$$F_{CD} = -F_{BC}$$

$$+\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}}F_{BC} - \frac{1}{\sqrt{5}}F_{CD} - P = 0$$

$$F_{BC} = \frac{\sqrt{5}}{2}P$$

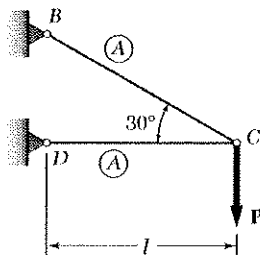
$$F_{CD} = -\frac{\sqrt{5}}{2}P$$

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2EA} [F_{BC}^2 L_{BC} + F_{CD}^2 L_{CD}]$$

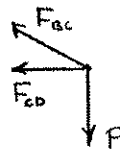
$$= \frac{1}{2EA} [(\frac{\sqrt{5}}{2}P)^2 (\frac{\sqrt{5}}{2}l) + (\frac{\sqrt{5}}{2}P)^2 (\frac{\sqrt{5}}{2}l)]$$

$$U = 1.398 \frac{P^2 l}{EA}$$

Problem 11.18



Joint C



$$+\uparrow \Sigma F_y = 0:$$

$$\frac{1}{2}F_{BC} - P = 0 \quad F_{BC} = 2P$$

$$+\rightarrow \Sigma F_x = 0:$$

$$-F_{CD} - \frac{\sqrt{3}}{2}F_{BC} = 0 \quad F_{CD} = -\sqrt{3}P$$

$$U = \Sigma \frac{F^2 L}{2EA} = \frac{1}{2E} \Sigma \frac{F^2 L}{A}$$

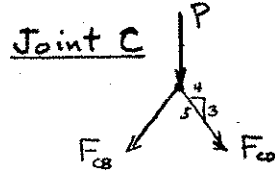
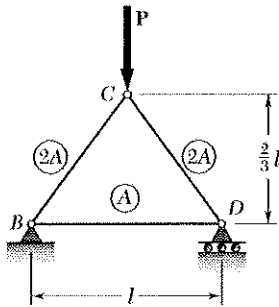
Member	F	L	A	$F^2 L / A$
BC	2P	$\frac{2}{\sqrt{3}}l$	A	$\frac{8}{\sqrt{3}}P^2 l / A$
CD	$-\sqrt{3}P$	l	A	$3P^2 l / A$
Σ				$7.62P^2 l / A$

$$U = \frac{1}{2E} (7.62 \frac{P^2 l}{A})$$

$$U = 3.81 \frac{P^2 l}{EA}$$

Problem 11.19

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.



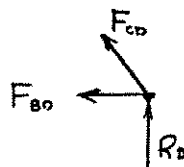
$$+\rightarrow \sum F_x = 0; \frac{3}{5} F_{CB} - \frac{3}{5} F_{CD} = 0$$

$$F_{CB} = F_{CD}$$

$$+\uparrow \sum F_y = 0; -P - 2 \cdot \frac{4}{5} F_{CD} = 0$$

$$F_{CB} = F_{CD} = -\frac{5}{8} P$$

Joint D



$$+\rightarrow \sum F_x = 0;$$

$$-F_{BD} - \frac{3}{5} F_{CD} = 0$$

$$F_{BD} = -\frac{3}{5} \cdot \frac{5}{8} P = -\frac{3}{8} P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

$$= \frac{1}{2E} \left(\frac{175}{384} \frac{P^2 l}{A} \right)$$

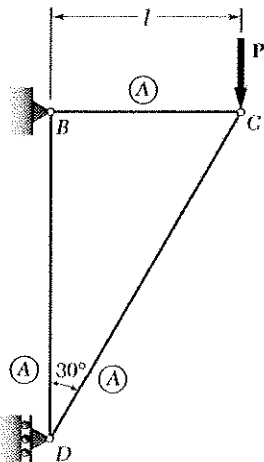
$$= \frac{175}{768} \frac{P^2 l}{EA}$$

$$U = 0.233 \frac{P^2 l}{EA}$$

Member	F	L	A	$F^2 L / A$
CB	$-\frac{5}{8} P$	$\frac{5}{6} l$	$2A$	$\frac{175}{768} \frac{P^2 l}{A}$
CD	$-\frac{5}{8} P$	$\frac{5}{6} l$	$2A$	$\frac{175}{768} \frac{P^2 l}{A}$
BD	$-\frac{3}{8} P$	l	A	$\frac{9}{256} \frac{P^2 l}{A}$
Σ				$\frac{179}{384} \frac{P^2 l}{A}$

Problem 11.20

11.17 through 11.20 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied.



Joint C

$$\uparrow \sum F_y = 0: -\frac{\sqrt{3}}{2} F_{CD} - P = 0$$

$$F_{CD} = -\frac{2}{\sqrt{3}} P$$

$$+\rightarrow \sum F_x = 0: -F_{BC} - \frac{1}{2} F_{CD} = 0$$

$$F_{BC} = \frac{1}{\sqrt{3}} P$$

Joint D

$$+\uparrow \sum F_y = 0:$$

$$F_{BD} + \frac{\sqrt{3}}{2} F_{CD} = 0$$

$$F_{BD} = P$$

Member	F	L	A	$F^2 L / A$
BC	$\frac{1}{\sqrt{3}} P$	l	A	$\frac{1}{3} P^2 l / A$
CD	$-\frac{2}{\sqrt{3}} P$	$2l$	A	$\frac{8}{3} P^2 l / A$
BD	P	$\sqrt{3} l$	A	$\sqrt{3} P^2 l / A$
Σ				$4.732 P^2 l / A$

$$U = \sum \frac{1}{2} \frac{F^2 L}{EA}$$

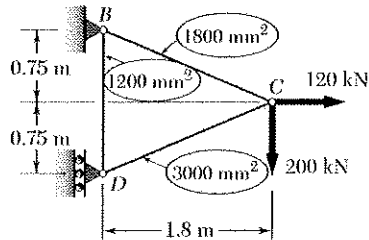
$$= \frac{1}{2E} \sum \frac{F^2 L}{A}$$

$$= \frac{1}{2E} (4.732 \frac{P^2 l}{A})$$

$$U = 2.37 \frac{P^2 l}{EA}$$

Problem 11.21

11.21 In the truss shown, all members are made of aluminum and have the uniform cross-sectional area shown. Using $E = 72 \text{ GPa}$, determine the strain energy of the truss for the loading shown.



$$l_{BC} = l_{CD} = \sqrt{1.8^2 + 0.75^2} = 1.95 \text{ m}$$

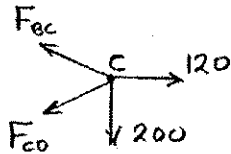
Joint C

$$+\rightarrow \Sigma F_x = 0: -\frac{1.8}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} + 120 = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0: \frac{0.75}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} - 200 = 0 \quad (2)$$

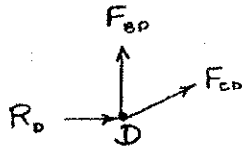
Solving (1) and (2) simultaneously,

$$F_{BC} = 325 \text{ kN} \quad F_{CD} = -195 \text{ kN}$$



Joint D

$$+\uparrow \Sigma F_y = 0: F_{BD} + \frac{0.75}{1.95} F_{CD} = 0 \quad F_{BD} = 75 \text{ kN}$$

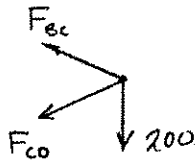
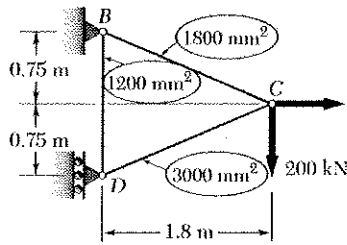


$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10^{-6} m^2)	$F^2 L / A$ (N^2 / m)
BC	325	1.95	1800	114.43×10^{12}
BD	75	1.5	1200	7.03×10^{12}
CD	-195	1.95	3000	24.72×10^{12}
Σ				146.18×10^{12}

$$U = \frac{146.18 \times 10^{12}}{(2)(72 \times 10^9)} = 1.015 \times 10^3 \text{ N-m} \quad U = 1015 \text{ J}$$

Problem 11.22



11.22 Solve Prob. 11.21, assuming that the 120-kN load is removed.

11.21 In the truss shown, all members are made of aluminum and have the uniform cross-sectional area shown. Using $E = 72 \text{ GPa}$, determine the strain energy of the truss for the loading shown.

$$l_{BC} = l_{CD} = \sqrt{1.8^2 + 0.75^2} = 1.95 \text{ m}$$

Joint C

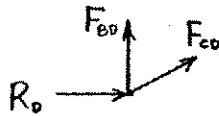
$$+\rightarrow \sum F_x = 0: -\frac{1.8}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} = 0$$

$$+\uparrow \sum F_y = 0: \frac{0.75}{1.95} F_{BC} - \frac{0.75}{1.95} F_{CD} - 200 = 0$$

Solving (1) and (2) simultaneously,

$$F_{BC} = 260 \text{ kN} \quad F_{CD} = -260 \text{ kN}$$

Joint D



$$+\uparrow \sum F_y = 0: F_{BD} + \frac{0.75}{1.95} F_{CD} = 0 \quad F_{BD} = 100 \text{ kN}$$

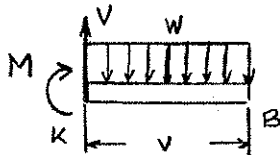
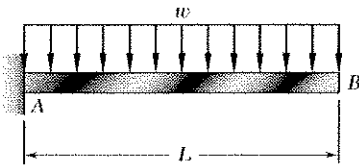
$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10^6 m^2)	$F^2 L / A$ (N^2/m)
BC	260	1.95	1800	73.23×10^{12}
BD	100	1.5	1200	12.50×10^{12}
CD	-260	1.95	3000	43.94×10^{12}
Σ				129.67×10^{12}

$$U = \frac{129.67 \times 10^{12}}{(2)(72 \times 10^9)} = 900 \text{ N}\cdot\text{m} \quad U = 900 \text{ J}$$

Problem 11.23

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\oplus \sum M_K = 0: -M - (wv)\left(\frac{v}{2}\right) = 0$$

$$M = -\frac{1}{2} wv^2$$

$$U = \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \left(\frac{1}{2} wv^2\right)^2 dv$$

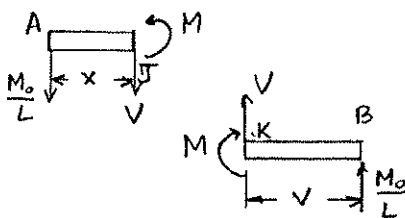
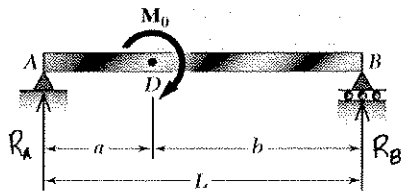
$$= \frac{w^2}{8EI} \int_0^L v^4 dv = \frac{w^2}{8EI} \left[\frac{v^5}{5} \right]_0^L$$

$$= \frac{w^2 L^5}{40EI}$$

$$U = \frac{w^2 L^5}{40EI}$$

Problem 11.24

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$+\circlearrowleft \sum M_B = 0: -R_A L - M_0 = 0 \quad R_A = \frac{M_0}{L} \downarrow$$

$$+\circlearrowleft \sum M_A = 0: R_B L - M_0 = 0 \quad R_B = \frac{M_0}{L} \uparrow$$

$$\text{A to D: } +\circlearrowleft \sum M_J = 0: \frac{M_0 x}{L} + M = 0$$

$$M = -\frac{M_0 x}{L}$$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_0^2}{2EIL^2} \int_0^a x^2 dx = \frac{M_0^2 a^3}{6EIL^2}$$

$$\text{D to B: } +\circlearrowleft \sum M_K = 0: -M + \frac{M_0 v}{L}$$

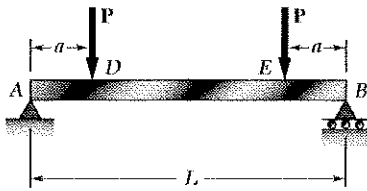
$$M = \frac{M_0 v}{L}$$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{M_0^2}{2EIL^2} \int_0^b v^2 dv = \frac{M_0^2 b^3}{6EIL^2}$$

$$\text{Total: } U = U_{AD} + U_{DB} = \frac{M_0^2 (a^3 + b^3)}{6EIL^2}$$

Problem 11.25

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



Symmetric beam and loading: $R_A = R_B$

$$+\uparrow \Sigma F_y = 0: R_A + R_B - 2P = 0 \quad R_A = R_B = P$$

Over portion AD: $M = R_A x = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2}{2EI} \left[\frac{x^3}{3} \right]_0^a = \frac{P^2 a^3}{6EI}$$

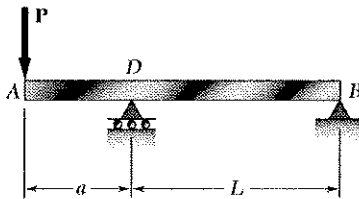
Over portion DE: $M = Pa$ $U_{DE} = \frac{P^2 a^2 (L - 2a)}{2EI}$

Over portion EB: By symmetry $U_{EB} = U_{AD} = \frac{P^2 a^3}{6EI}$

Total: $U = U_{AD} + U_{DE} + U_{EB} = \frac{P^2 a^2}{6EI} (3L - 4a)$ $\blacktriangleleft$

Problem 11.26

11.23 through 11.26 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\Sigma M_D = 0:$$

$$aP + LR_B = 0 \quad R_B = -\frac{aP}{L} = \frac{aP}{L} \downarrow$$

Over portion AD:

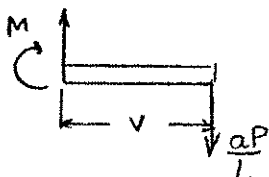
$$M = -Px$$



$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a P^2 x^2 dx = \frac{P^2}{2EI} \left[\frac{x^3}{3} \right]_0^a = \frac{P^2 a^3}{6EI}$$

Over portion DB:

$$M = -\frac{aP}{L} v$$



$$U_{DB} = \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \frac{a^2 P^2}{L^2} v^2 dv = \frac{P^2 a^2}{2EIL^2} \int_0^L v^2 dv = \frac{P^2 a^2}{2EIL^2} \left[\frac{v^3}{3} \right]_0^L = \frac{P^2 a^2 L}{6EI}$$

Total: $U = U_{AD} + U_{DB}$

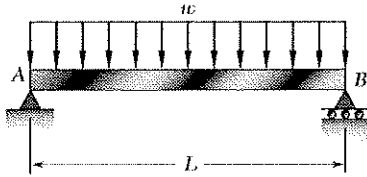
$$U = \frac{P^2 a^2}{6EI} (a + L) \quad \blacktriangleleft$$

Problem 11.27

11.27 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain-energy density in the beam is

$$u_{\max} = \frac{45 U}{8 V}$$

where U is the strain energy of the beam and V is its volume.



$$+\circlearrowleft M_B = 0: -R_A L + (wL) \frac{L}{2} = 0 \quad R_A = \frac{1}{2} wL$$

$$M = R_A x - \frac{1}{2} wL^2 = \frac{1}{2} w(Lx - x^2)$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{w^2}{8EI} \int_0^L (L^2 x^2 - 2Lx^3 + x^4) dx = \frac{w^2}{8EI} \left[\frac{L^2 x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5} \right]_0^L$$

$$= \frac{w^2 L^5}{8EI} \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right] = \frac{w^2 L^5}{240EI}$$

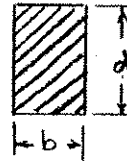
$$M_{\max} = \frac{1}{2} w \left[L \cdot \frac{L}{2} - \left(\frac{L}{2} \right)^2 \right] = \frac{1}{8} wL^2$$

$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{wL^2 c}{8I}$$

$$U_{\max} = \frac{\sigma_{\max}^2}{2E} = \frac{w^2 L^4 c^2}{128 EI^2}$$

$$\frac{U}{U_{\max}} = \frac{8LI}{15c^2} = \frac{8L(\frac{1}{2}bd^3)}{15(\frac{d}{2})^2} = \frac{8}{45} Lbd = \frac{8}{45} V$$

$$U_{\max} = \frac{45}{8} \frac{U}{V}$$

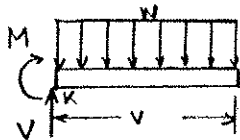
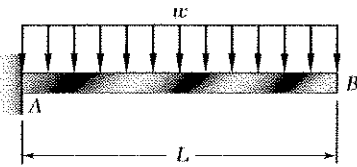


Problem 11.28

11.28 Assuming that the prismatic beam AB has a rectangular cross section, show that for the given loading the maximum value of the strain-energy density in the beam is

$$u_{\max} = 15 \frac{U}{V}$$

where U is the strain energy of the beam and V is its volume.



$$+\circlearrowleft \sum M_K = 0: \quad -M - (wv) \frac{v}{2} = 0$$

$$M = -\frac{1}{2} w v^2$$

$$U = \int_0^L \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^L \left(\frac{1}{2} w v^2 \right)^2 dv = \frac{w^2}{8EI} \int_0^L v^4 dv = \frac{w^2}{8EI} \frac{v^5}{5} \Big|_0^L = \frac{w^2 L^5}{40EI}$$

$$M_{\max} = \frac{1}{2} w L^2$$

$$\sigma_{\max} = \frac{M_{\max} c}{I}$$

$$U_{\max} = \frac{\sigma_{\max}^2}{2E} = \frac{M_{\max}^2 c^2}{2EI^2} = \frac{\frac{1}{4} w^2 L^4 c^2}{2EI^2} = \frac{w^2 L^4 c^2}{8EI^2}$$

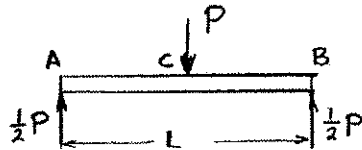
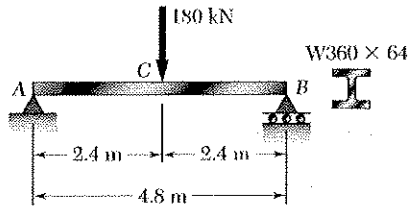
$$\frac{U}{U_{\max}} = \frac{LI}{5c^2} = \frac{L \left(\frac{1}{12} b d^3 \right)}{5 \left(\frac{d}{2} \right)^2} = \frac{1}{15} L b d = \frac{1}{15} V$$

$$U_{\max} = 15 \frac{U}{V}$$



Problem 11.29

11.29 and 11.31 Using $E = 200 \text{ GPa}$, determine the strain energy due to bending for the steel beam and loading shown.



Over portion AC: $M = \frac{1}{2}Px$

$$U_{AC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx = \frac{P^2}{8EI} \int_0^{\frac{L}{2}} x^2 dx$$

$$= \frac{P^2}{8EI} \left[\frac{x^3}{3} \right]_0^{\frac{L}{2}} = \frac{P^2 L^3}{192EI}$$

By symmetry, $U_{CB} = U_{AC} = \frac{P^2 L^3}{192EI}$

Total: $U = U_{AC} + U_{CB} = \frac{P^2 L^3}{96EI}$

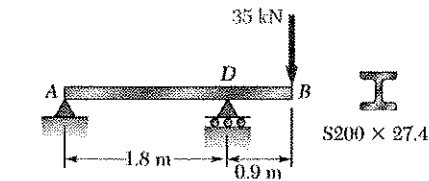
Data: $P = 180 \times 10^3 \text{ N}$, $L = 4.8 \text{ m}$, $E = 200 \times 10^9 \text{ Pa}$
 $I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6} \text{ m}^4$

$$U = \frac{(180 \times 10^3)^2 (4.8)^3}{(96)(200 \times 10^9)(178 \times 10^{-6})} = 1048 \text{ N} \cdot \text{m}$$

$$U = 1048 \text{ J}$$

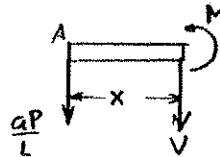
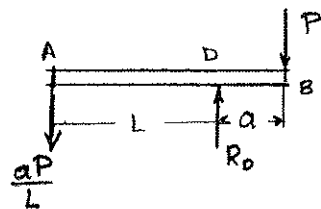
Problem 11.30

11.30 and 11.32 Using $E = 200 \text{ GPa}$, determine the strain energy due to bending for the steel beam and loading shown.



$$\sum M_D = 0 \quad -R_A L - aP = 0 \quad R_A = \frac{aP}{L} \downarrow$$

Over portion AD: $M = -\frac{aP}{L}x$



$$\begin{aligned} U_{AD} &= \int_0^L \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^L \left(\frac{aP}{L}x \right)^2 dx \\ &= \frac{P^2 a^2}{2EI L^2} \int_0^L x^2 dx \\ &= \frac{P^2 a^2 L}{6EI} \end{aligned}$$

Over portion DB: $M = -Px$



$$U_{DB} = \int_0^a \frac{M^2}{2EI} dv = \frac{1}{2EI} \int_0^a P^2 x^2 dx = \frac{P^2 a^3}{6EI}$$

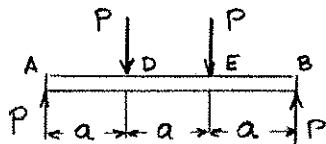
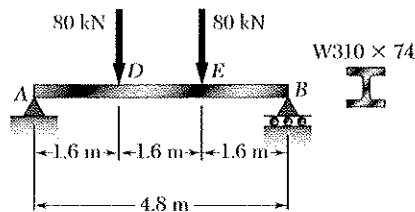
$$\text{Total: } U = U_{AD} + U_{DB} = \frac{P^2 a^2}{6EI} (a + L)$$

Data: $P = 35000$, $L = 1.8 \text{ m}$, $a = 0.9 \text{ m}$, $E = 2006 \text{ Pa}$
 $I = 23.9 \times 10^{-6} \text{ m}^4$

$$U = \frac{(35000)^2 (0.9)^2 (1.8 + 0.9)}{(6)(200 \times 10^9)(23.9 \times 10^{-6})} = 93.4 \text{ J}$$

Problem 11.31

11.29 and 11.31 Using $E = 200 \text{ GPa}$, determine the strain energy due to bending for the steel beam and loading shown.



Over portion AD: $M = Px$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{1}{2EI} \int_0^a (Px)^2 dx$$

$$= \frac{P^2}{2EI} \left. \frac{x^3}{3} \right|_0^a = \frac{P^2 a^3}{6EI}$$

Over portion DE: $M = Pa$

$$U_{DE} = \frac{(Pa)^2 a}{2EI} = \frac{P^2 a^3}{2EI}$$

By symmetry, $U_{EB} = U_{AD} = \frac{P^2 a^3}{6EI}$

$$U = U_{AD} + U_{DE} + U_{EB} = \frac{5}{6} \frac{P^2 a^3}{EI}$$

Data: $P = 80 \times 10^3 \text{ N}$, $a = 1.6 \text{ m}$, $E = 200 \times 10^9 \text{ Pa}$

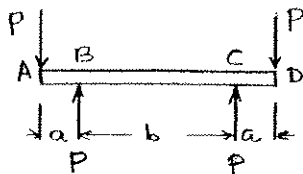
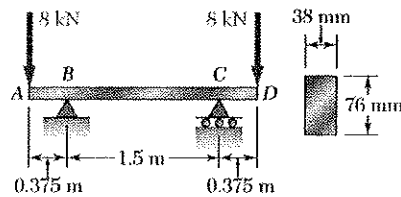
$$I = 165 \times 10^6 \text{ mm}^4 = 165 \times 10^{-6} \text{ m}^4$$

$$U = \frac{5}{6} \frac{(80 \times 10^3)^2 (1.6)^3}{(200 \times 10^9)(165 \times 10^{-6})} = 662 \text{ N}\cdot\text{m}$$

$$U = 662 \text{ J}$$

Problem 11.32

11.31 and 11.32 Using $E = 200$ GPa, determine the strain energy due to bending for the steel beam and loading shown.



Over A to B: $M = -Px$

$$U_{AB} = \int_0^a \frac{M^2 dx}{2EI} = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

Over B to C: $-M = Pa = \text{constant}$

$$U_{BC} = \frac{M^2 b}{2EI} = \frac{P^2 a^2 b}{2EI}$$

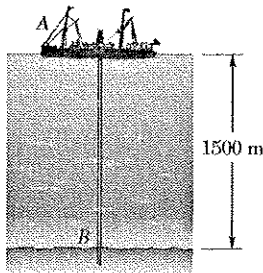
By symmetry, $U_{CD} = U_{AB} = \frac{P^2 a^3}{6EI}$

$$\text{Total } U = U_{AB} + U_{BC} + U_{CD} = \frac{P^2 a^2 (2a + 3b)}{6EI}$$

Data: $P = 8 \text{ kN}$ $a = 0.375 \text{ m}$ $b = 1.5 \text{ m}$ $I = \frac{1}{12} (76)^3 (38) = 1.39 \times 10^6 \text{ mm}^4$

$$U = \frac{(8 \times 10^3)^2 (0.375)^2 [(2)(0.375) + (3)(1.5)]}{(6)(200 \times 10^9)(1.39 \times 10^{-6})} = 28.3 \text{ J}$$

Problem 11.33



11.33 The ship at A has just started to drill for oil on the ocean floor at a depth of 1500 m. The steel drill pipe has an outer diameter of 200 mm and a uniform wall thickness of 12 mm. Knowing that the top of the drill pipe rotates through two complete revolutions before the drill bit at B starts to operate and using $G = 77$ GPa, determine the maximum strain energy acquired by the drill pipe.

$$\phi = 2 \text{ rev} = (2)(2\pi) = 4\pi \text{ radians}$$

$$L = 1500 \text{ m}, \quad c_o = \frac{1}{2} d_o = 100 \text{ mm}, \quad c_i = c_o - t = 88 \text{ mm}$$

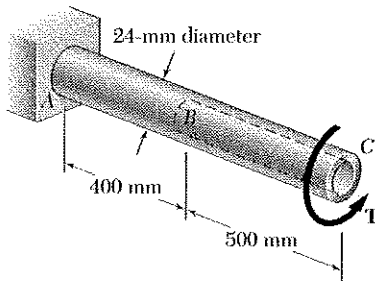
$$J = \frac{\pi}{2} (c_o^4 - c_i^4) = \frac{\pi}{2} (100^4 - 88^4) = 62.88 \times 10^6 \text{ mm}^4 = 62.88 \times 10^{-6} \text{ m}^4$$

$$\phi = \frac{TL}{GJ}$$

$$T = \frac{GJ\phi}{L}$$

$$U = \frac{T^2 L}{2GJ} = \frac{\left(\frac{GJ\phi}{L}\right)^2 L}{2GJ} = \frac{GJ\phi^2}{2L} = \frac{(77 \times 10^9)(62.88 \times 10^{-6})(4\pi)^2}{(2)(1500)} = 254.86 \times 10^3 \text{ J} = 254.9 \text{ kJ}$$

Problem 11.34



11.34 Rod AC is made of aluminum and is subjected to a torque T applied at C . Knowing that $G = 73 \text{ GPa}$ and that portion BC of the rod is hollow and has an inner diameter of 16 mm, determine the strain energy of the rod for a maximum shearing stress of 120 MPa.

$$C_o = \frac{d_o}{2} = 12 \text{ mm}, \quad C_i = \frac{d_i}{2} = 8 \text{ mm}$$

$$J_{AB} = \frac{\pi}{2} C_o^4 = \frac{\pi}{2} (12)^4 = 32.572 \times 10^3 \text{ mm}^4 = 32.572 \times 10^{-9} \text{ m}^4$$

$$J_{BC} = \frac{\pi}{2} (C_o^4 - C_i^4) = \frac{\pi}{2} (12^4 - 8^4) = 26.138 \times 10^3 \text{ mm}^4 = 26.138 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{T C}{J_{\min}} \quad T = \frac{J_{\min} \tau_{\max}}{C} = \frac{(26.138 \times 10^{-9})(120 \times 10^6)}{12 \times 10^{-3}} = 261.38 \text{ N}\cdot\text{m}$$

$$U_{AB} = \frac{T^2 L_{AB}}{2 G J_{AB}} = \frac{(261.38)^2 (400 \times 10^{-3})}{(2)(73 \times 10^9)(32.572 \times 10^{-9})} = 5.747 \text{ J}$$

$$U_{BC} = \frac{T^2 L_{BC}}{2 G J_{BC}} = \frac{(261.38)^2 (500 \times 10^{-3})}{(2)(73 \times 10^9)(26.138 \times 10^{-9})} = 8.951 \text{ J}$$

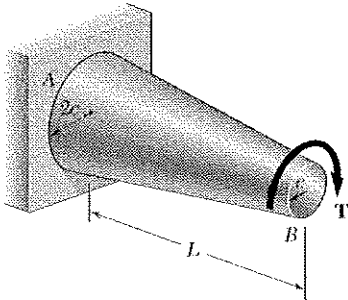
$$\text{Total: } U = U_{AB} + U_{BC} \quad U = 14.70 \text{ J} \quad \blacktriangleleft$$

Problem 11.35

11.35 Show by integration that the strain energy in the tapered rod AB is

$$U = \frac{7}{48} \frac{T^2 L}{G J_{\min}}$$

where $J_{\min}$ is the polar moment of inertia of the rod at end B .



$$r = \frac{C x}{L}$$

$$J = \frac{\pi}{2} r^4 = \frac{\pi}{2} \frac{C^4}{L^4} x^4, \quad J_{\min} = \frac{\pi}{2} C^4$$

$$U = \int_L^{2L} \frac{T^2 dx}{2 G J} = \int_L^{2L} \frac{T^2}{2 G \left(\frac{\pi}{2} \frac{C^4}{L^4} x^4 \right)} dx$$

$$= \frac{T^2 L^4}{2 G J_{\min}} \int_L^{2L} \frac{dx}{x^4}$$

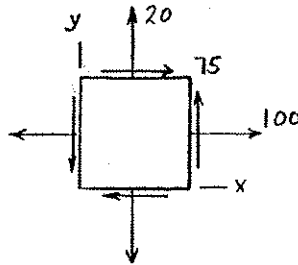
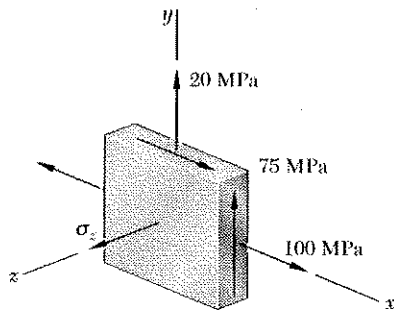
$$= \frac{T^2 L^4}{2 G J_{\min}} \left(-\frac{1}{3x^3} \right) \Big|_L^{2L}$$

$$U = \frac{T^2 L^4}{2 G J_{\min}} \left(-\frac{1}{3(2L)^3} + \frac{1}{3L^3} \right)$$

$$U = \frac{7}{48} \frac{T^2 L}{G J_{\min}} \quad \blacktriangleleft$$

Problem 11.36

11.36 The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine whether yield occurs when (a) $\sigma_z = +45$ MPa, (b) $\sigma_z = -45$ MPa.

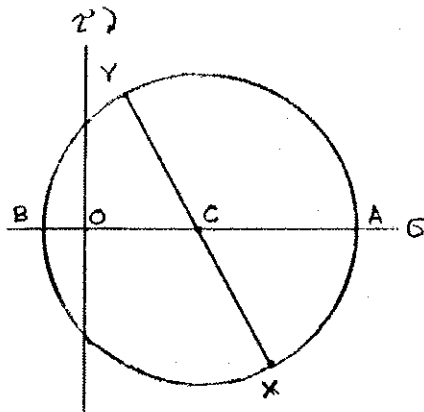


$$\sigma_{ave} = \frac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

$$\tau_{xy} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$



$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\sigma_c = \sigma_z$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 \stackrel{?}{<} 2\sigma_Y^2$$

(a) $\sigma_c = \sigma_z = +45 \text{ MPa}$

$$(145 + 25)^2 + (-25 - 45)^2 + (45 - 145)^2 \stackrel{?}{<} 2(160)^2 = 51200$$

$$28900 + 4900 + 10000 = 43800 < 51200 \quad (\text{No yield})$$

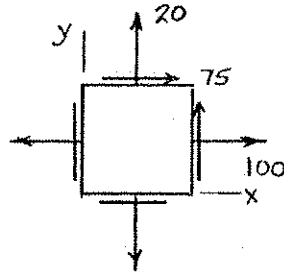
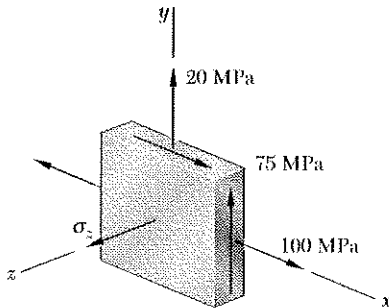
(b) $\sigma_c = \sigma_z = -45 \text{ MPa}$

$$(145 + 25)^2 + (-25 + 45)^2 + (-45 - 145)^2 \stackrel{?}{<} 51200$$

$$28900 + 400 + 36100 = 65400 > 51200 \quad (\text{Yield occurs})$$

Problem 11.37

11.37 The state of stress shown occurs in a machine component made of a brass for which $\sigma_Y = 160$ MPa. Using the maximum-distortion-energy criterion, determine the range of values of σ_z for which yield does not occur.



$$\sigma_{ave} = \frac{1}{2}(100 + 20) = 60 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{100 - 20}{2} = 40 \text{ MPa}$$

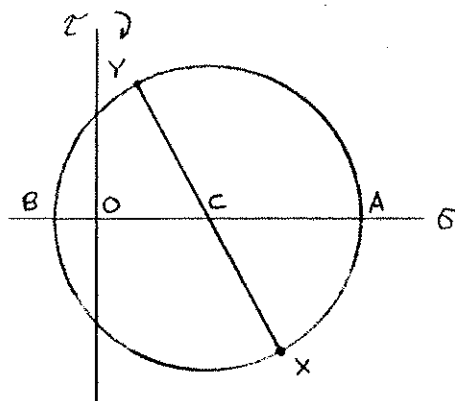
$$\tau_{xy} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + 75^2} = 85 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 145 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -25 \text{ MPa}$$

$$\sigma_c = \sigma_z$$



$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\sigma_Y^2$$

$$(145 + 25)^2 + (-25 - \sigma_z)^2 + (\sigma_z - 145)^2 = (2)(160)^2$$

$$28900 + (625 + 50\sigma_z + \sigma_z^2) + (\sigma_z^2 - 290\sigma_z + 21025) = 51200$$

$$2\sigma_z^2 - 240\sigma_z - 650 = 0$$

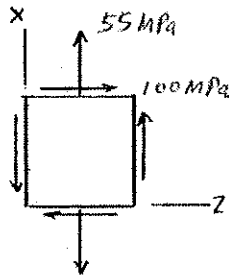
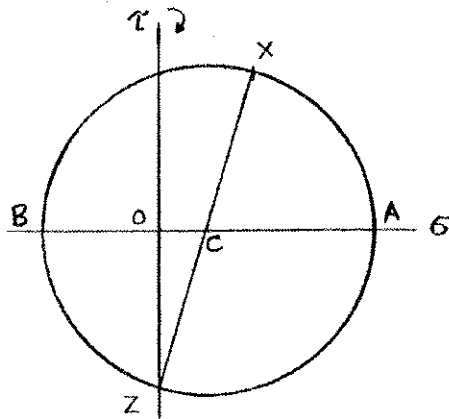
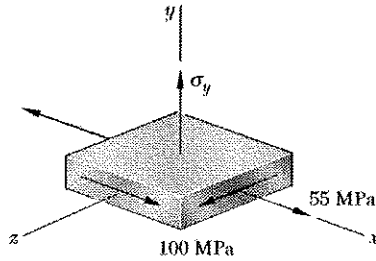
$$\sigma_z = \frac{240 \pm \sqrt{240^2 + (4)(2)(650)}}{(2)(2)} = 60 \pm 62.65$$

$$\sigma_z = 122.65 \text{ MPa}, -2.65 \text{ MPa}$$

$$-2.65 \text{ MPa} < \sigma_z < 122.65 \text{ MPa}$$

Problem 11.38

11.38 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_Y = 450$ MPa. Using the maximum-distortion-energy criterion, determine the factor of safety associated with the yield strength when (a) $\sigma_Y = +110$ MPa, (b) $\sigma_Y = -110$ MPa.



$$\sigma_{ave} = \frac{1}{2}(0 + 55) = 27.5 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{55 - 0}{2} = 27.5 \text{ MPa}$$

$$\tau_{xz} = 100 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \sqrt{27.5^2 + 100^2} = 103.7 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 131.2 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -76.2 \text{ MPa}$$

$$\sigma_c = \sigma_y$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_Y}{F.S.}\right)^2$$

(a) $\sigma_c = \sigma_y = 110 \text{ MPa}$

$$(131.2 + 76.2)^2 + (-76.2 - 110)^2 + (110 - 131.2)^2 = 2\left(\frac{450}{F.S.}\right)^2$$

$$43014.8 + 34670.4 + 449.4 = \frac{405000}{(F.S.)^2}$$

$$F.S. = 2.28$$

(b) $\sigma_c = \sigma_y = -110 \text{ MPa}$

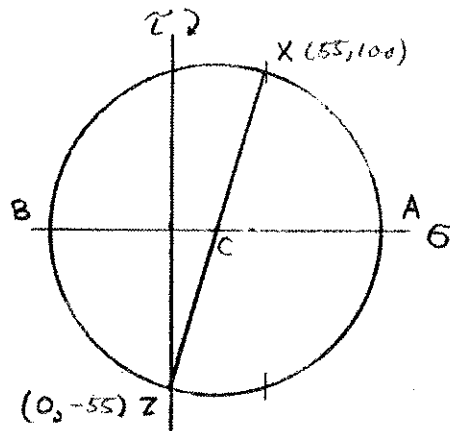
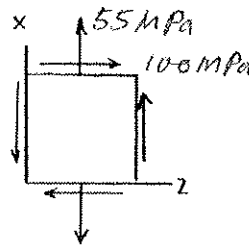
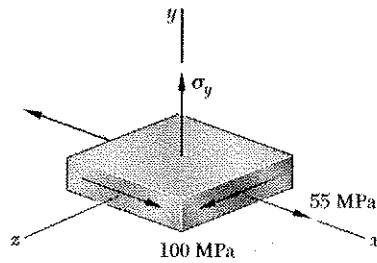
$$(131.2 + 76.2)^2 + (-76.2 + 110)^2 + (-110 - 131.2)^2 = 2\left(\frac{450}{F.S.}\right)^2$$

$$43014.8 + 1142.4 + 58177.4 = \frac{405000}{(F.S.)^2}$$

$$F.S. = 1.99$$

Problem 11.39

11.39 The state of stress shown occurs in a machine component made of a grade of steel for which $\sigma_y = 450$ MPa. Using the maximum-distortion-energy criterion, determine the range of values of σ_x for which the factor of safety associated with the yield strength is equal to or larger than 2.2.



$$\sigma_{ave} = \frac{1}{2} (0 + 55) = 27.5 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_z}{2} = \frac{55 - 0}{2} = 27.5 \text{ MPa}$$

$$\tau_{xz} = 100 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= \sqrt{27.5^2 + 100^2} = 103.7 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 131.2 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 76.2 \text{ MPa}$$

$$\sigma_c = \sigma_y$$

$$(\sigma_a - \sigma_b)^2 + (\sigma_b - \sigma_c)^2 + (\sigma_c - \sigma_a)^2 = 2\left(\frac{\sigma_y}{F.S.}\right)^2$$

$$(131.2 - 76.2)^2 + (-76.2 - \sigma_y)^2 + (\sigma_y - 131.2)^2 = 2\left(\frac{450}{2.2}\right)^2$$

$$43014.8 + (5806.4 + 152.4\sigma_y + \sigma_y^2) + (\sigma_y^2 - 262.4\sigma_y + 17213.4) = 83677.7$$

$$2\sigma_y^2 - 110\sigma_y - 17643 = 0$$

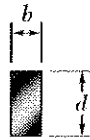
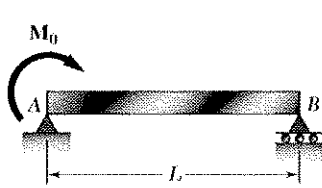
$$\sigma_y = \frac{110 \pm \sqrt{110^2 + (4)(2)(17643)}}{(2)(2)} = 27.5 \pm 97.9$$

$$\sigma_y = 125.4 \text{ MPa}, -70.4 \text{ MPa}$$

$$-70.4 \leq \sigma_y \leq 125.4 \text{ MPa} \quad \blacktriangleleft$$

Problem 11.40

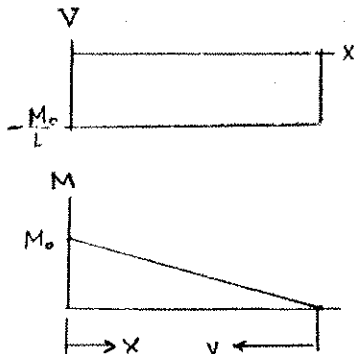
11.40 Determine the strain energy of the prismatic beam AB , taking into account the effect of both normal and shearing stresses.



Reactions $R_A = \frac{M_0}{L} \downarrow$, $R_B = \frac{M_0}{L} \uparrow$

Shear: $V = -\frac{M_0}{L}$

Bending moment: $M = \frac{M_0}{L} x$



For bending

$$U_1 = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^L x^2 dx = \frac{M_0^2 L^3}{6EI L^2} = \frac{M_0^2 L}{6EI}$$

For shear

$$\tau_{xy} = \frac{3}{2} \frac{V}{A} \left(1 - \frac{y^2}{c^2}\right) \quad c = \frac{1}{2}d$$

$$u = \frac{\tau_{xy}^2}{2G} = \frac{9V^2}{8GA^2} \left(1 - \frac{y^2}{c^2}\right)^2 = \frac{9M_0^2}{8G(bd)^2 L^2} \left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right)$$

$$\begin{aligned} U_2 &= \int u dV = \int_0^L \int_{-c}^c u b dy dx = \frac{9M_0^2 b}{8G b^2 d^2 L^2} \int_0^L \int_{-c}^c \left(1 - 2\frac{y^2}{c^2} + \frac{y^4}{c^4}\right) dy dx \\ &= \frac{9M_0^2}{8G b d^2 L^2} \int_0^L \left(y - \frac{2}{3} \frac{y^3}{c^2} + \frac{1}{5} \frac{y^5}{c^4}\right) \Big|_{-c}^c dx = \frac{9M_0^2}{8G b d^2 L^2} \int_0^L \left(2c - \frac{4}{3}c + \frac{2}{5}c\right) dx \\ &= \frac{9M_0^2}{8G b d^2 L^2} \left(\frac{16}{15}c\right) L = \frac{6}{5} \frac{M_0^2 c}{G b d^2 L} = \frac{3}{5} \frac{M_0^2}{G b d L} \end{aligned}$$

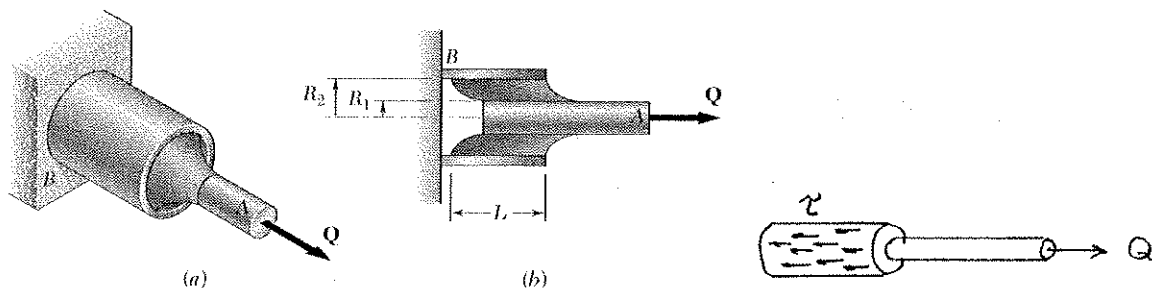
Total: $U = U_1 + U_2 = \frac{M_0^2 L}{6EI} + \frac{3}{5} \frac{M_0^2}{G b d L}$

With $I = \frac{1}{12} b d^3$,

$$U = \frac{2M_0^2 L}{E b d^3} + \frac{3}{5} \frac{M_0^2}{G b d L} = \frac{2M_0^2 L}{E b d^3} \left\{ 1 + \frac{3}{10} \cdot \frac{E}{G} \cdot \frac{d^2}{L^2} \right\}$$

Problem 11.41

*11.41 A vibration isolation support is made by bonding a rod A , of radius R_1 , and a tube B , of inner radius R_2 to a hollow rubber cylinder. Denoting by G the modulus of rigidity of the rubber, determine the strain energy of the hollow rubber cylinder for the loading shown.



$$+\rightarrow \sum F_x = 0: -\tau(2\pi r L) + Q = 0$$

$$\tau = \frac{Q}{2\pi r L}$$

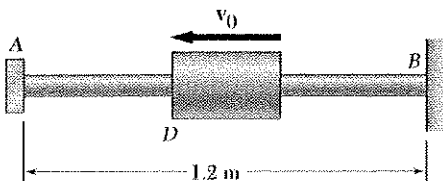
$$u = \frac{\tau^2}{2G} = \frac{Q^2}{8\pi^2 r^2 L^2 G}$$

$$U = \int u dV = \frac{Q^2}{8\pi^2 G L^2} \int \frac{dV}{r^2} = \frac{Q^2}{8\pi^2 G L^2} \int_0^L \int_{R_1}^{R_2} \frac{2\pi r dr}{r^2} dx$$

$$= \frac{Q^2}{4\pi G L^2} \int_0^L \int_{R_1}^{R_2} \frac{dr}{r} dx = \frac{Q^2}{4\pi G L^2} \int_0^L (\ln r \Big|_{R_1}^{R_2}) dx = \frac{Q^2}{4\pi G L} \ln \frac{R_2}{R_1}$$

Problem 11.42

11.42 A 6-kg collar has a speed $v_0 = 4.5$ m/s when it strikes a small plate attached to end A of the 20-mm-diameter rod AB . Using $E = 200$ GPa, determine the equivalent static load, (b) the maximum stress in the rod, (c) the maximum deflection of end A .



$$U_m = \frac{1}{2} m v_0^2 = \frac{1}{2} (6) (4.5)^2 = 60.75 \text{ J}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (20 \times 10^{-3})^2 = 314.159 \times 10^{-6} \text{ m}^2$$

$$U_m = \frac{P^2 L}{2EA}$$

$$(a) \quad P = \sqrt{\frac{2EAU_m}{L}} = \sqrt{\frac{(2)(200 \times 10^9)(314.159 \times 10^{-6})(60.75)}{1.2}}$$

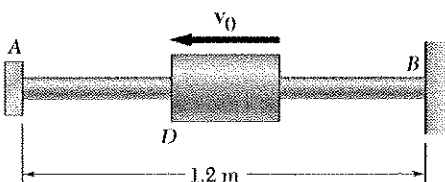
$$= 79.76 \times 10^3 \text{ N} \quad P = 79.8 \text{ kN} \quad \blacktriangleleft$$

$$(b) \quad \sigma_{max} = \frac{P}{A} = \frac{79.76 \times 10^3}{314.159 \times 10^{-6}} = 254 \times 10^6 \text{ Pa} \quad \sigma_{max} = 254 \text{ MPa} \quad \blacktriangleleft$$

$$(c) \quad \delta = \frac{PL}{EA} = \frac{(79.76 \times 10^3)(1.2)}{(200 \times 10^9)(314.159 \times 10^{-6})} = 1.523 \times 10^{-3} \text{ m} \quad \delta = 1.523 \text{ mm} \quad \blacktriangleleft$$

Problem 11.43

11.43 A 5-kg collar D moves along the uniform rod AB and has a speed $v_0 = 6$ m/s when it strikes a small plate attached to end A of the rod. Using $E = 200$ GPa and knowing that the allowable stress in the rod is 250 MPa, determine the smallest diameter that can be used for the rod.



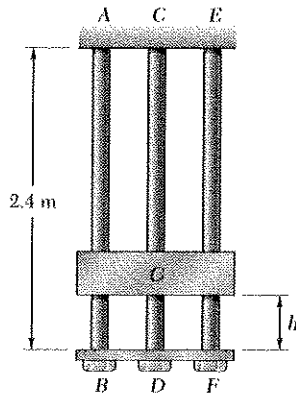
$$U_m = \frac{1}{2} m v_0^2 = \frac{1}{2} (5) (6)^2 = 90 \text{ J}$$

$$U_m = \frac{P_m^2 L}{2EA} = \frac{(A \sigma_{max})^2 L}{2EA}$$

$$A = \frac{2EU_m}{\sigma_{max}^2 L} = \frac{(2)(200 \times 10^9)(90)}{(250 \times 10^6)^2 (1.2)} = 480 \times 10^{-6} \text{ m}^2$$

$$\frac{\pi}{4} d^2 = A \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(480 \times 10^{-6})}{\pi}} = 24.7 \times 10^{-3} \text{ m} \quad d = 24.7 \text{ mm} \quad \blacktriangleleft$$

Problem 11.44



11.44 The 45-kg collar C is released from rest in the position shown and is stopped by Plate BDF that is attached the 22-mm-diameter steel rod CD and to the 16-mm-diameter steel rods AB and EF . Knowing that for the grade of steel used $\sigma_{all} = 165 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the largest allowable distance h .

Let Δ_m be the elongation.

$$\Delta_m = \frac{\sigma_{AB} L}{E} = \frac{\sigma_{CD} L}{E} = \frac{\sigma_{EF} L}{E}$$

$$\sigma_{AB} = \sigma_{CD} = \sigma_{EF} = 165 \text{ MPa}$$

$$L = 2.4 \text{ m}$$

$$\Delta_m = \frac{(165 \times 10^6)(2.4)}{200 \times 10^9} = 1.98 \times 10^{-3} \text{ m}$$

$$\text{For each rod } U = \frac{F_m^2 L}{2EA} = \frac{(EA \Delta_m / L)^2 L}{2EA} = \frac{EA \Delta_m^2}{2L}$$

$$\text{Rod } CD: A_{CD} = \frac{\pi}{4} (22)^2 = 380 \text{ mm}^2$$

$$U_{CD} = \frac{(200 \times 10^9)(380 \times 10^{-6})(1.98 \times 10^{-3})^2}{(2)(2.4)} = 62.07 \text{ J}$$

$$\text{Rods } AB \text{ and } EF: A_{AB} = A_{EF} = \frac{\pi}{4} (16)^2 = 201 \text{ mm}^2$$

$$U_{AB} = U_{EF} = \frac{(200 \times 10^9)(201 \times 10^{-6})(1.98 \times 10^{-3})^2}{(2)(2.4)} = 32.83 \text{ J}$$

$$\text{Total } U_m = U_{AB} + U_{CD} + U_{EF} = 127.73 \text{ J}$$

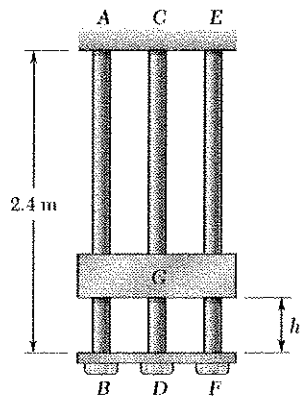
$$\text{Falling distance is } h + \Delta_m. \quad W = 45 \times 9.81 = 441.45 \text{ N}$$

$$W(h + \Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{127.73}{441.45} = 0.28934 \text{ m}$$

$$h = 0.28934 - 0.00198 = 0.28736 \text{ m} = 287 \text{ mm} \quad \blacktriangleleft$$

Problem 11.45



11.45 Solve Prob. 11.44, assuming that the 22-mm-diameter steel rod CD is replaced by a 22-mm-diameter rod made of an aluminum alloy for which $\sigma_{all} = 140 \text{ MPa}$ and $E = 73 \text{ GPa}$.

11.44 The 45-kg collar C is released from rest in the position shown and is stopped by Plate BDF that is attached the 22-mm-diameter steel rod CD and to the 16-mm-diameter steel rods AB and EF. Knowing that for the grade of steel used $\sigma_{all} = 165 \text{ MPa}$ and $E = 200 \text{ GPa}$, determine the largest allowable distance h .

Let Δ_m be the elongation.

$$L = 2.4 \text{ m}$$

$$\Delta_m = \frac{\sigma_{AB} L}{E_{AB}} = \frac{\sigma_{CD} L}{E_{CD}} = \frac{\sigma_{EF} L}{E_{EF}}$$

$$\text{If } \sigma_{AB} = 165 \text{ MPa}, \quad \Delta_m = \frac{(165 \times 10^6)(2.4)}{200 \times 10^9} = 1.98 \times 10^{-3} \text{ m}$$

$$\text{If } \sigma_{CD} = 140 \text{ MPa}, \quad \Delta_m = \frac{(140 \times 10^6)(2.4)}{73 \times 10^9} = 4.603 \times 10^{-3} \text{ m}$$

$$\text{Smaller value governs} \quad \Delta_m = 1.98 \times 10^{-3} \text{ m}$$

$$\text{For each rod} \quad U = \frac{F^2 L}{2EA} = \frac{(EA \Delta_m / L)^2 L}{2EA} = \frac{EA \Delta_m^2}{2L}$$

$$\text{Rod CD:} \quad A_{CD} = \frac{\pi}{4} (22)^2 = 380 \text{ mm}^2, \quad E_{CD} = 73 \times 10^9 \text{ Pa}$$

$$U_{CD} = \frac{(73 \times 10^9)(380 \times 10^{-6})(1.98 \times 10^{-3})^2}{(2)(2.4)} = 22.66 \text{ J}$$

$$\text{Rods AB and EF:} \quad A_{AB} = A_{EF} = \frac{\pi}{4} (16)^2 = 201 \text{ mm}^2$$

$$U_{AB} = U_{EF} = \frac{(200 \times 10^9)(201 \times 10^{-6})(1.98 \times 10^{-3})^2}{(2)(2.4)} = 32.83 \text{ J}$$

$$\text{Total} \quad U_m = U_{AB} + U_{CD} + U_{EF} = 88.32 \text{ J}$$

$$\text{Falling distance is } h + \Delta_m \quad W = 45 \times 9.81 = 441.45 \text{ N}$$

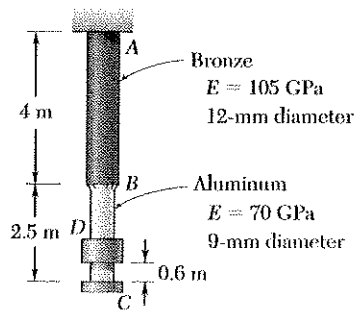
$$W(h + \Delta_m) = U_m$$

$$h + \Delta_m = \frac{U_m}{W} = \frac{88.32}{441.45} = 0.2 \text{ m}$$

$$h = 0.2 - 0.00198 = 0.198 \text{ m} = 198 \text{ mm}$$

Problem 11.46

11.46 Collar D is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC . Determine the mass of the collar for which the maximum normal stress in portion BC is 125 MPa.



Portion BC: $\sigma_m = 125 \times 10^6 \text{ Pa}$

$$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

$$P_m = \sigma_m A_{BC} = 7952 \text{ N}$$

Corresponding strain energy.

$$U_{BC} = \frac{P_m^2 L_{BC}}{2 E_{BC} A_{BC}} = \frac{(7952)^2 (2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2 E_{AB} A_{AB}} = \frac{(7952)^2 (4)}{(2)(105 \times 10^9)(113.907 \times 10^{-6})} = 10.574 \text{ J}$$

$$U_m = U_{BC} + U_{AB} = 28.324 \text{ J}$$

Corresponding elongation Δ_m . $\frac{1}{2} P_m \Delta_m = U_m$

$$\Delta_m = \frac{2 U_m}{P_m} = \frac{(2)(28.324)}{7952} = 7.12 \times 10^{-3} \text{ m}$$

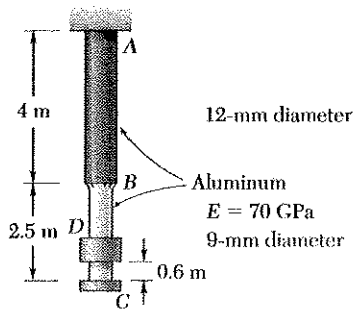
Falling distance: $h = 0.6 + 7.12 \times 10^{-3} = 0.60712 \text{ m}$

Work of weight = U_m $Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{28.324}{(9.81)(0.60712)}$$

$$m = 4.76 \text{ kg}$$

Problem 11.47



11.47 Solve Prob. 11.46, assuming that both portions of rod ABC are made of aluminum.

11.46 Collar D is released from rest in the position shown and is stopped by a small plate attached at end C of the vertical rod ABC . Determine the mass of the collar for which the maximum normal stress in portion BC is 125 MPa.

Portion BC : $\sigma_m = 125 \times 10^6 \text{ Pa}$

$$A_{BC} = \frac{\pi}{4}(9)^2 = 63.617 \text{ mm}^2 = 63.617 \times 10^{-6} \text{ m}^2$$

$$P_m = \sigma_m A_{BC} = 7952 \text{ N}$$

Corresponding strain energy.

$$U_{BC} = \frac{P_m^2 L_{BC}}{2EA_{BC}} = \frac{(7952)^2 (2.5)}{(2)(70 \times 10^9)(63.617 \times 10^{-6})} = 17.750 \text{ J}$$

$$A_{AB} = \frac{\pi}{4}(12)^2 = 113.907 \text{ mm}^2 = 113.907 \times 10^{-6} \text{ m}^2$$

$$U_{AB} = \frac{P_m^2 L_{AB}}{2EA_{AB}} = \frac{(7952)^2 (4)}{(2)(70 \times 10^9)(113.907 \times 10^{-6})} = 15.861 \text{ J}$$

Total: $U_m = U_{BC} + U_{AB} = 33.611 \text{ J}$

Corresponding elongation Δ_m . $\frac{1}{2} P_m \Delta_m = U_m$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(33.611)}{7952} = 8.45 \times 10^{-3} \text{ m}$$

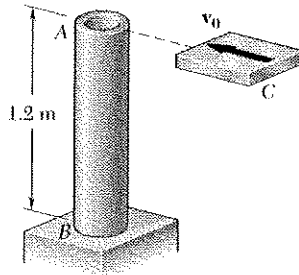
Falling distance. $h = 0.6 + \Delta_m = 0.60845 \text{ m}$

Work of weight = U_m . $Wh = mgh = U_m$

$$m = \frac{U_m}{gh} = \frac{33.611}{(9.81)(0.60845)} =$$

$$m = 5.63 \text{ kg} \rightarrow$$

Problem 11.48



11.48 The post AB consists of a steel pipe of 80-mm-diameter and 6-mm wall thickness. A 6-kg block C moving horizontally with at velocity v_0 hits the post squarely at A . Using $E = 200$ GPa, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 180 MPa.

$$C_o = \frac{1}{2}d_o = \frac{1}{2}(80) = 40 \text{ mm} \quad C_i = C_o - t = 20 - 6 = 34 \text{ mm}$$

$$I = \frac{\pi}{4}(C_o^4 - C_i^4) = \frac{\pi}{4}(40^4 - 34^4) = 961.06 \times 10^3 \text{ mm}^4 \\ = 961.06 \times 10^{-9} \text{ m}^4$$

$$\sigma_m = 180 \times 10^6 \text{ Pa} = \frac{M_m C}{I}$$

$$M_m = \frac{I \sigma_m}{C} = \frac{(961.06 \times 10^{-9})(180 \times 10^6)}{40 \times 10^{-3}} = 4.325 \times 10^3 \text{ N}\cdot\text{m}$$

$$P_m = \frac{M_m}{L} = \frac{4.325 \times 10^3}{1.2} = 3.604 \times 10^3 \text{ N}$$

By Appendix D, Case 1

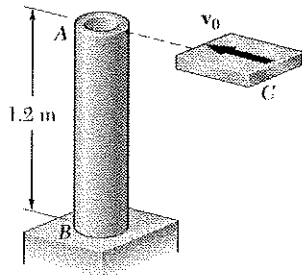
$$y_m = \frac{P_m L^3}{3EI} = \frac{(3.604 \times 10^3)(1.2)^3}{(3)(200 \times 10^9)(961.06 \times 10^{-9})} = 10.8 \times 10^{-3} \text{ m}$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2}(3.604 \times 10^3)(10.8 \times 10^{-3}) = 19.4616 \text{ J}$$

$$\frac{1}{2} m v_0^2 = U_m$$

$$v_0 = \sqrt{\frac{2U_m}{m}} = \sqrt{\frac{(2)(19.4616)}{6}} = 2.55 \text{ m/s}$$

Problem 11.49



11.49 Solve Prob. 11.48, assuming that the post AB consists of a solid steel rod of 80-mm diameter.

11.48 The post AB consists of a steel pipe of 80-mm-diameter and 6-mm wall thickness. A 6-kg block C moving horizontally with at velocity v_0 hits the post squarely at A . Using $E = 200$ GPa, determine the largest speed v_0 for which the maximum normal stress in the pipe does not exceed 180 MPa.

$$C = \frac{1}{2}d = \frac{1}{2}(80) = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$I = \frac{\pi}{4}C^4 = \frac{\pi}{4}(40 \times 10^{-3})^4 = 2.01062 \times 10^{-6} \text{ m}^4$$

$$\sigma_m = 180 \times 10^6 \text{ Pa} = \frac{M_m C}{I}$$

$$M_m = \frac{I \sigma_m}{C} = \frac{(2.01062 \times 10^{-6})(180 \times 10^6)}{40 \times 10^{-3}} = 9.048 \times 10^3 \text{ N}\cdot\text{m}$$

$$P_m = \frac{M_m}{L} = \frac{9.048 \times 10^3}{1.2} = 7.5398 \times 10^3 \text{ N}$$

By Appendix D, Case 1 $y_m = \frac{P_m L^3}{3EI}$

$$y_m = \frac{(7.5398 \times 10^3)(1.2)^3}{(3)(200 \times 10^9)(2.01062 \times 10^{-6})} = 10.8 \times 10^{-3} \text{ m}$$

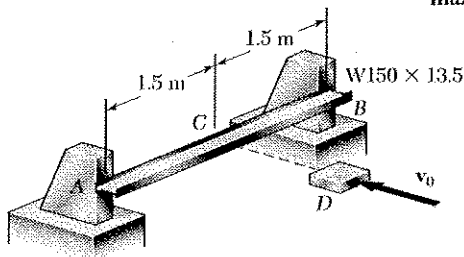
$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2}(7.5398 \times 10^3)(10.8 \times 10^{-3}) = 40.715 \text{ J}$$

$$\frac{1}{2} m v_0^2 = U_m$$

$$v_0 = \sqrt{\frac{2U_m}{m}} = \sqrt{\frac{(2)(40.715)}{6}} = 3.68 \text{ m/s}$$

Problem 11.50

11.50 The steel beam AB is struck squarely at its midpoint C by a 45-kg block moving horizontally with a speed $v_0 = 2$ m/s. Using $E = 200$ GPa, determine the equivalent static load, (b) the maximum normal stress in the beam, (c) the maximum deflection of the midpoint C of the beam.



From Appendix C, for $W150 \times 13.5$

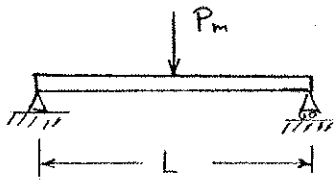
$$I_x = 6.87 \times 10^6 \text{ mm}^4 = 6.87 \times 10^{-6} \text{ m}^4$$

$$S_x = 91.6 \times 10^3 \text{ mm}^3 = 91.6 \times 10^{-6} \text{ m}^3$$

$$\text{Kinetic energy: } T = \frac{1}{2} m v_0^2 = \frac{1}{2} (45) (2)^2 = 90 \text{ J}$$

From Appendix D, Case 4

$$|y_m| = \frac{P L^3}{48 E I}, \quad M_{\max} = \frac{P L}{4}$$



$$U = \frac{1}{2} P_m |y_m| = \frac{P_m^2 L^3}{96 E I} = T$$

$$(a) \quad P_m = \sqrt{\frac{96 E I T}{L^3}} = \sqrt{\frac{(96)(200 \times 10^9)(6.87 \times 10^{-6})(90)}{(3.0)^3}} = 20.968 \times 10^3 \text{ N} = 21.0 \text{ kN}$$

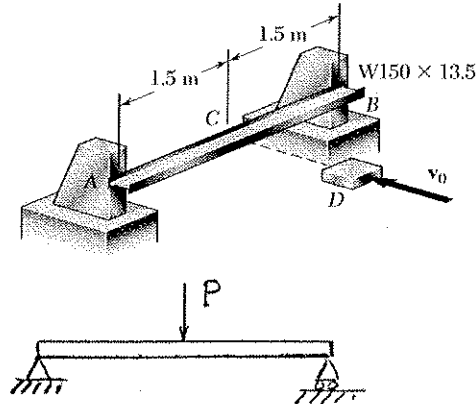
$$(b) \quad \sigma_m = \frac{M_{\max}}{S} = \frac{P_m L}{4 S} = \frac{(20.968 \times 10^3)(3.0)}{(4)(91.6 \times 10^{-6})} = 171.7 \times 10^6 \text{ Pa} = 171.7 \text{ MPa}$$

$$(c) \quad |y_m| = \frac{2U}{P_m} = \frac{(2)(90)}{20.968 \times 10^3} = 8.58 \times 10^{-3} \text{ m} = 8.58 \text{ mm}$$

Problem 11.51

11.51 Solve Prob. 11.50, assuming that the W150 × 13.5 rolled-steel beam is rotated by 90° about its longitudinal axis so that its web is vertical.

11.50 The steel beam AB is struck squarely at its midpoint C by a 45-kg block moving horizontally with a speed $v_0 = 2$ m/s. Using $E = 200$ GPa, determine the equivalent static load, (b) the maximum normal stress in the beam, (c) the maximum deflection of the midpoint C of the beam.



From Appendix C, for W 150 × 13.5

$$I_y = 0.918 \times 10^6 \text{ mm}^4 = 0.918 \times 10^{-6} \text{ m}^4$$

$$S_y = 18.4 \times 10^3 \text{ mm}^3 = 18.4 \times 10^{-6} \text{ m}^3$$

$$\text{Kinetic energy: } T = \frac{1}{2} m v_0^2 = \frac{1}{2} (45)(2)^2 = 90 \text{ J}$$

From Appendix D, Case 4

$$|y_m| = \frac{P L^3}{48 E I} \quad M_{\max} = \frac{P L}{4}$$

$$U = \frac{1}{2} P_m |y_m| = \frac{P_m^2 L^3}{96 E I} = T$$

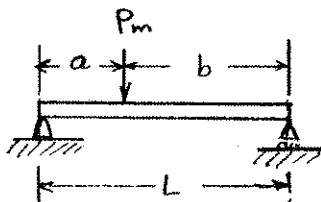
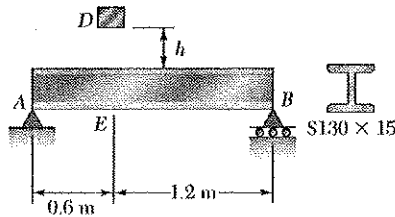
$$(a) \quad P_m = \sqrt{\frac{96 E I T}{L^3}} = \sqrt{\frac{(96)(200 \times 10^9)(0.918 \times 10^{-6})(90)}{(3.0)^3}} = 7.665 \times 10^3 \text{ N} = 7.67 \text{ kN}$$

$$(b) \quad \sigma_m = \frac{M_{\max}}{S} = \frac{P_m L}{4 S} = \frac{(7.665 \times 10^3)(3.0)}{(4)(18.4 \times 10^{-6})} = 312 \times 10^6 \text{ Pa} = 312 \text{ MPa}$$

$$(c) \quad |y_m| = \frac{2U}{P_m} = \frac{(2)(90)}{7.665 \times 10^3} = 23.5 \times 10^{-3} \text{ m} = 23.5 \text{ mm}$$

Problem 11.52

11.52 The 20-kg block D is dropped from a height $h = 0.18$ m onto the steel beam AB . Knowing that $E = 200$ GPa, determine (a) the maximum deflection at point E , (b) the maximum normal stress in the beam.



$$I_x = 5107 \times 10^{-6} \text{ m}^4 \quad S_x = 79.8 \times 10^{-6} \text{ m}^3$$

$$a = 0.6 \text{ m} \quad b = 1.2 \text{ m}$$

$$L = 1.8 \text{ m} \quad h = 0.18 \text{ m}$$

$$R_A = \frac{P_m b}{L}, \quad R_B = \frac{P_m a}{L}, \quad M_m = M_E = \frac{P_m a b}{L}$$

From Appendix D Case 5

$$y_E = \frac{P_m a^2 b^2}{3 E I L} = \frac{P_m (0.6)^2 (1.2)^2}{(3)(200 \times 10^9)(5107 \times 10^{-6})(1.8)} = 94.67 \times 10^{-9} P_m$$

$$P_m = 10.563 \times 10^6 y_E$$

$$U_m = \frac{1}{2} P_m y_m = 5.2815 \times 10^6 y_E^2$$

Work of falling weight: $W(h + y_E) = (20 \times 9.81)(0.18 + y_E) = 35.3 + 196 y_E$

Equating work and energy, $35.3 + 196 y_E = 5.2815 \times 10^6 y_E^2$

$$y_E^2 - 37.1 \times 10^{-6} y_E - 6.68 \times 10^{-6} = 0$$

$$(a) \quad y_E = \frac{1}{2} \left[37.1 \times 10^{-6} + \sqrt{(37.1 \times 10^{-6})^2 - (4)(-6.68 \times 10^{-6})} \right] = 4.01 \times 10^{-3} \text{ m}$$

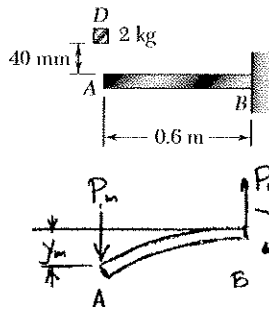
$$(b) \quad P_m = (10.563 \times 10^6)(0.00401) = 42357 \text{ N} \quad = 4 \text{ mm}$$

$$M_m = \frac{(42357)(0.6)(1.2)}{1.8} = 16942.8 \text{ Nm}$$

$$\sigma_m = \frac{M_m}{S_x} = \frac{16942.8}{79.8 \times 10^{-6}} = 212.3 \text{ MPa}$$

Problem 11.53

11.53 and 11.54 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (a) the maximum deflection of end A , (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.



$$I = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} \left(\frac{16}{2} \right)^4 = 3.2170 \times 10^3 \text{ mm}^4 = 3.2170 \times 10^{-9} \text{ m}^4$$

$$c = \frac{d}{2} = 8 \text{ mm} = 8 \times 10^{-3} \text{ m} \quad L_{AB} = 0.6 \text{ m}$$

Appendix D, Case 1

$$y_m = \frac{P_m L_{AB}^3}{3EI} \quad M_m = P_m L_{AB}$$

$$P_m = \frac{3EI}{L_{AB}^3} y_m = \frac{(3)(200 \times 10^9)(3.217 \times 10^{-9})}{(0.6)^3} = 8.9361 \times 10^3 y_m$$

$$U_m = \frac{1}{2} P_m y_m = \frac{1}{2} (8.9361 \times 10^3) y_m^2 = 4.4681 \times 10^3 y_m^2$$

$$\begin{aligned} \text{Work of dropped weight: } mg(h + y_m) &= (2)(9.81)(0.040 + y_m) \\ &= 0.7848 + 19.62 y_m \end{aligned}$$

Equating work and energy,

$$0.7848 + 19.62 y_m = 4.4681 \times 10^3 y_m^2$$

$$y_m^2 - 4.3911 \times 10^{-3} y_m - 175.645 \times 10^{-6} = 0$$

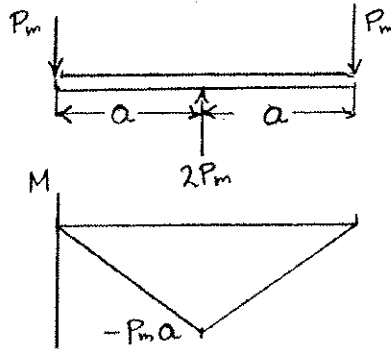
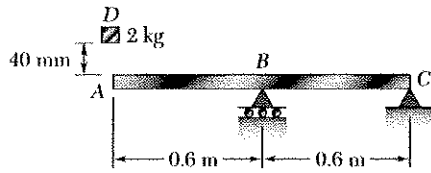
$$\begin{aligned} (a) \quad y_m &= \frac{1}{2} \left\{ 4.3911 \times 10^{-3} + \sqrt{(4.3911 \times 10^{-3})^2 + (4)(175.645 \times 10^{-6})} \right\} \\ &= 15.629 \times 10^{-3} \text{ m} \quad y_m = 15.63 \text{ mm} \quad \blacktriangleleft \end{aligned}$$

$$P_m = (8.9361 \times 10^3)(15.629 \times 10^{-3}) = 139.66 \text{ N}$$

$$(b) \quad M_m = -P_m L_{AB} = -(139.66)(0.6) \quad |M_m| = 83.8 \text{ N}\cdot\text{m} \quad \blacktriangleleft$$

$$\begin{aligned} (c) \quad \sigma_m &= \frac{|M_m|c}{I} = \frac{(83.8)(8 \times 10^{-3})}{3.2170 \times 10^{-9}} = 208 \times 10^6 \text{ Pa} \\ \sigma_m &= 208 \text{ MPa} \quad \blacktriangleleft \end{aligned}$$

Problem 11.54



11.53 and 11.54 The 2-kg block *D* is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (a) the maximum deflection of end *A*, (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.

$$I = \frac{\pi}{4} \left(\frac{d}{2} \right)^4 = \frac{\pi}{4} \left(\frac{16}{2} \right)^4 = 3.2170 \times 10^3 \text{ mm}^4 = 3.2170 \times 10^{-9} \text{ m}^4$$

$$c = \frac{d}{2} = 8 \text{ mm} = 8 \times 10^{-3} \text{ m} \quad a = 0.6 \text{ m}$$

Over AB: $M = -P_m x$ $M_m = -P_m a$

$$U_{AB} = \int_0^a \frac{P_m^2 x^2}{2EI} dx = \frac{P_m^2 a^3}{6EI}$$

$$= \frac{(0.6)^3}{(6)(200 \times 10^9)(3.2170 \times 10^{-9})} P_m^2$$

$$= 55.953 \times 10^{-6} P_m^2$$

By symmetry of bending moment diagram,

$$U_{BC} = U_{AB} = 55.953 \times 10^{-6} P_m^2$$

$$U_m = U_{AB} + U_{BC} = 111.906 \times 10^{-6} P_m^2$$

$$\frac{1}{2} P_m y_m = U_m = 111.906 \times 10^{-6} P_m^2 \quad P_m = 4.4681 \times 10^3 y_m$$

$$U_m = \frac{1}{2} P_m y_m = 2.2340 \times 10^3 y_m^2$$

Work of dropped weight: $mg(h + y_m) = (2)(9.81)(0.040 + y_m)$

$$= 0.7848 + 19.62 y_m$$

Equating work and energy,

$$0.7848 + 19.62 y_m = 2.2340 \times 10^3 y_m^2$$

$$y_m^2 - 8.7825 \times 10^{-3} y_m - 351.298 \times 10^{-6} = 0$$

$$(a) \quad y_m = \frac{1}{2} \left\{ 8.7825 \times 10^{-3} + \sqrt{(8.7825 \times 10^{-3})^2 + (4)(351.298 \times 10^{-6})} \right\}$$

$$= 23.636 \times 10^{-3} \text{ m} = 23.6 \text{ mm} \quad y_m = 23.6 \text{ mm} \leftarrow$$

$$P_m = (4.4681 \times 10^3)(23.636 \times 10^{-3}) = 105.61 \text{ N}$$

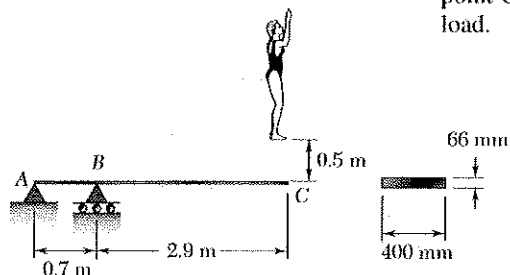
$$(b) \quad M_m = -(105.61)(0.6) = -64.4 \text{ N}\cdot\text{m} \quad |M_m| = 64.4 \text{ N}\cdot\text{m} \leftarrow$$

$$(c) \quad \sigma_m = \frac{|M_m|c}{I} = \frac{(64.4)(8 \times 10^{-3})}{3.2170 \times 10^{-9}} = 157.6 \times 10^6 \text{ Pa}$$

$$\sigma_m = 157.6 \text{ MPa} \leftarrow$$

Problem 11.55

11.55 A 72-kg diver jumps from a height of 0.5 m onto end C of a diving board having the uniform cross section shown. Assuming that the diver's legs remain rigid and using $E = 12$ GPa, determine (a) the maximum deflection at point C, (b) the maximum normal stress in the board, (c) the equivalent static load.

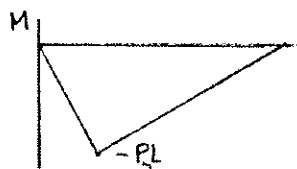
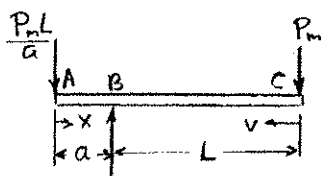


$$W = 72 \times 9.81 = 706.32 \text{ N}$$

$$I = \frac{1}{12} (0.4)(0.066)^3 = 9.583 \times 10^{-6} \text{ m}^4$$

$$L = 2.9 \text{ m}, \quad a = 0.7 \text{ m}$$

$$c = \frac{1}{2} (0.066) = 0.033 \text{ m}$$



Over portion AB: $M = -\frac{P_m L}{a} x$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \frac{P_m^2 L^2}{2EI a^2} \int_0^a x^2 dx = \frac{P_m^2 L^2 a}{6EI}$$

Over portion BC: $M = -P_m v$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dv = \frac{P_m^2}{2EI} \int_0^L v^2 dv = \frac{P_m^2 L^3}{6EI}$$

Total $U = U_{AB} + U_{BC} = \frac{P_m^2 L^2 (a+L)}{6EI}$

$$\frac{1}{2} P_m y_m = U_m \quad y_m = \frac{2U_m}{P_m} = \frac{P_m L^2 (a+L)}{3EI}$$

$$P_m = \frac{3EI}{L^2 (a+L)} y_m = \frac{(3)(12 \times 10^9)(9.583 \times 10^{-6})}{(2.9)^2 (2.9 + 0.7)} y_m = 11394.8 y_m$$

$$U_m = \frac{1}{2} P_m y_m = 5697.4 y_m^2$$

Work of weight = $W(h + y_m) = (706.32)(0.5 + y_m) = 353.16 + 706.32 y_m$

Equating: $353.16 + 706.32 y_m = 5697.4 y_m^2$

$$y_m^2 - 0.124 y_m - 0.062 = 0$$

(a) $y_m = \frac{1}{2} \left\{ 0.124 + \sqrt{0.124^2 + (4)(0.062)} \right\} =$

$$y_m = 318.6 \text{ mm} \blacktriangleleft$$

(c) $P_m = (11394.8)(0.3186) = 3630 \text{ N}$

$$P_m = 3.63 \text{ kN} \blacktriangleleft$$

$$M_m = -(3630)(2.9) = 10527 \text{ N m}$$

(b) $\sigma_m = \frac{M_m c}{I} = \frac{(10527)(0.033)}{9.583 \times 10^{-6}} = 36.25 \text{ MPa}$

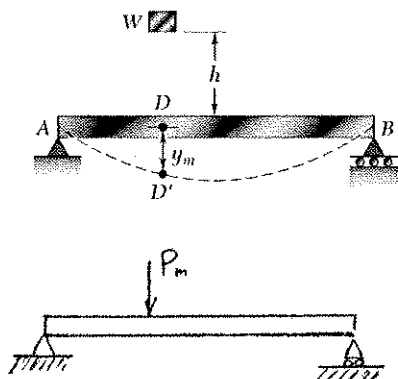
$$\sigma_m = 36.25 \text{ MPa} \blacktriangleleft$$

Problem 11.56

11.56 A block of weight W is dropped from a height h onto the horizontal beam AB and hits it at point D . (a) Show that the maximum deflection y_m at point D can be expressed as

$$y_m = y_{st} \left(1 + \sqrt{1 + \frac{2h}{y_{st}}} \right)$$

where y_{st} represents the deflection at D caused by a static load W applied at that point and where the quantity in parenthesis is referred to as the *impact factor*. (b) Compute the impact factor for the beam and the impact of Prob. 11.53.



Work of falling weight: $Work = W(h + y_m)$

Strain energy: $U = \frac{1}{2} P y_m = \frac{1}{2} k y_m^2$

where k is the spring constant for a load applied at point D .

Equating work and energy,

$$W(h + y_m) = \frac{1}{2} k y_m^2$$

$$y_m^2 - \frac{2W}{k} y_m - \frac{2W}{k} h = 0$$

$$y_m^2 - 2y_{st} y_m - 2y_{st} h = 0 \quad \text{where } y_{st} = \frac{W}{k}$$

$$(a) \quad y_m = \frac{2y_{st} + \sqrt{4y_{st}^2 + 8y_{st}h}}{2} = y_{st} \left(1 + \sqrt{1 + \frac{2h}{y_{st}}} \right)$$

For Prob. 11.53

$$W = mg = (2)(9.81) = 19.62 \text{ N}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = \frac{\pi}{4} \left(\frac{16}{2} \right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$$

$$L = 0.6 \text{ m}$$

$$h = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

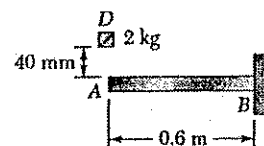
Using Appendix D Case 1

$$y_{st} = \frac{WL^3}{3EI}$$

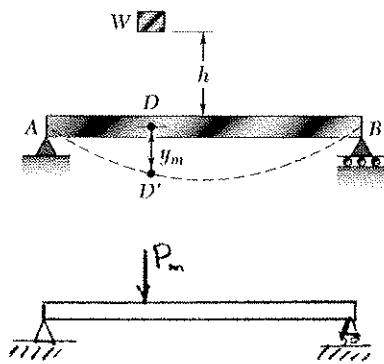
$$y_{st} = \frac{(19.62)(0.6)^3}{(3)(200 \times 10^9)(3.217 \times 10^{-9})} = 2.196 \times 10^{-3} \text{ m}$$

$$\frac{2h}{y_{st}} = \frac{(2)(40 \times 10^{-3})}{2.196 \times 10^{-3}} = 36.44$$

$$(b) \quad \text{impact factor} = 1 + \sqrt{1 + 36.44} = 7.12$$

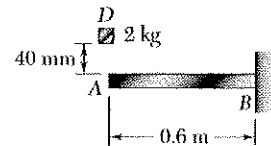


Problem 11.57



11.57 A block of weight W is dropped from a height h onto the horizontal beam AB and hits point D . (a) Denoting by y_m the exact value of the maximum deflection at D and by y'_m the value obtained by neglecting the effect of this deflection on the change in potential energy of the block, show that the absolute value of the relative error is $(y'_m - y_m)/y_m$, never exceeding $y'_m/2h$. (b) Check the result obtained in part (a) by solving part (a) of Prob. 11.53 without taking y_m into account when determining the change in potential energy of the load, and comparing the answer obtained in this way with the exact answer to that problem.

11.53 and 11.54 The 2-kg block D is dropped from the position shown onto the end of a 16-mm-diameter rod. Knowing that $E = 200$ GPa, determine (a) the maximum deflection of end A , (b) the maximum bending moment in the rod, (c) the maximum normal stress in the rod.



$$U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2 \quad \text{where } k \text{ is the spring constant for a load at point } D.$$

$$\text{Work of falling weight:} \quad \begin{array}{l} \text{exact: } \text{Work} = W(h + y_m) \\ \text{approximate: } \text{Work} \approx Wh \end{array}$$

$$\text{Equating work and energy,} \quad \begin{array}{l} \frac{1}{2} k y_m^2 = W(h + y_m) \quad (1) \text{ exact} \\ \frac{1}{2} k y_m'^2 = Wh \quad (2) \text{ approximate} \end{array}$$

where y'_m is the approximate value for y_m .

$$\text{Subtracting,} \quad \frac{1}{2} k (y_m^2 - y_m'^2) = W y_m$$

$$y_m^2 - y_m'^2 = (y_m - y_m')(y_m + y_m') = \frac{2W}{k} y_m$$

$$\text{Relative error: } \frac{y_m - y_m'}{y_m} = \frac{2W}{k(y_m + y_m')}$$

$$\text{But, } \frac{2W}{k} = \frac{y_m'^2}{h} \quad \text{from equation (2).}$$

$$(a) \text{ Relative error} = \frac{y_m - y_m'}{y_m} = \frac{y_m'^2}{h(y_m + y_m')} < \frac{y_m'}{2h}$$

$$(b) \text{ From the solution to Prob. 11.53, } y_m = 15.63 \text{ mm}$$

$$\text{Approximate solution: } W = mg = (2)(9.81) = 19.62 \text{ N}$$

$$E = 200 \times 10^9 \text{ Pa} \quad I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{16}{2}\right)^4 = 3.217 \times 10^3 \text{ mm}^4 = 3.217 \times 10^{-9} \text{ m}^4$$

$$L = 0.6 \text{ m}, \quad h = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$$

$$k = \frac{3EI}{L^3} = \frac{(3)(200 \times 10^9)(3.217 \times 10^{-9})}{(0.6)^3} = 8.936 \times 10^3 \text{ N/m}$$

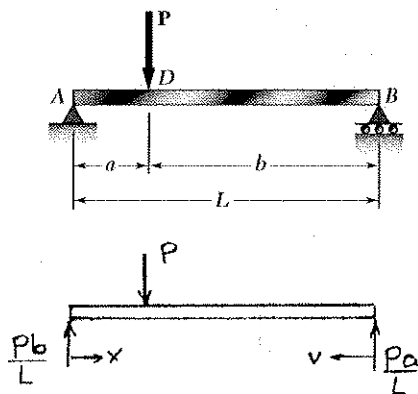
$$y_m'^2 = \frac{2Wh}{k} = \frac{(2)(19.62)(40 \times 10^{-3})}{8.936 \times 10^3} = 175.65 \times 10^{-6} \text{ m}^2$$

$$y_m' = 13.25 \times 10^{-3} \text{ m} = 13.25 \text{ mm}$$

$$\text{relative error} = \frac{15.63 - 13.25}{15.63} = 0.152 \quad \frac{y_m'}{2h} = 0.166$$

Problem 11.58

11.58 and 11.59 Using the method of work and energy, determine the deflection at point D caused by the load P.



$$\text{Reactions: } R_A = \frac{Pb}{L}, \quad R_B = \frac{Pa}{L}$$

$$\text{Over AD: } M = R_A x = \frac{Pbx}{L}$$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2 b^2}{2EI L^2} \int_0^a x^2 dx = \frac{P^2 b^2 a^3}{6EI L^2}$$

$$\text{Over DB: } M = R_B v = \frac{Pav}{L}$$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EI L^2} \int_0^b v^2 dv = \frac{P^2 a^2 b^3}{6EI L^2}$$

$$\text{Total: } U = U_{AD} + U_{DB} = \frac{P^2 a^2 b^2 (a+b)}{6EI L^2} = \frac{P^2 a^2 b^2}{6EI L}$$

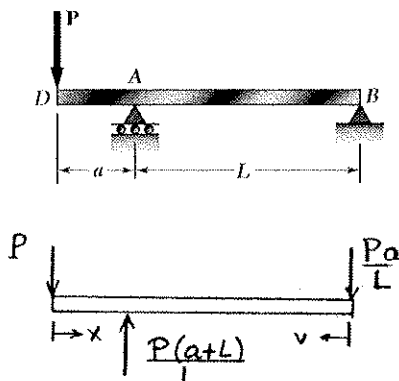
$$\frac{1}{2} P S_D = U$$

$$S_D = \frac{2U}{P}$$

$$S_D = \frac{Pa^2 b^2}{3EI} \downarrow$$

Problem 11.59

11.58 and 11.59 Using the method of work and energy, determine the deflection at point D caused by the load P.



$$\sum M_A = 0: Pa + R_B L = 0 \quad R_B = -\frac{Pa}{L}$$

$$\text{Over portion DA: } M = -Px$$

$$U_{DA} = \int_0^a \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^a x^2 dx = \frac{P^2 a^3}{6EI}$$

$$\text{Over portion AB: } M = -\frac{Pa v}{L}$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv = \frac{P^2 a^2}{2EI L^2} \int_0^L v^2 dv = \frac{Pa^2 L}{6EI}$$

$$\text{Total: } U = U_{DA} + U_{AB} = \frac{P^2 a^2 (a+L)}{6EI}$$

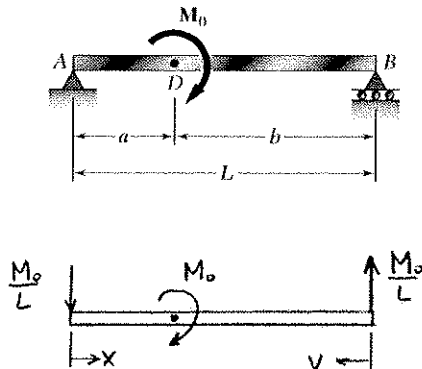
$$\frac{1}{2} P S_D = U$$

$$S_D = \frac{2U}{P}$$

$$S_D = \frac{Pa^2(a+L)}{3EI} \downarrow$$

Problem 11.60

11.60 and 11.61 Using the method of work and energy, determine the slope at point D caused by the couple M_0 .



Reactions: $R_A = \frac{M_0}{L} \downarrow$ $R_B = \frac{M_0}{L} \uparrow$

Over portion AD: $M = -\frac{M_0 x}{L}$

$$U_{AD} = \int_0^a \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^a x^2 dx$$

$$= \frac{M_0^2 a^3}{6EI L^2}$$

Over portion DB: $M = \frac{M_0 v}{L}$

$$U_{DB} = \int_0^b \frac{M^2}{2EI} dv = \frac{M_0^2}{2EI L^2} \int_0^b v^2 dv$$

$$= \frac{M_0^2 b^3}{6EI L^2}$$

Total: $U = U_{AD} + U_{DB} = \frac{M_0^2 (a^3 + b^3)}{6EI L^2}$

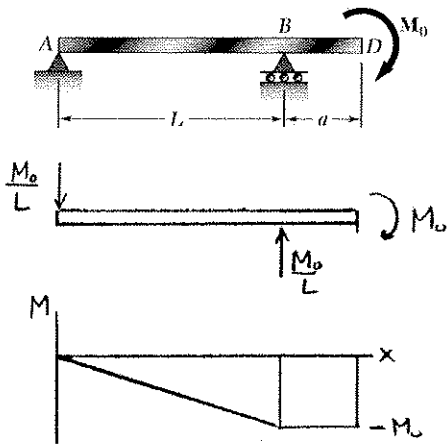
$$\frac{1}{2} M_0 \theta_D = U$$

$$\theta_D = \frac{2U}{M_0}$$

$$\theta_D = \frac{M_0 (a^3 + b^3)}{3EI L^2} \quad \swarrow$$

Problem 11.61

11.60 and 11.61 Using the method of work and energy, determine the slope at point D caused by the couple M_0 .



Reactions: $R_A = \frac{M_0}{L} \downarrow$ $R_B = \frac{M_0}{L} \uparrow$

Over portion AB: $M = -\frac{M_0 x}{L}$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2}{2EI L^2} \int_0^L x^2 dx$$

$$= \frac{M_0^2 L}{6EI}$$

Over portion BD: $M = -M_0$

$$U_{BD} = \frac{M_0^2 a}{2EI}$$

Total: $U = U_{AB} + U_{BD} = \frac{M_0^2 (L + 3a)}{6EI}$

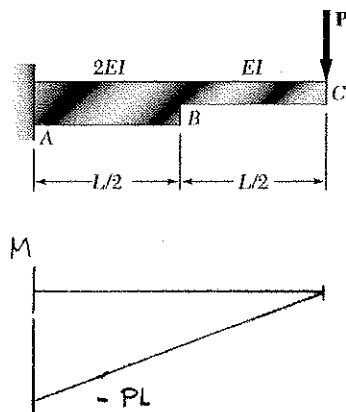
$$\frac{1}{2} M_0 \theta_D = U$$

$$\theta_D = \frac{2U}{M_0}$$

$$\theta_D = \frac{M_0 (L + 3a)}{3EI} \quad \swarrow$$

Problem 11.62

11.62 and 11.63 Using the method of work and energy, determine the deflection at point C caused by the load P.



Bending moment. $M = -Pv$

Over AB:

$$U_{AB} = \int_{L/2}^L \frac{M^2}{4EI} dv = \frac{P^2}{4EI} \int_{L/2}^L v^2 dv$$

$$= \frac{P^2}{12EI} \left[L^3 - \left(\frac{L}{2}\right)^3 \right] = \frac{7}{96} \frac{P^2 L^3}{EI}$$

Over BC: $U_{BC} = \int_0^{L/2} \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_0^{L/2} v^2 dv$

$$= \frac{1}{48} \frac{P^2 L^3}{EI}$$

Total: $U = U_{AB} + U_{BC} = \frac{3}{32} \frac{P^2 L^3}{EI}$

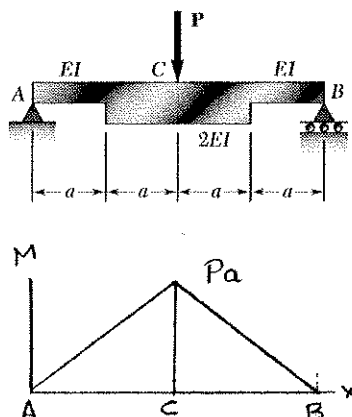
$$\frac{1}{2} P \delta_c = U$$

$$\delta_c = \frac{2U}{P}$$

$$\delta_c = \frac{3 PL^3}{16 EI} \downarrow \blacktriangleleft$$

Problem 11.63

11.62 and 11.63 Using the method of work and energy, determine the deflection at point C caused by the load P.



Symmetric beam and loading: $R_A = R_B = \frac{1}{2} P$

From A to C: $M = R_A x = \frac{1}{2} Px$

$$U_{AC} = \int_0^a \frac{M^2}{2EI} dx + \int_a^{2a} \frac{M^2}{4EI} dx$$

$$= \frac{P^2}{8EI} \int_0^a x^2 dx + \frac{P^2}{16EI} \int_a^{2a} x^2 dx$$

$$= \frac{P^2 a^3}{24EI} + \frac{P^2}{48EI} [(2a)^3 - a^3] = \frac{3}{16} \frac{P^2 a^3}{EI}$$

By symmetry, $U_{CB} = U_{AB} = \frac{3}{16} \frac{P^2 a^3}{EI}$

Total: $U = U_{AB} + U_{BC} = \frac{3}{8} \frac{P^2 a^3}{EI}$

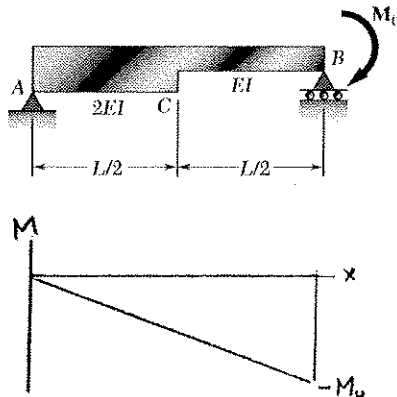
$$\frac{1}{2} P \delta_c = U$$

$$\delta_c = \frac{2U}{P}$$

$$\delta_c = \frac{3 Pa^3}{4 EI} \downarrow \blacktriangleleft$$

Problem 11.64

11.64 Using the method of work and energy, determine the slope at point B caused by the couple M_0 .



$$\sum M_A = 0: -R_B L - M_0 = 0 \quad R_B = -\frac{M_0}{L}$$

$$M = R_B x = -\frac{M_0}{L} x$$

Over portion AC: $U_{AC} = \int_0^{\frac{L}{2}} \frac{M^2}{2(2EI)} dx$

$$U_{AC} = \frac{M_0^2}{4EIL^2} \int_0^{\frac{L}{2}} x^2 dx = \frac{1}{96} \frac{M_0^2 L}{EI}$$

Over portion CB: $U_{CB} = \int_{\frac{L}{2}}^L \frac{M^2}{2EI} dx$

$$U_{CB} = \frac{M_0^2}{2EIL^2} \int_{\frac{L}{2}}^L x^2 dx = \frac{M_0^2}{6EIL^2} \left[L^3 - \left(\frac{L}{2}\right)^3 \right]$$

$$= \frac{7}{48} \frac{M_0^2 L}{EI}$$

Total: $U = U_{AC} + U_{CB} = \frac{5}{32} \frac{M_0^2 L}{EI}$

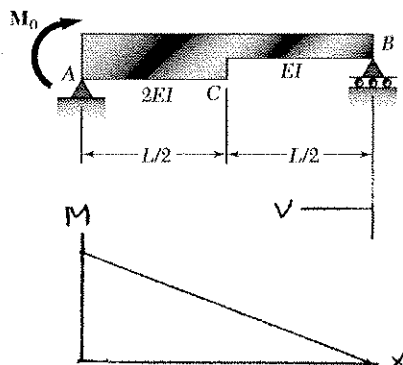
$$\frac{1}{2} M_0 \theta_B = U$$

$$\theta_B = \frac{2U}{M_0}$$

$$\theta_B = \frac{5M_0 L}{16EI} \quad \swarrow$$

Problem 11.65

11.65 Using the method of work and energy, determine the slope at point A caused by the couple M_0 .



$$R_B = \frac{M_0}{L}$$

$$M = R_B v = \frac{M_0}{L} v$$

Over AC: $U_{AC} = \int_{\frac{L}{2}}^L \frac{M^2}{2(2EI)} dv$

$$U_{AC} = \frac{M_0^2}{4EIL^2} \int_{\frac{L}{2}}^L v^2 dv = \frac{M_0^2}{12EIL^2} \left[L^3 - \left(\frac{L}{2}\right)^3 \right]$$

$$= \frac{7}{96} \frac{M_0^2 L}{EI}$$

Over CB: $U_{CB} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv$

$$U_{CB} = \frac{M_0^2}{2EIL^2} \int_0^{\frac{L}{2}} v^2 dv = \frac{1}{48} \frac{M_0^2 L}{EI}$$

Total: $U = U_{AC} + U_{CB} = \frac{3}{32} \frac{M_0^2 L}{EI}$

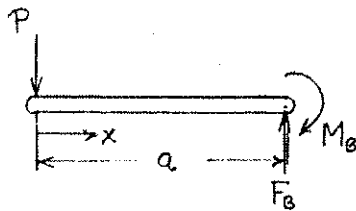
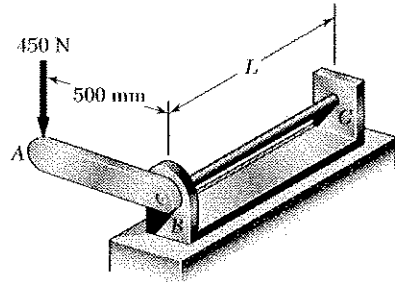
$$\frac{1}{2} M_0 \theta_A = U$$

$$\theta_A = \frac{2U}{M_0}$$

$$\theta_A = \frac{3M_0 L}{16EI} \quad \swarrow$$

Problem 11.66

11.66 The 20-mm-diameter steel rod BC is attached to the lever AB and to the fixed support C . The uniform steel lever is 10 mm thick and 30 mm deep. Using the method of work and energy, determine the deflection of point A when $L = 600$ mm. Use $E = 200$ GPa and $G = 77.2$ GPa.



Member AB. (Bending)

$$I = \frac{1}{12} (10)(30)^3 = 22.5 \times 10^3 \text{ mm}^4 = 22.5 \times 10^{-9} \text{ m}^4$$

$$a = 500 \text{ mm} = 0.500 \text{ m}$$

$$M_B = Pa = (450)(0.500) = 225 \text{ N}\cdot\text{m}$$

$$M = Px$$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \int_0^a \frac{P^2 x^2}{2EI} dx = \frac{P^2 a^3}{6EI}$$

$$= \frac{(450)^2 (0.500)^3}{(6)(200 \times 10^9)(22.5 \times 10^{-9})} = 0.9375 \text{ J}$$

Member BC. (Torsion)

$$T = M_B = 225 \text{ N}\cdot\text{m} \quad c = \frac{1}{2}d = 10 \text{ mm} \quad J = \frac{\pi}{2} c^4 = 15.708 \times 10^3 \text{ mm}^4 = 15.708 \times 10^{-9} \text{ m}^4$$

$$L = 600 \text{ mm} = 0.600 \text{ m}$$

$$U_{BC} = \frac{T^2 L}{2GJ} = \frac{(225)^2 (0.600)}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 12.5242 \text{ J}$$

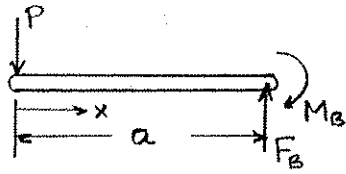
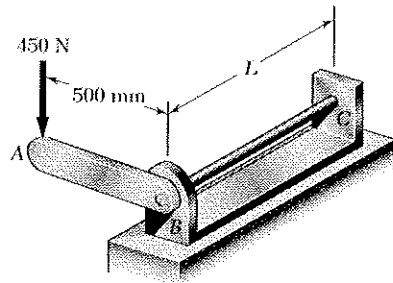
$$\text{Total: } U = U_{AB} + U_{BC} = 0.9375 + 12.5242 = 13.4617 \text{ J}$$

$$\frac{1}{2} P \delta_A = U \quad \delta_A = \frac{2U}{P} = \frac{(2)(13.4617)}{450} = 59.8 \times 10^{-3} \text{ m}$$

$$\delta_A = 59.8 \text{ mm} \quad \blacktriangleleft$$

Problem 11.67

11.67 The 20-mm-diameter steel rod BC is attached to the lever AB and to the fixed support C . The uniform steel lever is 10 mm thick and 30 mm deep. Using the method of work and energy, determine the length L of the rod BC for which the deflection at point A is 40 mm. Use $E = 200$ GPa and $G = 77.2$ GPa.



Member AB. (Bending)

$$I = \frac{1}{12}(10)(30)^3 = 22.5 \times 10^3 \text{ mm}^4 = 22.5 \times 10^{-9} \text{ m}^4$$

$$a = 500 \text{ mm} = 0.500 \text{ m}$$

$$M_B = Pa = (450)(0.500) = 225 \text{ N}\cdot\text{m}$$

$$M = Px$$

$$U_{AB} = \int_0^a \frac{M^2}{2EI} dx = \int_0^a \frac{P^2 x^2}{2EI} dx = \frac{P^2 a^3}{6EI}$$

$$= \frac{(450)^2 (0.500)^3}{(6)(200 \times 10^9)(22.5 \times 10^{-9})} = 0.9375 \text{ J}$$

Member BC (Torsion)

$$T = M_B = 225 \text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 10 \text{ mm} \quad J = \frac{\pi}{2}c^4 = 15.708 \times 10^3 \text{ mm}^4 = 15.708 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T^2 L}{2GJ} = \frac{(225)^2 L}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 20.874 L$$

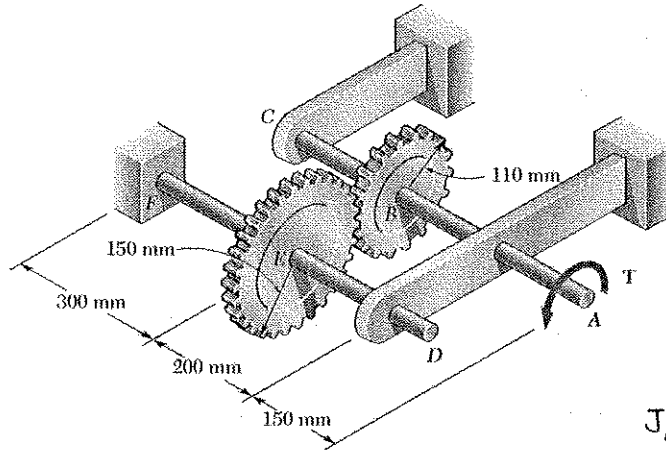
$$\frac{1}{2} P \delta_A = U_{AB} + U_{BC}$$

$$\frac{1}{2}(450)(40 \times 10^{-3}) = 0.9375 + 20.874 L$$

$$L = 0.386 \text{ m} = 386 \text{ mm}$$

Problem 11.68

11.68 Two steel shafts, each of 22-mm diameter, are connected by the gears shown. Knowing that $G = 77$ GPa and that shaft DF is fixed at F , determine the angle through which end A rotates when a $135\text{-N}\cdot\text{m}$ torque is applied at A . Ignore the strain energy due to the bending of the shafts.



Work-energy equation:

$$\frac{1}{2} T_A \phi_A = U$$

$$\phi_A = \frac{2U}{T_A}$$

Portion AB of shaft ABC:

$$T_{AB} = T_A = 135 \text{ N}\cdot\text{m}$$

$$L_{AB} = 0.2 + 0.15 = 0.35 \text{ m}$$

$$J_{AB} = \frac{\pi}{2} \left(\frac{d}{2} \right)^4 = \frac{\pi}{2} (11)^4 = 22998 \text{ mm}^4$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2GJ_{AB}} = \frac{(135)^2 (0.35)}{(2)(77 \times 10^9)(22998 \times 10^{-12})} = 1.8 \text{ J}$$

Portion BC of shaft ABC: $U_{BC} = 0$

$$\text{Gear B: } F_{BE} = \frac{T_B}{r_B} = \frac{T_{AB}}{r_B} = \frac{135}{0.11} = 1227.3 \text{ N}$$

$$\text{Gear E: } T_E = r_E F_{BE} = (0.15)(1227.3) = 184 \text{ N}\cdot\text{m}$$

Portion DE of shaft DEF: $U_{DE} = 0$

Portion EF of shaft DEF: $T_{EF} = T_E = 184 \text{ N}\cdot\text{m}$

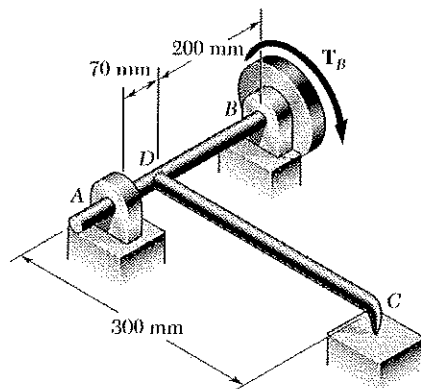
$$L_{EF} = 12 \text{ in. } J_{EF} = J_{AB} = 22998 \text{ mm}^4$$

$$U_{EF} = \frac{T_{EF}^2 L_{EF}}{2GJ_{EF}} = \frac{(184)^2 (0.3)}{(2)(77 \times 10^9)(22998 \times 10^{-12})} = 2.87 \text{ J}$$

$$\text{Total: } U = U_{AB} + U_{BC} + U_{DE} + U_{EF} = 4.67 \text{ J}$$

$$\phi_A = \frac{2U}{T_A} = \frac{(2)(4.67)}{135} = 69.19 \times 10^{-3} \text{ rad} = 3.96^\circ$$

Problem 11.69



11.69 The 20-mm-diameter steel rod CD is welded to the 20-mm-diameter steel shaft AB as shown. End C of rod CD is touching the rigid surface shown when a couple T_B is applied to a disk attached to shaft AB . Knowing that the bearings are self-aligning and exert no couples on the shaft, determine the angle of rotation of the disk when $T_B = 400 \text{ N}\cdot\text{m}$. Use $E = 200 \text{ GPa}$ and $G = 77.2 \text{ GPa}$. (Consider the strain energy due to both bending and twisting in shaft AB and to bending in arm CD .)

$$\sum M_{AB} = 0: \quad r_{CD} F_C = T_B \quad F_C = \frac{T_B}{r_{CD}}$$

$$F_C = \frac{400}{300 \times 10^{-3}} = 1333.3 \text{ N}, \quad F_D = 1333.3 \text{ N}$$

Bending of rod CD :

$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi}{4} \left(\frac{20}{2}\right)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$M = F_C x$$

$$U = \int_0^{L_{CD}} \frac{(F_C x)^2}{2EI} dx = \frac{F_C^2 L_{CD}^3}{6EI}$$

$$= \frac{(1333.3)^2 (300 \times 10^{-3})^3}{(6)(200 \times 10^9)(7.854 \times 10^{-9})} = 5.093 \text{ J}$$

Bending of shaft ADB :

$$\sum M_B = 0: \quad -F_A L_{AB} + F_D b = 0 \quad F_A = \frac{F_D b}{L_{AB}}$$

$$\sum M_A = 0: \quad +F_A L_{AB} - F_D a = 0 \quad F_A = \frac{F_D a}{L_{AB}}$$

$$U = \frac{1}{2EI} \left\{ \int_0^a \left(\frac{F_D b}{L_{AB}}\right)^2 dx + \int_0^b \left(\frac{F_D a}{L_{AB}}\right)^2 dx \right\} = \frac{F_D^2 a^2 b^2}{6EI L_{AB}}$$

$$I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = (270 \times 10^{-3}) \text{ m}$$

$$U = \frac{(1333.3)^2 (70 \times 10^{-3})^2 (200 \times 10^{-3})^2}{(6)(200 \times 10^9)(7.854 \times 10^{-9})(270 \times 10^{-3})} = 0.137 \text{ J}$$

Torsion: Only portion DB carries torque. $J = 2J = 15.708 \times 10^{-9} \text{ m}^4$

$$U = \frac{T_B^2 L_{DB}}{2GJ} = \frac{(400)^2 (200 \times 10^{-3})}{(2)(77.2 \times 10^9)(15.708 \times 10^{-9})} = 13.194 \text{ J}$$

$$\text{Total: } U = 5.093 + 0.137 + 13.194 = 18.424 \text{ J}$$

$$\frac{1}{2} T_B \phi_B = U$$

$$\phi_B = \frac{2U}{T_B} = \frac{(2)(18.424)}{400} = 92.1 \times 10^{-3} \text{ rad}$$

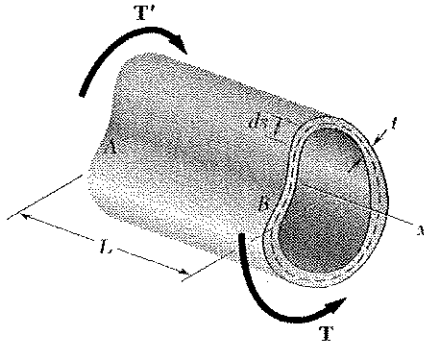
$$\phi_B = 5.28^\circ$$

Problem 11.70

11.70 The thin-walled hollow cylindrical member AB has a noncircular cross section of nonuniform thickness. Using the expression given in Eq. (3.53) of Sec. 3.13, and the expression for the strain-energy density given in Eq. (11.19), show that the angle of twist of member AB is

$$\phi = \frac{TL}{402G} \oint \frac{ds}{t}$$

where ds is an element of the center line of the wall cross section and O is the area enclosed by that center line.



From equation (3.53), $\tau = \frac{T}{2ta}$

Strain energy density:

$$U = \frac{\tau^2}{2G} = \frac{T^2}{8Gt^2a^2}$$

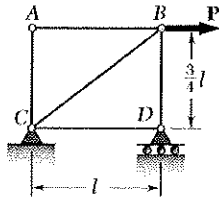
$$U = \int_0^L \oint U t ds dx$$

$$= \int_0^L \frac{T^2}{8Ga^2} \oint \frac{ds}{t} dx = \frac{T^2L}{8Ga^2} \oint \frac{ds}{t}$$

$$\text{Work of torque} = \frac{1}{2}T\phi = \frac{T^2L}{8Ga^2} \oint \frac{ds}{t} \quad \phi = \frac{TL}{4Ga^2} \oint \frac{ds}{t}$$

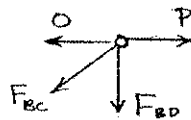
Problem 11.71

11.71 and 11.72 Each member of the truss shown has a uniform cross-sectional area A . Using the method of work and energy, determine the horizontal deflection of the point of application of the load P .



Members AB , AC and CD are zero force members

Joint B



$$+\rightarrow \sum F_x = 0:$$

$$P - \frac{4}{5} F_{BC} = 0$$

$$F_{BC} = \frac{5}{4} P$$

$$+\uparrow \sum F_y = 0:$$

$$-F_{BD} - \frac{3}{5} F_{BC} = 0$$

$$F_{BD} = -\frac{3}{4} P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$= \frac{19}{16} \frac{P^2 l}{EA}$$

$$\text{Work of } P = \frac{1}{2} P \Delta = U$$

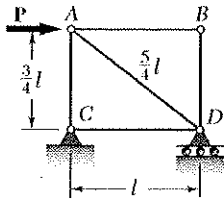
$$\Delta = \frac{2U}{P} = \frac{19}{8} \frac{Pl}{EA}$$

Member	F	L	$F^2 L$
AB	0	l	0
AC	0	$\frac{3}{4} l$	0
CD	0	l	0
BC	$\frac{5}{4} P$	$\frac{5}{4} l$	$\frac{125}{64} P^2 l$
BD	$-\frac{3}{4} P$	$\frac{3}{4} l$	$\frac{27}{64} P^2 l$
Σ			$\frac{19}{8} P^2 l$

$$\Delta = 2.375 \frac{Pl}{EA} \rightarrow \blacktriangleleft$$

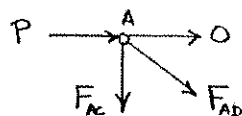
Problem 11.72

11.71 and 11.72 Each member of the truss shown has a uniform cross-sectional area A . Using the method of work and energy, determine the horizontal deflection of the point of application of the load P .



Members AB and BD are zero force members.

Joint A



$$+\rightarrow \sum F_x = 0:$$

$$\frac{4}{5} F_{AD} + P = 0$$

$$F_{AD} = -\frac{5}{4} P$$

$$+\uparrow \sum F_y = 0:$$

$$-F_{AC} - \frac{3}{5} F_{AD} = 0$$

$$F_{AC} = \frac{3}{4} P$$

$$\text{Joint D } +\rightarrow \sum F_x = 0:$$

$$\frac{4}{5} \cdot \frac{5}{4} P - F_{CD} = 0$$

$$F_{CD} = P$$

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$= \frac{27}{16} \frac{P^2 l}{EA}$$

Member	F	L	$F^2 L$
AB	0	l	0
BD	0	$\frac{3}{4} l$	0
AD	$-\frac{5}{4} P$	$\frac{5}{4} l$	$\frac{125}{64} P^2 l$
CD	P	l	$P^2 l$
AC	$\frac{3}{4} P$	$\frac{3}{4} l$	$\frac{27}{64} P^2 l$
Σ			$\frac{27}{8} P^2 l$

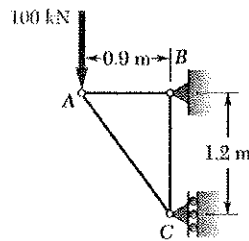
$$\text{Work of } P = \frac{1}{2} P \Delta = U$$

$$\Delta = \frac{2U}{P} = \frac{27}{8} \frac{Pl}{EA}$$

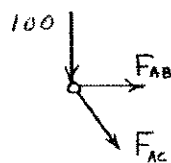
$$\Delta = 3.375 \frac{Pl}{EA} \rightarrow \blacktriangleleft$$

Problem 11.73

11.73 Each member of the truss shown is made of steel and has a uniform cross-sectional area of 1945 mm^2 . Using $E = 200 \text{ GPa}$, determine the vertical deflection of joint A caused by the application of the 100-kN load.



Joint A



$$+\uparrow \sum F_y = 0$$

$$-100 - \frac{0.9}{1.2} F_{AC} = 0$$

$$F_{AC} = -133.3 \text{ kN}$$

$$+\rightarrow \sum F_x = 0$$

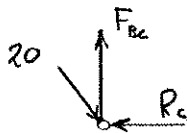
$$\frac{0.9}{1.5} F_{AC} + F_{AB} = 0$$

$$F_{AB} = 80 \text{ kN}$$

Joint C

$$+\uparrow \sum F_y = 0 \quad F_B - \frac{0.9}{1.2} (133.3) = 0$$

$$F_{BC} = 100 \text{ kN}$$



$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$A = 1945 \text{ mm}^2$$

Member	F (kN)	L (m)	F ² L (kN ² m)
AB	80	0.9	5760
AC	-133.3	1.5	26653
BC	100	1.2	12000
Σ			44413

$$U = \frac{44413 \times 10^6}{(2)(200 \times 10^9)(1945 \times 10^{-6})} = 57.09 \text{ J}$$

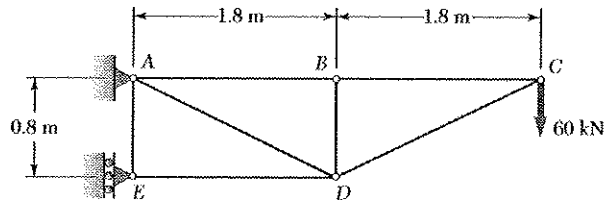
$$\frac{1}{2} P \Delta = U$$

$$\Delta = \frac{2U}{P} = \frac{(2)(57.09)}{100000} = 1.14 \times 10^{-3} \text{ m}$$

$$= 1.14 \text{ mm} \downarrow$$

Problem 11.74

11.74 Each member of the truss shown is made of steel and has a uniform cross-sectional area of 3220 mm². Using $E = 200$ GPa, determine the vertical deflection of joint C caused by the application of the 60-kN load.



Members BD and AE are zero force members.

For entire truss $\sum M_A = 0$:

$$0.8 R_D - (60)(3.6) = 0$$

$$R_D = 270 \text{ kN}$$

$$F_{ED} = -R_D = -270 \text{ kN}$$

For equilibrium of joint E

Joint C

$$+\uparrow \sum F_y = 0$$

$$-\frac{0.8}{1.97} F_{CD} - 60 = 0$$

$$F_{CD} = -147.75$$

$$+\rightarrow \sum F_x = 0$$

$$-\frac{1.8}{1.97} F_{CD} - F_{BC} = 0$$

$$F_{BC} = 135 \text{ kN}$$

Joint D

$$+\rightarrow \sum F_x = 0$$

$$-270 - \frac{1.8}{1.97} (F_{AD} + 147.75) = 0$$

$$F_{AD} = 147.75 \text{ kN}$$

Joint B $\sum F_x = 0$:

$$-F_{AB} + F_{BC} = 0$$

$$F_{AB} = 135 \text{ kN}$$

$$\text{Strain energy: } U_m = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

Member	F (kN)	L (m)	F ² L (kN ² m)
AB	135	1.8	32805
BC	135	1.8	32805
CD	-147.75	1.97	43005
DE	-120	1.8	25920
BD	0	0.8	0
AE	0	0.8	0
AD	147.75	1.97	43005
Σ			177540

$$\text{Data: } E = 200 \times 10^6 \text{ kPa}$$

$$A = 3220 \text{ mm}^2$$

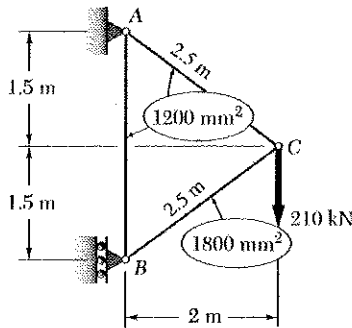
$$U_m = \frac{177540}{(2)(200 \times 10^6)(3220 \times 10^{-6})} = 0.2757 \text{ kJ}$$

$$\frac{1}{2} P_m \Delta_m = U$$

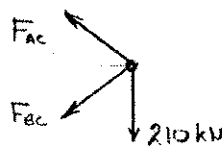
$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(0.2757)}{60} = 0.00919 \text{ m} = 9.2 \text{ mm} \downarrow$$

Problem 11.75

11.75 Members of the truss shown are made of steel and have the cross-sectional areas shown. Using $E = 200 \text{ GPa}$, determine the vertical deflection of joint C caused by the application of the 210-kN load.



Joint C



$$+\rightarrow \Sigma F_x = 0:$$

$$-\frac{4}{5} F_{AC} - \frac{4}{5} F_{BC} = 0$$

$$+\uparrow \Sigma F_y = 0:$$

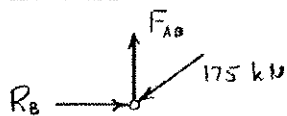
$$\frac{3}{5} F_{AC} - \frac{3}{5} F_{BC} - 210 = 0$$

Solving simultaneously,

$$F_{AC} = 175 \text{ kN}$$

$$F_{BC} = -175 \text{ kN}$$

Joint B



$$+\uparrow \Sigma F_y = 0:$$

$$F_{AB} - \left(\frac{3}{5}\right)(175) = 0$$

$$F_{AB} = 105 \text{ kN}$$

$$U_m = \Sigma \frac{F^2 L}{2EA}$$

Member	F (kN)	L (m)	A (10^{-6} m^2)	$F^2 L / A$ (N^2/m)
AB	105	3.0	1200	27.5625×10^{12}
AC	175	2.5	1200	63.8021×10^{12}
BC	-175	2.5	1800	42.5347×10^{12}
Σ				133.8993×10^{12}

$$U_m = \frac{1}{2E} \Sigma \frac{F^2 L}{A} = \frac{133.8993 \times 10^{12}}{(2)(200 \times 10^9)} = 334.75 \text{ J}$$

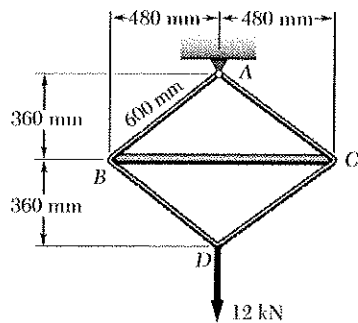
$$\frac{1}{2} P_m \Delta_m = U_m$$

$$\Delta_m = \frac{2U_m}{P_m} = \frac{(2)(334.75)}{210 \times 10^3} = 3.19 \times 10^{-3} \text{ m}$$

$$\Delta_m = 3.19 \text{ mm} \downarrow$$

Problem 11.76

11.76 The steel rod BC has a 24-mm diameter and the steel cable $ABDCA$ as a 12-mm diameter. Using $E = 200$ GPa, determine the deflection of joint D caused by the 12-kN load.



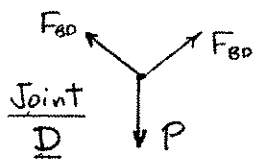
Owing to symmetry, $F_{AB} = F_{BD} = F_{DC} = F_{CA}$

$$U_{AB} = U_{BD} = U_{DC} = U_{CA}$$

$$U = 4U_{BD} + U_{BC} = 4 \frac{F_{BD}^2 L_{BD}}{2EA_{BD}} + \frac{F_{BC}^2 L_{BC}}{2EA_{BC}}$$

Let P be the load at D .

$$S_D = \frac{\partial U}{\partial P} = 4 \frac{F_{BD} L_{BD}}{EA_{BD}} \frac{\partial F_{BD}}{\partial P} + \frac{F_{BC} L_{BC}}{EA_{BC}} \frac{\partial F_{BC}}{\partial P}$$

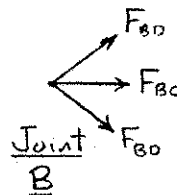


$$+\uparrow \Sigma F_y = 0:$$

$$2 \frac{3}{5} F_{BD} - P = 0$$

$$F_{BD} = \frac{5}{2} P$$

$$\frac{\partial F_{BD}}{\partial P} = \frac{5}{2}$$



$$+\rightarrow \Sigma F_x = 0:$$

$$F_{BC} + (2) \frac{4}{5} F_{BD} = 0$$

$$F_{BC} = -\frac{8}{5} F_{BD} = -\frac{4}{3} P$$

$$\frac{\partial F_{BC}}{\partial P} = -\frac{4}{3}$$

$$S_D = 4 \left(\frac{5}{2} \right)^2 \frac{P L_{BD}}{EA_{BD}} + \left(-\frac{4}{3} \right)^2 \frac{P L_{BC}}{EA_{BC}} = \frac{P}{E} \left\{ \frac{25}{9} \frac{L_{BD}}{A_{BD}} + \frac{16}{9} \frac{L_{BC}}{A_{BC}} \right\}$$

$$\text{Data: } P = 12 \times 10^3 \text{ N}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$L_{BD} = 600 \times 10^{-3} \text{ m}$$

$$A_{BD} = \frac{\pi}{4} (12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$L_{BC} = 960 \times 10^{-3} \text{ m}$$

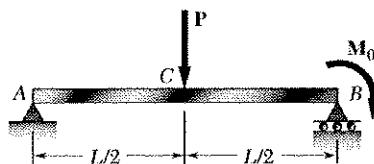
$$A_{BC} = \frac{\pi}{4} (24)^2 = 452.39 \text{ mm}^2 = 452.39 \times 10^{-6} \text{ m}^2$$

$$S_D = \frac{12 \times 10^3}{200 \times 10^9} \left\{ \frac{25}{9} \cdot \frac{600 \times 10^{-3}}{113.097 \times 10^{-6}} + \frac{16}{9} \cdot \frac{960 \times 10^{-3}}{452.39 \times 10^{-6}} \right\} = 1.111 \times 10^{-3} \text{ m}$$

$$S_D = 1.111 \text{ mm} \downarrow$$

Problem 11.77

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load P is applied first, (b) if the couple M is applied first.



From Appendix D, Case 4.

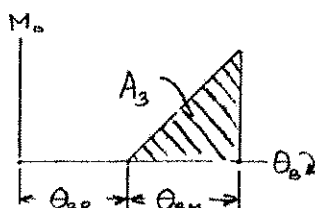
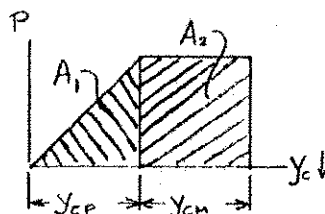
$$\downarrow y_c = \frac{PL^3}{48EI} \quad \curvearrowleft \theta_B = -\frac{PL^2}{16EI}$$

From Appendix D, Case 7.

$$\downarrow y_c = \frac{M_0}{6EI} \left((L/2)^3 - L^2(L/2) \right) = -\frac{M_0 L^2}{16EI}$$

$$\curvearrowright \theta_B = \frac{M_0 L}{3EI}$$

(a) First P , then M_0 .

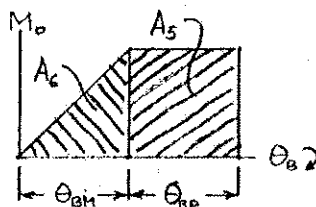
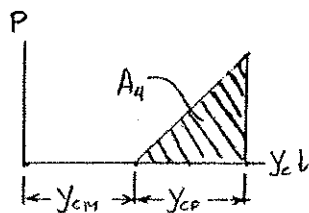


$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2} P y_{CP} + P y_{CM} + \frac{1}{2} M_0 \theta_{BM}$$

$$= \frac{P^2 L^3}{96EI} - \frac{PM_0 L^2}{16EI} + \frac{M_0^2 L}{6EI}$$

(b) First M_0 , then P .



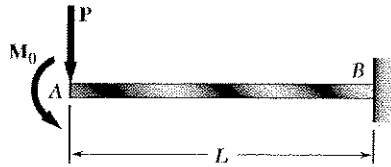
$$U = A_4 + A_5 + A_6$$

$$= \frac{1}{2} P y_{CP} + M_0 \theta_{BP} + \frac{1}{2} M_0 \theta_{BM}$$

$$= \frac{P^2 L^3}{96EI} - \frac{M_0 P L^2}{16EI} + \frac{M_0^2 L}{6EI}$$

Problem 11.78

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load P is applied first, (b) if the couple M is applied first.



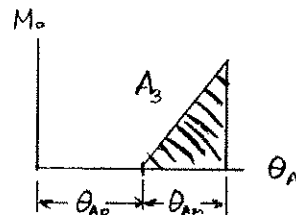
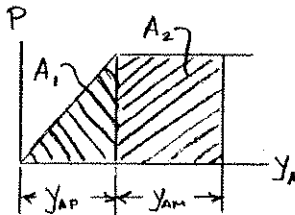
From Appendix D, Case 1.

$$y_{AP} = \frac{PL^3}{3EI} \quad \theta_{AP} = \frac{PL^2}{2EI}$$

From Appendix D, Case 3.

$$y_{AM} = \frac{M_0 L^2}{2EI} \quad \theta_{AM} = \frac{M_0 L}{EI}$$

(a) First P , then M_0 .

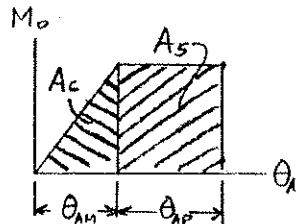
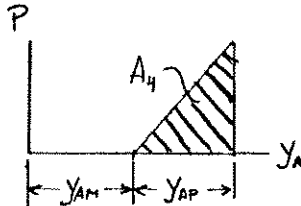


$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2} P y_{AP} + P y_{AM} + \frac{1}{2} M_0 \theta_{AM}$$

$$= \frac{P^2 L^3}{6EI} + \frac{P M_0 L^2}{2EI} + \frac{M_0^2 L}{2EI}$$

(b) First M_0 , then P .



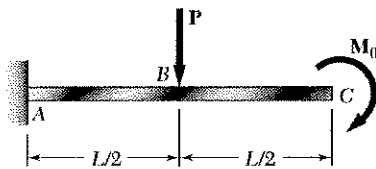
$$U = A_4 + A_5 + A_6$$

$$= \frac{1}{2} P y_{AP} + M_0 \theta_{AP} + \frac{1}{2} M_0 \theta_{AM}$$

$$= \frac{P^2 L^3}{6EI} + \frac{M_0 P L^2}{2EI} + \frac{M_0^2 L}{2EI}$$

Problem 11.79

11.77 through 11.79 Using the information in Appendix D, compute the work of the loads as they are applied to the beam (a) if the load P is applied first, (b) if the couple M is applied first.



Appendix D Cases 1 and 3.

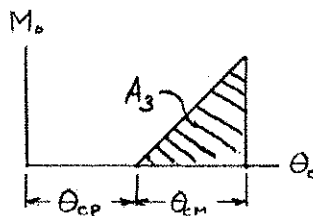
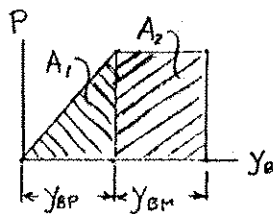
$$y_{BP} = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI}$$

$$\theta_{CP} = \frac{P(L/2)^2}{2EI} = \frac{PL^2}{8EI}$$

$$y_{BM} = \frac{M_0(L/2)^2}{2EI} = \frac{M_0L^2}{8EI}$$

$$\theta_{BM} = \frac{M_0L}{EI}$$

(a) First P , then M_0 .

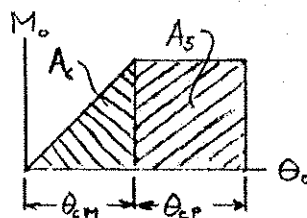
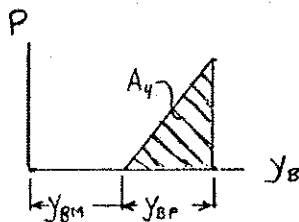


$$U = A_1 + A_2 + A_3$$

$$= \frac{1}{2}Py_{BP} + Py_{BM} + \frac{1}{2}M_0\theta_{CM}$$

$$= \frac{P^2L^3}{48EI} + \frac{PM_0L^2}{8EI} + \frac{M_0^2L}{2EI}$$

(b) First M_0 , then P .



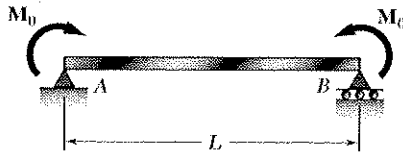
$$U = A_4 + A_5 + A_6$$

$$= \frac{1}{2}Py_{BP} + M_0\theta_{CP} + \frac{1}{2}M_0\theta_{CM}$$

$$= \frac{P^2L^3}{48EI} + \frac{M_0PL^2}{8EI} + \frac{M_0^2L}{2EI}$$

Problem 11.80

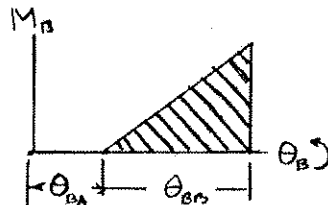
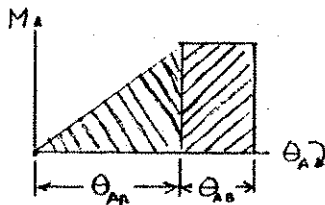
11.80 through 11.82 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.



(a) Label the couples M_A and M_B .

Using Appendix D, Case 7.

$$C\theta_{AA} = \frac{M_A L}{3EI} \quad C\theta_{BA} = \frac{M_A L}{6EI} \quad C\theta_{BB} = \frac{M_B L}{3EI} \quad C\theta_{AB} = \frac{M_B L}{6EI}$$



Apply M_A first, then M_B .

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} M_A \theta_{AA} + M_A \theta_{AB} + \frac{1}{2} M_B \theta_{BB} = \frac{1}{6} \frac{M_A^2 L}{EI} + \frac{1}{6} \frac{M_A M_B L}{EI} + \frac{1}{6} \frac{M_B^2 L}{EI}$$

$$\text{With } M_A = M_B = M_0$$

$$U = \frac{1}{2} \frac{M_0^2 L}{EI}$$

$$U = \frac{M_0^2 L}{2EI}$$

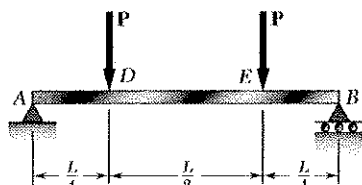
(b) Bending moment

$$M = M_0$$

$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{M_0^2 L}{2EI}$$

$$U = \frac{M_0^2 L}{2EI}$$

Problem 11.81



11.80 through 11.82 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.

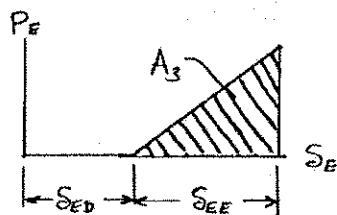
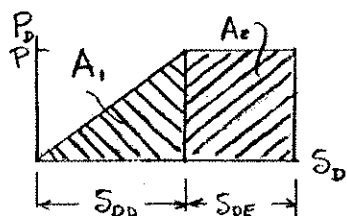
(a) Label the forces P_D and P_E .

Using Appendix D Case 5,

$$S_{EE} = \frac{P_E a^2 b^2}{3EI L} = \frac{P_E \left(\frac{3L}{4}\right)^2 \left(\frac{L}{4}\right)^2}{3EI L} = \frac{3}{256} \frac{P_E L^3}{EI}$$

$$S_{DE} = \frac{P_E b}{6EI L} \left[(L^2 - b^2)x - x^3 \right] = \frac{P_E \left(\frac{L}{4}\right)}{6EI L} \left[\left(L^2 - \left(\frac{L}{4}\right)^2 \right) \left(\frac{L}{4}\right) - \left(\frac{L}{4}\right)^3 \right] = \frac{7}{768} \frac{P_E L^3}{EI}$$

$$\text{Likewise, } S_{DD} = \frac{3}{256} \frac{P_D L^3}{EI} \quad \text{and} \quad S_{ED} = \frac{7}{768} \frac{P_D L^3}{EI}$$



Let P_D be applied first.

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} P_D S_{DD} + P_D S_{DE} + \frac{1}{2} P_E S_{EE} = \frac{3}{512} \frac{P_D^2 L^3}{EI} + \frac{7}{768} \frac{P_D P_E L^3}{EI} + \frac{3}{512} \frac{P_E^2 L^3}{EI}$$

$$\text{With } P_D = P_E = P \quad U = \frac{1}{48} \frac{P^2 L^3}{EI}$$

$$U = \frac{P^2 L^3}{48 EI}$$

(b) Reactions: $R_A = R_B = P$

Over portion AD: $(0 < x < \frac{L}{4})$; $M = Px$

$$U_{AD} = \int_0^{\frac{L}{4}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{\frac{L}{4}} x^2 dx = \frac{P^2}{2EI} \cdot \frac{1}{3} \left(\frac{L}{4}\right)^3 = \frac{1}{384} \frac{P^2 L^3}{EI}$$

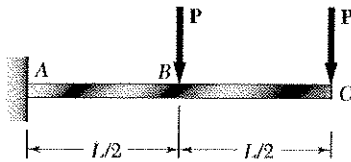
$$\text{Over portion DE: } M = \frac{PL}{4} \quad U_{DE} = \frac{M^2 \left(\frac{L}{2}\right)}{2EI} + \frac{P^2 L^2}{2EI} \cdot \frac{1}{16} \cdot \frac{L}{2} = \frac{P^2 L^3}{64 EI}$$

$$\text{Over portion EB: By symmetry, } U_{EB} = U_{AD} = \frac{1}{384} \frac{P^2 L^3}{EI}$$

$$\text{Total: } U = U_{AD} + U_{DE} + U_{EB} = \left(\frac{1}{384} + \frac{1}{64} + \frac{1}{384} \right) \frac{P^2 L^3}{EI} = \frac{1}{48} \frac{P^2 L^3}{EI}$$

Problem 11.82

11.80 through 11.82 For the beam and loading shown, (a) compute the work of the loads as they are applied successively to the beam, using the information provided in Appendix D, (b) compute the strain energy of the beam by the method of Sec. 11.4 and show that it is equal to the work obtained in part a.



(a) Label the forces P_B and P_C .

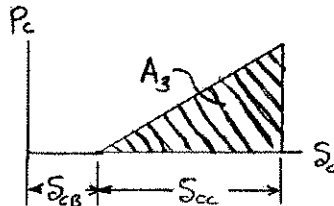
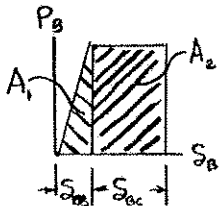
Using Appendix D Case 1

$$\delta_{BB} = \frac{P_B (L/2)^3}{3EI} = \frac{1}{24} \frac{P_B L^3}{EI}$$

$$\delta_{CB} = \delta_{BB} + \frac{L}{2} \theta_B = \frac{1}{24} \frac{P_B L^3}{EI} + \frac{L}{2} \frac{P_B (L/2)^2}{2EI} = \left(\frac{1}{24} + \frac{1}{16} \right) \frac{P_B L^3}{EI} = \frac{5}{48} \frac{P_B L^3}{EI}$$

$$\delta_{CC} = \frac{1}{3} \frac{P_C L^3}{EI}$$

$$\delta_{BC} = \frac{P_C}{6EI} (3Lx^2 - x^3) = \frac{P_C}{6EI} \left(3L \left(\frac{L}{2} \right)^2 - \left(\frac{L}{2} \right)^3 \right) = \frac{5}{48} \frac{P_C L^3}{EI}$$



Apply P_B first, then P_C

$$U = A_1 + A_2 + A_3$$

$$U = \frac{1}{2} P_B \delta_{BB} + P_B \delta_{BC} + \frac{1}{2} P_C \delta_{CC} = \frac{1}{48} \frac{P_B L^3}{EI} + \frac{5}{48} \frac{P_B P_C L^3}{EI} + \frac{1}{6} \frac{P_C^2 L^3}{EI}$$

$$\text{With } P_B = P_C = P \quad U = \left(\frac{1}{48} + \frac{5}{48} + \frac{1}{6} \right) \frac{P^2 L^3}{EI} \quad U = \frac{7P^2 L^3}{24EI}$$

(b) Over AB: $M = Pv + P(v - \frac{L}{2}) = P(2v - \frac{L}{2})$

$$\begin{aligned} U_{AB} &= \int_{\frac{L}{2}}^L \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_{\frac{L}{2}}^L \left(4v^2 - 2Lv + \frac{1}{4} L^2 \right) dv \\ &= \frac{P^2}{2EI} \left\{ \frac{4}{3} \left[L^3 - \left(\frac{L}{2} \right)^3 \right] - 2L \cdot \frac{1}{2} \left[L^2 - \left(\frac{L}{2} \right)^2 \right] + \frac{1}{4} L^2 \left[L - \frac{L}{2} \right] \right\} \\ &= \frac{P^2}{2EI} \left\{ \frac{7}{6} L^3 - \frac{3}{4} L^3 + \frac{1}{8} L^3 \right\} = \frac{13}{48} \frac{P^2 L^3}{EI} \end{aligned}$$

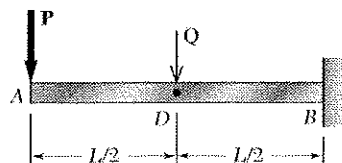
Over BC: $M = Pv \quad U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv = \frac{P^2}{2EI} \int_0^{\frac{L}{2}} v^2 dv = \frac{P^2}{2EI} \cdot \frac{1}{3} \left(\frac{L}{2} \right)^3$

$$= \frac{P^2 L^3}{48EI}$$

Total: $U = U_{AB} + U_{BC} = \left(\frac{13}{48} + \frac{1}{48} \right) \frac{P^2 L^3}{EI} \quad U = \frac{7P^2 L^3}{24EI}$

Problem 11.83

11.83 and 11.84 For the prismatic beam shown, determine the deflection of point D.



Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_D = \frac{\partial U}{\partial Q} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx$$

Over portion AD: $(0 < x < \frac{L}{2})$ $M = -Px$, $\frac{\partial M}{\partial Q} = 0$

Over portion DB: $(\frac{L}{2} < x < L)$ $M = -Px - Q(x - \frac{L}{2})$, $\frac{\partial M}{\partial Q} = -(x - \frac{L}{2})$

Set $Q = 0$.

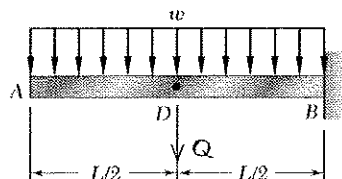
$$S_D = \frac{1}{EI} \int_0^{\frac{L}{2}} (-Px)(0) dx + \frac{1}{EI} \int_{\frac{L}{2}}^L (-Px) \left[-(x - \frac{L}{2}) \right] dx$$

$$= \frac{P}{EI} \int_{\frac{L}{2}}^L (x^2 - \frac{L}{2}x) dx = \frac{P}{EI} \left\{ \frac{1}{3}L^3 - \frac{1}{3}(\frac{L}{2})^3 - (\frac{L}{2})\frac{1}{2}L^2 + \frac{L}{2}\frac{1}{2}(\frac{L}{2})^2 \right\}$$

$$= \left(\frac{1}{3} - \frac{1}{24} - \frac{1}{4} + \frac{1}{16} \right) \frac{PL^3}{EI} \quad S_D = \frac{5PL^3}{48EI} \downarrow$$

Problem 11.84

11.83 and 11.84 For the prismatic beam shown, determine the deflection of point D.



Add force Q at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_D = \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx$$

Over portion AD: $(0 < x < \frac{L}{2})$ $M = -\frac{1}{2}wx^2$ $\frac{\partial M}{\partial Q} = 0$

Over portion DB: $(\frac{L}{2} < x < L)$ $M = -\frac{1}{2}wx^2 - Q(x - \frac{L}{2})$ $\frac{\partial M}{\partial Q} = -(x - \frac{L}{2})$

Set $Q = 0$.

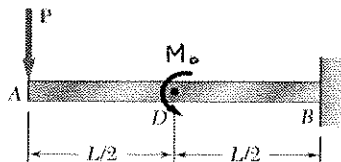
$$S_D = \frac{1}{EI} \int_0^{\frac{L}{2}} (-\frac{1}{2}wx^2)(0) dx + \frac{1}{EI} \int_{\frac{L}{2}}^L (-\frac{1}{2}wx^2) \left[-(x - \frac{L}{2}) \right] dx$$

$$= \frac{w}{2EI} \int_{\frac{L}{2}}^L (x^3 - \frac{L}{2}x^2) dx = \frac{w}{2EI} \left\{ \frac{1}{4}L^4 - \frac{1}{4}(\frac{L}{2})^4 - (\frac{L}{2})\frac{1}{3}L^3 + (\frac{L}{2})\frac{1}{3}(\frac{L}{2})^3 \right\}$$

$$= \frac{1}{2} \left(\frac{1}{4} - \frac{1}{64} - \frac{1}{6} + \frac{1}{48} \right) \frac{wL^4}{EI} = \frac{17}{384} \frac{wL^4}{EI} \quad S_D = 0.0443 \frac{wL^4}{EI} \downarrow$$

Problem 11.85

11.85 and 11.86 For the prismatic beam shown, determine the slope at point D.



Add couple M_0 at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\theta_D = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial M_0} dx$$

Over portion AD: ($0 < x < \frac{L}{2}$)

$$M = -Px$$

$$\frac{\partial M}{\partial M_0} = 0$$

Over portion DB: ($\frac{L}{2} < x < L$)

$$M = -Px - M_0$$

$$\frac{\partial M}{\partial M_0} = -1$$

$$\text{Set } M_0 = 0. \quad \theta_D = \frac{1}{EI} \int_{\frac{L}{2}}^L (-Px)(-1) dx + \frac{1}{EI} \int_0^{\frac{L}{2}} (-Px)(0) dx$$

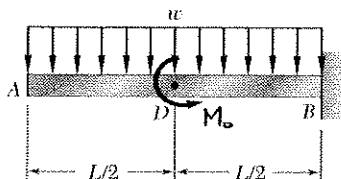
$$= \frac{P}{EI} \int_{\frac{L}{2}}^L x dx = \frac{P}{EI} \left[\frac{1}{2} L^2 - \frac{1}{2} \left(\frac{L}{2} \right)^2 \right]$$

$$= \left(\frac{1}{2} - \frac{1}{8} \right) \frac{PL^2}{EI}$$

$$\theta_D = \frac{3PL^2}{8EI} \quad \swarrow \quad \blacktriangleleft$$

Problem 11.86

11.85 and 11.86 For the prismatic beam shown, determine the slope at point D.



Add couple M_0 at point D.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$\theta_D = \frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial M_0} dx$$

Over portion AD: ($0 < x < \frac{L}{2}$)

$$M = -\frac{1}{2}wx^2$$

$$\frac{\partial M}{\partial M_0} = 0$$

Over portion DB: ($\frac{L}{2} < x < L$)

$$M = -\frac{1}{2}wx^2 - M_0$$

$$\frac{\partial M}{\partial M_0} = -1$$

$$\text{Set } M_0 = 0. \quad \theta_D = \frac{1}{EI} \int_{\frac{L}{2}}^L \left(-\frac{1}{2}wx^2 \right) (-1) dx + \frac{1}{EI} \int_0^{\frac{L}{2}} \left(-\frac{1}{2}wx^2 \right) (0) dx$$

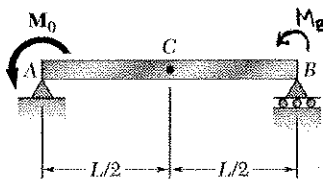
$$= \frac{w}{2EI} \int_{\frac{L}{2}}^L x^2 dx = \frac{w}{2EI} \left[\frac{1}{3} L^3 - \frac{1}{3} \left(\frac{L}{2} \right)^3 \right]$$

$$= \frac{1}{6} \left(1 - \frac{1}{8} \right) \frac{wL^3}{EI}$$

$$\theta_D = \frac{7wL^3}{48EI} \quad \swarrow \quad \blacktriangleleft$$

Problem 11.87

11.87 For the prismatic beam shown, determine the slope at point B.



Add couple M_B at point B as shown.

Reactions. $R_A = \frac{1}{L}(M_0 + M_B) \uparrow$

Strain energy. $U = \int_0^L \frac{M^2}{2EI} dx$

Slope at point B. $\theta_B = \frac{\partial U}{\partial M_B}$

$$M = R_A x - M_0 = (M_0 + M_B) \frac{x}{L} - M_0$$

$$\frac{\partial M}{\partial M_B} = \frac{x}{L} \quad \text{With } M_B = 0 \quad M = M_0 \left(\frac{x}{L} - 1 \right)$$

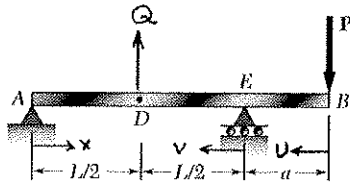
$$\begin{aligned} \frac{\partial U}{\partial M_B} &= \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_B} dx = \frac{M_0}{EI} \int_0^L \left(\frac{x}{L} - 1 \right) \frac{x}{L} dx = \frac{M_0}{EIL^2} \int_0^L (x - L)x dx \\ &= \frac{M_0}{EIL^2} \int_0^L (x^2 - Lx) dx = \frac{M_0}{EI} \left(\frac{x^3}{3} - \frac{Lx^2}{2} \right) \Big|_0^L = -\frac{M_0 L}{6EI} \end{aligned}$$

$$\theta_B = -\frac{M_0 L}{6EI}$$

$$\theta_B = \frac{M_0 L}{6EI} \quad \swarrow \blacktriangleleft$$

Problem 11.88

11.88 and 11.89 For the prismatic beam shown, determine the deflection at point D.



Add force Q at point D.

$$\text{Reactions: } R_A = -\frac{Pa}{L} - \frac{1}{2}Q, \quad R_E = \frac{P(a+L)}{L} - \frac{1}{2}Q$$

$$U = U_{AD} + U_{DE} + U_{EB}; \quad S_D = \frac{\partial U}{\partial Q}$$

$$\text{Over portion AD: } U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = -\frac{Pa}{L}x - \frac{1}{2}Qx, \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}x$$

$$\begin{aligned} \text{Set } Q = 0. \quad \frac{\partial U_{AD}}{\partial Q} &= \frac{1}{EI} \int_0^{L/2} \left(-\frac{Pa}{L}x\right) \left(-\frac{1}{2}x\right) dx = \frac{Pa}{2EIL} \int_0^{L/2} x^2 dx \\ &= \frac{Pa}{2EIL} \frac{1}{3} \left(\frac{L}{2}\right)^3 = \frac{PaL^2}{48EI} \end{aligned}$$

$$\text{Over portion DE: } U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv, \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}v$$

$$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{1}{2}Qv - P(a+v) = \frac{Pa}{L}v - Pa - \frac{1}{2}Qv$$

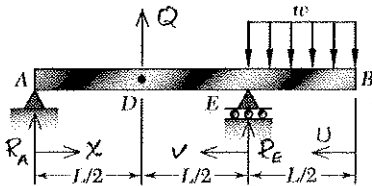
$$\begin{aligned} \text{Set } Q = 0. \quad \frac{\partial U_{DE}}{\partial Q} &= \frac{1}{EI} \int_0^{L/2} \left(\frac{Pa}{L}v - Pa\right) \left(-\frac{1}{2}v\right) dv = \frac{Pa}{2EIL} \int_0^{L/2} (-v^2 + Lv) dv \\ &= \frac{Pa}{2EIL} \left[-\frac{1}{3}\left(\frac{L}{2}\right)^3 + (L)\frac{1}{2}\left(\frac{L}{2}\right)^2\right] = \frac{Pa}{2EIL} \left[-\frac{L^3}{24} + \frac{L^3}{8}\right] \\ &= \frac{1}{24} \frac{PaL^2}{EI} \end{aligned}$$

$$\text{Over portion EB: } M = -Pv, \quad \frac{\partial M}{\partial Q} = 0, \quad \frac{\partial U_{EB}}{\partial Q} = 0$$

$$S_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{PaL^2}{48EI} + \frac{PaL^2}{24EI} + 0 \quad S_D = \frac{PaL^2}{16EI} \uparrow \blacktriangleleft$$

Problem 11.89

11.88 and 11.89 For the prismatic beam shown, determine the deflection at point D.



Add force Q at point D.

$$\text{Reactions: } R_A = -\frac{1}{8}wL - \frac{1}{2}Q, \quad R_E = \frac{5}{8}wL - \frac{1}{2}Q$$

$$U = U_{AD} + U_{DE} + U_{EB}; \quad S_D = \frac{\partial U}{\partial Q}$$

Over portion AD: $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = -\frac{1}{8}wLx - \frac{1}{2}Qx, \quad \frac{\partial M}{\partial x} = -\frac{1}{2}x$

Set $Q = 0. \quad M = -\frac{1}{8}wLx$

$$\frac{\partial U_{AD}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dx = \int_0^{L/2} \left(-\frac{wLx}{8EI} \right) \left(-\frac{1}{2}x \right) dx = \frac{wL}{16EI} \int_0^{L/2} x^2 dx = \frac{wL^4}{384EI}$$

Over portion DE: $U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv$

$$M = R_E v - \frac{wL}{2} \left(v + \frac{L}{4} \right) = \frac{1}{8}wL(v - L) - \frac{1}{2}Qv \quad \frac{\partial M}{\partial Q} = -\frac{1}{2}v$$

Set $Q = 0. \quad M = -\frac{wL}{8}(L - v)$

$$\begin{aligned} \frac{\partial U}{\partial Q} &= \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dv = \frac{wL}{16EI} \int_0^{L/2} (L - v)v dv = \frac{wL}{16EI} \left(\frac{Lv^2}{2} - \frac{v^3}{3} \right) \bigg|_0^{L/2} \\ &= \frac{wL^4}{16EI} \left(\frac{1}{8} - \frac{1}{24} \right) = \frac{wL^4}{192EI} \end{aligned}$$

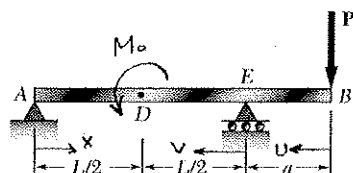
Over portion EB: $M = -\frac{1}{2}wu^2 \quad \frac{\partial M}{\partial Q} = 0 \quad U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} du$

$$\frac{\partial U_{EB}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} du = 0$$

$$S_D = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q} = \frac{wL^4}{384EI} + \frac{wL^4}{192EI} + 0 \quad S_D = \frac{wL^4}{128EI} \uparrow \blacktriangleleft$$

Problem 11.90

11.90 and 11.91 For the prismatic beam shown, determine the slope at point D.



Add couple M_o at point D.

$$\text{Reactions: } R_A = -\frac{Pa}{L} + \frac{M_o}{L}, \quad R_B = \frac{P(a+L)}{L} - \frac{M_o}{L}$$

$$U = U_{AD} + U_{DE} + U_{EB} \quad \theta_D = \frac{\partial U}{\partial M_o}$$

Over portion AD: $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = -\frac{Pa}{L}x + \frac{M_o}{L}x, \quad \frac{\partial M}{\partial M_o} = \frac{1}{L}x$

$$\begin{aligned} \text{Set } M_o = 0. \quad \frac{\partial U_{AD}}{\partial M_o} &= \frac{1}{EI} \int_0^{L/2} \left(-\frac{Pa}{L}x\right)\left(\frac{1}{L}x\right) dx = -\frac{Pa}{EIL^2} \int_0^{L/2} x^2 dx \\ &= -\frac{Pa}{EIL^2} \cdot \frac{1}{3}\left(\frac{L}{2}\right)^3 = -\frac{PaL}{24EI} \end{aligned}$$

Over portion DE: $U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv \quad \frac{\partial M}{\partial M_o} = -\frac{1}{L}v$

$$M = R_E v - P(a+v) = \frac{P(a+L)}{L}v - \frac{M_o}{L}v - P(a+v) = \frac{Pa}{L}v - Pa - \frac{M_o}{L}v$$

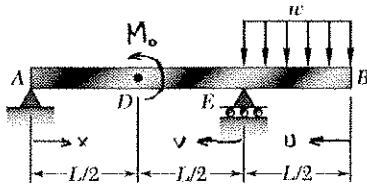
$$\begin{aligned} \text{Set } M_o = 0. \quad \frac{\partial U_{DE}}{\partial M_o} &= \frac{1}{EI} \int_0^{L/2} \left(\frac{Pa}{L}v - Pa\right)\left(-\frac{1}{L}v\right) dv = -\frac{Pa}{EIL^2} \int_0^{L/2} (v^2 - Lv) dv \\ &= -\frac{Pa}{EIL^2} \left[\frac{1}{3}\left(\frac{L}{2}\right)^3 - L \cdot \frac{1}{2}\left(\frac{L}{2}\right)^2 \right] = -\frac{Pa}{EIL^2} \left[\frac{1}{24}L^3 - \frac{1}{8}L^3 \right] \\ &= \frac{1}{12} \frac{PaL}{EI} \end{aligned}$$

Over portion EB: $M = -Pv \quad \frac{\partial M}{\partial M_o} = 0 \quad \frac{\partial U_{EB}}{\partial M_o} = 0$

$$\text{Total: } \theta_D = \frac{\partial U_{AD}}{\partial M_o} + \frac{\partial U_{DE}}{\partial M_o} + \frac{\partial U_{EB}}{\partial M_o} = -\frac{1}{24} \frac{PaL}{EI} + \frac{1}{12} \frac{PaL}{EI} + 0 \quad \theta_D = \frac{PaL}{24EI} \quad \swarrow$$

Problem 11.91

11.90 and 11.91 For the prismatic beam shown, determine the slope at point D.



Add couple M_0 at point D.

$$\text{Reactions: } R_A = \frac{M_0}{L} - \frac{wL}{8}, \quad R_E = -\frac{M_0}{L} + \frac{5wL}{8}$$

$$U = U_{AD} + U_{DE} + U_{EB}; \quad \theta_D = \frac{\partial U}{\partial M_0}$$

$$\text{Over portion AD: } U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx, \quad M = R_A x = \frac{M_0 x}{L} - \frac{wLx}{8}, \quad \frac{\partial M}{\partial M_0} = \frac{x}{L}$$

$$\text{Set } M_0 = 0, \quad M = -\frac{wLx}{8}$$

$$\frac{\partial U_{AD}}{\partial M_0} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \int_0^{L/2} \left(-\frac{wLx}{8EI}\right) \left(\frac{x}{L}\right) dx = -\frac{w}{8EI} \int_0^{L/2} x^2 dx = -\frac{wL^3}{192EI}$$

$$\text{Over portion DE: } U_{DE} = \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$M = R_E v - \frac{wL}{2} \left(v + \frac{L}{4}\right) = -\frac{M_0 v}{L} + \frac{wL}{8} (v - L), \quad \frac{\partial M}{\partial M_0} = -\frac{v}{L}$$

$$\text{Set } M_0 = 0, \quad M = -\frac{1}{8} wL (L - v)$$

$$\frac{\partial U_{DE}}{\partial M_0} = \frac{w}{8EI} \int_0^{L/2} (L - v) v dv = \frac{w}{8EI} \left(\frac{Lv^2}{2} - \frac{v^3}{3}\right) \Big|_0^{L/2} = \frac{wL^3}{96EI}$$

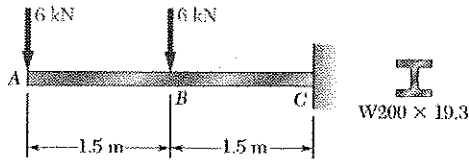
$$\text{Over portion EB: } U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} dv, \quad M = -\frac{1}{2} wv^2, \quad \frac{\partial M}{\partial M_0} = 0$$

$$\frac{\partial U_{EB}}{\partial M_0} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial M_0} dv = 0$$

$$\theta_D = \frac{\partial U_{AD}}{\partial M_0} + \frac{\partial U_{DE}}{\partial M_0} + \frac{\partial U_{EB}}{\partial M_0} = -\frac{wL^3}{192EI} + \frac{wL^3}{96EI} + 0 = \frac{wL^3}{192EI} \quad \swarrow$$

Problem 11.92

11.92 For the beam and loading shown, determine the deflection of point A. Use $E = 200 \text{ GPa}$.



Add force Q at point A.

Units: forces in kN, lengths in m

$$E = 200 \text{ GPa} \quad I = 16.4 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(16.4 \times 10^{-6}) = 3.28 \times 10^6 \text{ Nm}^2$$

$$U = \int_0^{1.5} \frac{M^2}{2EI} dx + \int_0^{1.5} \frac{M^2}{2EI} dv \quad \delta_A = \frac{\partial U}{\partial Q} = \frac{1}{EI} \left\{ \int_0^{1.5} M \frac{\partial M}{\partial Q} dx + \int_0^{1.5} M \frac{\partial M}{\partial Q} dv \right\}$$

Over AB: $M = -6x$, $\frac{\partial M}{\partial Q} = 0$ $\int_0^{1.5} M \frac{\partial M}{\partial Q} dx = 0$

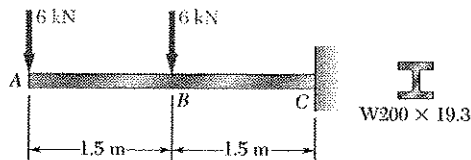
Over BC: $M = -6(v+1.5) - 6v - Qv = -12v - 9 - Qv$, $\frac{\partial M}{\partial Q} = -v$

$$\int_0^{1.5} M \frac{\partial M}{\partial Q} dv = \int_0^{1.5} (-12v - 9 - Qv)(-v) dv = \int_0^{1.5} (12v^2 + 9v) dv = (4)(1.5)^3 + (4.5)(1.5)^3 = 28.6875$$

$$\delta_A = \frac{1}{EI} \{ 0 + 28.6875 \} = \frac{28.6875}{3.28 \times 10^3} = 8.75 \times 10^{-3} \text{ m} = 8.75 \text{ mm} \quad \downarrow$$

Problem 11.93

11.93 For the beam and loading shown, determine the deflection of point B. Use $E = 200 \text{ GPa}$.



Add force Q at point B.

Units: forces in kN, lengths in m

$$E = 200 \text{ GPa}, \quad I = 16.4 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(16.4 \times 10^{-6}) = 3.28 \times 10^6 \text{ Nm}^2$$

$$U = \int_0^3 \frac{M^2}{2EI} dx \quad \delta_B = \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^3 M \frac{\partial M}{\partial Q} dx$$

Over portion AB $0 < x < 1.5$, $M = -6x - Qx$ $\frac{\partial M}{\partial Q} = -x$

$$\int_0^{1.5} M \frac{\partial M}{\partial Q} dx = \int_0^{1.5} (-6x - Qx)(-x) dx = \int_0^{1.5} (6x^2 + Qx^2) dx = (6 \times \frac{1}{3})(1.5)^3 = 6.75$$

Over portion BC $1.5 < x < 3$ $M = -6x - 6(x-1.5) - Qx$

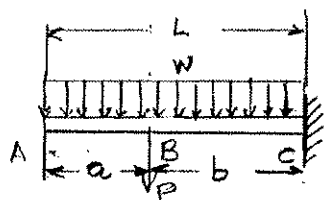
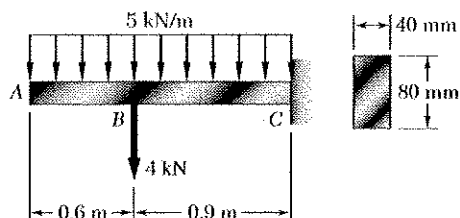
$$M = -12x + 9 - Qx \quad \frac{\partial M}{\partial Q} = -x$$

$$\int_{1.5}^3 M \frac{\partial M}{\partial Q} dx = \int_{1.5}^3 (-12x^2 + 9x - Qx^2) dx = (-12 \times \frac{1}{3})(3^3 - 1.5^3) - (9 \times \frac{1}{2})(3^2 - 1.5^2) = 64.125$$

$$\delta_B = \frac{1}{EI} \{ 6.75 + 64.125 \} = \frac{70.875}{3.28 \times 10^3} = 21.6 \times 10^{-3} \text{ m} = 21.6 \text{ mm} \quad \downarrow$$

Problem 11.94

11.94 and 11.95 For the beam and loading shown, determine the deflection at point B. Use $E = 200 \text{ GPa}$.



$$U = \int_0^a \frac{M^2}{2EI} dx + \int_a^L \frac{M^2}{2EI} dx$$

$$\delta_B = \frac{\partial U}{\partial P} = \int_0^a \frac{M}{EI} \frac{\partial M}{\partial P} dx + \int_a^L \frac{M}{EI} \frac{\partial M}{\partial P} dx$$

Portion AB: ($0 \leq x \leq a$)

$$M = -\frac{1}{2}wx^2 \quad \frac{\partial M}{\partial P} = 0$$

$$\int_0^a \frac{M}{EI} \frac{\partial M}{\partial P} dx = 0$$

Portion BC: ($a < x \leq L$)

$$M = -\frac{1}{2}wx^2 - P(x-a)$$

$$\frac{\partial M}{\partial P} = -(x-a)$$

$$\begin{aligned} \int_a^L \frac{M}{EI} \frac{\partial M}{\partial P} dx &= \frac{w}{2EI} \int_a^L x^2(x-a) dx + \frac{P}{EI} \int_a^L (x-a)^2 dx \\ &= \frac{w}{2EI} \int_a^L (x^3 - ax^2) dx + \frac{P}{EI} \int_0^b v^2 dv \\ &= \frac{w}{2EI} \left(\frac{L^4}{4} - \frac{aL^3}{3} - \frac{a^4}{4} + \frac{a^4}{3} \right) + \frac{Pb^3}{3EI} \end{aligned}$$

$$\delta_B = 0 + \frac{w}{2EI} \left(\frac{L^4}{4} - \frac{aL^3}{3} + \frac{a^4}{12} \right) + \frac{Pb^3}{3EI}$$

Data: $a = 0.6 \text{ m}$, $b = 0.9 \text{ m}$, $L = a + b = 1.5 \text{ m}$

$$w = 5 \times 10^3 \text{ N/m} \quad P = 4 \times 10^3 \text{ N}$$

$$I = \frac{1}{12} (40)(80)^3 = 1.70667 \times 10^6 \text{ mm}^4 = 1.70667 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(1.70667 \times 10^{-6}) = 341333 \text{ N}\cdot\text{m}^2$$

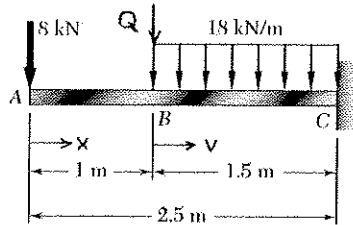
$$\delta_B = 0 + \frac{5 \times 10^3}{(2)(341333)} \left[\frac{(1.5)^4}{4} - \frac{(0.6)(1.5)^3}{3} + \frac{(0.6)^4}{12} \right] + \frac{(4 \times 10^3)(0.9)^3}{(3)(341333)}$$

$$= 7.25 \times 10^{-3} \text{ m}$$

$$\delta_B = 7.25 \text{ mm} \downarrow$$

Problem 11.95

11.94 and 11.95 For the beam and loading shown, determine the deflection at point B. Use $E = 200 \text{ GPa}$.



W250 $\times$ 22.3

Add force Q at point B.

Units: Forces in kN, lengths in m.

$$\text{Over AB: } M = -8x \quad \frac{\partial M}{\partial Q} = 0$$

$$\text{Over BC: } M = -8(v+1) - \frac{1}{2}(18)v^2 - Qv \quad \frac{\partial M}{\partial Q} = -v$$

$$E = 200 \times 10^9 \text{ Pa}, \quad I = 28.9 \times 10^6 \text{ mm}^4 = 28.9 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(28.9 \times 10^{-6}) = 5.78 \times 10^6 \text{ N}\cdot\text{m}^2 = 5780 \text{ kN}\cdot\text{m}^2$$

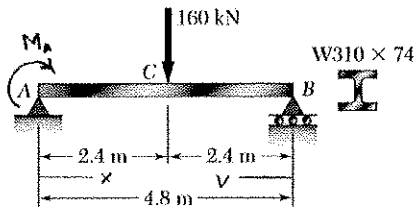
$$U = \int_0^1 \frac{M^2}{2EI} dx + \int_0^{1.5} \frac{M^2}{2EI} dv$$

$$\begin{aligned} \delta_B &= \frac{\partial U}{\partial Q} = \frac{1}{EI} \left\{ \int_0^1 M \frac{\partial M}{\partial Q} dx + \int_0^{1.5} M \frac{\partial M}{\partial Q} dv \right\} \\ &= \frac{1}{EI} \left\{ 0 + \int_0^{1.5} [-8(v+1) - \frac{1}{2}(18)v^2](-v) dv \right\} = \frac{1}{EI} \int_0^{1.5} (9v^3 + 8v^2 + 8v) dv \\ &= \frac{1}{EI} \left\{ \frac{9}{4}(1.5)^4 + \frac{8}{3}(1.5)^3 + \frac{8}{2}(1.5)^2 \right\} = \frac{29.391}{EI} = \frac{29.391}{5780} \end{aligned}$$

$$\delta_B = 5.08 \times 10^{-3} \text{ m} = 5.08 \text{ mm} \downarrow$$

Problem 11.96

11.96 For the beam and loading shown, determine the slope at end A.
Use $E = 200 \text{ GPa}$.



Add couple M_A at point A.

Units: forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}, I = 165 \times 10^6 \text{ mm}^4 = 165 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(165 \times 10^{-6}) = 33 \times 10^6 \text{ N}\cdot\text{m}^2 = 33000 \text{ kN}\cdot\text{m}^2$$

$$\text{Reactions: } R_A = 80 - \frac{M_A}{4.8} \quad R_B = 80 + \frac{M_A}{4.8}$$

$$U = U_{AB} + U_{BC} = \int_0^{2.4} \frac{M^2}{2EI} dx + \int_0^{2.4} \frac{M^2}{2EI} dv \quad \partial \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_A}$$

$$\text{Over AB: } M = M_A + R_A x = M_A + 80x - \frac{M_A}{4.8} x \quad \frac{\partial M}{\partial M_A} = \left(1 - \frac{x}{4.8}\right)$$

$$\text{Set } M_A = 0. \quad \frac{\partial U_{AB}}{\partial M_A} = \frac{1}{EI} \int_0^{2.4} (80x) \left(1 - \frac{x}{4.8}\right) dx = \frac{1}{EI} \int_0^{2.4} (80x - 16.6667x^2) dx$$

$$= \frac{1}{EI} \left\{ (80) \left(\frac{1}{2}\right) (2.4)^2 - (16.6667) \left(\frac{1}{3}\right) (2.4)^3 \right\} = \frac{153.6}{EI}$$

$$\text{Over BC: } M = R_B v = 80v + \frac{M_A}{4.8} v, \quad \frac{\partial M}{\partial M_A} = \frac{1}{4.8} v$$

$$\text{Set } M_A = 0. \quad \frac{\partial U_{BC}}{\partial M_A} = \frac{1}{EI} \int_0^{2.4} (80v) \left(\frac{1}{4.8} v\right) dv = \frac{16.6667}{EI} \int_0^{2.4} v^2 dv$$

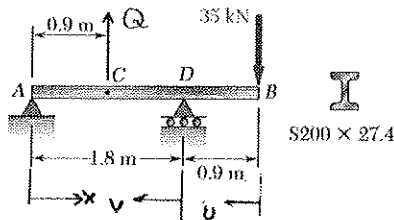
$$= \frac{(16.6667)(2.4)^3}{3EI} = \frac{76.8}{EI}$$

$$\partial \theta_A = \frac{1}{EI} \{ 153.6 + 76.8 \} = \frac{230.4}{33000}$$

$$\theta_A = 6.98 \times 10^{-3} \text{ rad} \quad \swarrow$$

Problem 11.97

11.97 For the beam and loading shown, determine the deflection at point C. Use $E = 200 \text{ GPa}$.



Units: Forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa} \quad I = 23.9 \times 10^{-6} \text{ m}^4$$

$$EI = 200 \times 23.9 = 4780 \text{ kN} \cdot \text{m}^2$$

Add dummy force Q at point C. Reactions $R_A = 17.5 + \frac{1}{2}Q \downarrow$, $R_D = 52.5 - \frac{1}{2}Q \uparrow$

$$U = U_{AC} + U_{CD} + U_{DB} \quad \delta_c = \frac{\partial U}{\partial Q} = \frac{\partial U_{AC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} + \frac{\partial U_{DB}}{\partial Q}$$

Over AC $0 < x < 0.9$ $M = -(17.5 + \frac{1}{2}Q)x$ $\frac{\partial M}{\partial Q} = -\frac{1}{2}x$ Set $Q = 0$.

$$\frac{\partial U_{AC}}{\partial Q} = \frac{1}{EI} \int_0^{0.9} (17.5x)(\frac{1}{2}x) dx = \frac{8.75}{EI} \int_0^{0.9} x^2 dx = \frac{(8.75)(0.9)^3}{3EI} = \frac{2.12625}{EI}$$

Over CD $0 < v < 0.9$ $M = R_D v - 35(v + 1.5) = 52.5v - \frac{1}{2}Q - 35v - 52.5$

$$= 17.5v - 52.5 - \frac{1}{2}Qv$$

$$\frac{\partial M}{\partial Q} = -\frac{1}{2}v \quad \text{Set } Q = 0$$

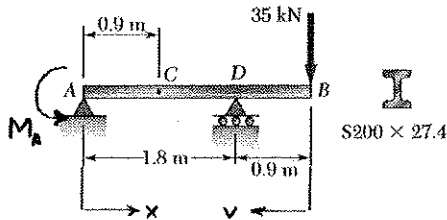
$$\begin{aligned} \frac{\partial U_{CD}}{\partial Q} &= \frac{1}{EI} \int_0^{0.9} (52.5 - 17.5v)(\frac{1}{2}v) dv = \frac{1}{EI} \int_0^{0.9} (26.25v - 8.75v^2) dv \\ &= \frac{1}{EI} \left\{ (26.25)(\frac{0.9^2}{2}) - (8.75)(\frac{0.9^3}{3}) \right\} \\ &= \frac{8.00625}{EI} \end{aligned}$$

Over DB $0 < v < 0.9$ $M = -35v$ $\frac{\partial M}{\partial Q} = 0$ $\frac{\partial U_{DB}}{\partial Q} = 0$

$$\delta_c = \frac{2.12625}{EI} + \frac{8.00625}{EI} + 0 = \frac{10.1325}{4780} = 2.12 \times 10^{-3} \text{ m} = 2.1 \text{ mm} \uparrow$$

Problem 11.98

11.98 For the beam and loading shown, determine the slope at end A. Use $E = 200 \text{ GPa}$.



Units: Forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa} \quad I = 23.9 \times 10^{-6} \text{ m}^4$$

$$EI = 200 \times 23.9 = 4780 \text{ kNm}^2$$

Add dummy couple M_A at end A. Reactions: $R_A = -17.5 + \frac{M_A}{1.8}$, $R_D = 52.5 - \frac{M_A}{1.8}$

$$U = U_{AD} + U_{DB} \quad \delta \theta_A = \frac{\partial U}{\partial M_A} = \frac{\partial U_{AD}}{\partial M_A} + \frac{\partial U_{DB}}{\partial M_A}$$

Over AD $0 < x < 1.8$ $M = -M_A + R_A x = -M_A - 17.5x + \frac{M_A}{1.8}x$

$$\frac{\partial M}{\partial M_A} = -\left(1 - \frac{x}{1.8}\right) \quad \text{Set } M_A = 0.$$

$$\begin{aligned} \frac{\partial U_{AD}}{\partial M_A} &= \frac{1}{EI} \int_0^{1.8} (17.5x)\left(1 - \frac{x}{1.8}\right) dx = \frac{1}{EI} \int_0^{1.8} \left(17.5x - \frac{17.5}{1.8}x^2\right) dx = \frac{1}{EI} \left\{ (17.5)\frac{(1.8)^2}{2} - \frac{17.5}{1.8} \frac{(1.8)^3}{3} \right\} \\ &= \frac{17.85}{EI} \end{aligned}$$

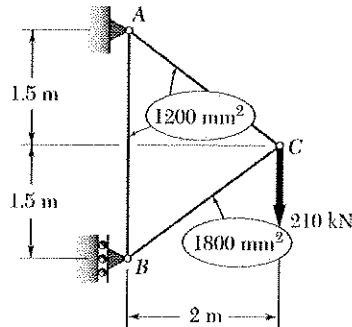
Over DB $0 < v < 0.9$ $M = -35v$ $\frac{\partial M}{\partial M_A} = 0$ $\frac{\partial U_{DB}}{\partial M_A} = 0$

$$\delta \theta_A = \frac{17.85}{EI} + 0 = \frac{17.85}{4780} = 3.73 \times 10^{-3} \text{ rad}$$

Problem 11.99

11.99 and 11.100 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

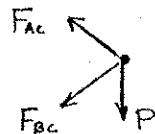
11.99 Vertical deflection of joint C.



Call the vertical load P . The vertical deflection of joint C is δ_P .

$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial P}$$

Joint C: $\rightarrow +\sum F_x = 0: -\frac{4}{5}F_{AC} - \frac{4}{5}F_{BC} = 0$



$\uparrow +\sum F_y = 0: \frac{3}{5}F_{AC} - \frac{3}{5}F_{BC} - P = 0$

Solving simultaneously,

$$F_{AC} = \frac{5}{6}P \quad F_{BC} = -\frac{5}{6}P$$

Joint B: $\uparrow +\sum F_y = 0:$

$F_{AB} - \frac{3}{5} \cdot \frac{5}{6}P = 0$
 $F_{AB} = \frac{1}{2}P$

Member	F	$\partial F / \partial P$	L (m)	A (10^{-6} m^2)	$F(\partial F / \partial P)L/A$
AB	$\frac{1}{2}P$	$\frac{1}{2}$	3	1200	$625 P$
AC	$\frac{5}{6}P$	$\frac{5}{6}$	2.5	1200	$1446.76 P$
BC	$-\frac{5}{6}P$	$-\frac{5}{6}$	2.5	1800	$964.51 P$
Σ					$3036.27 P$

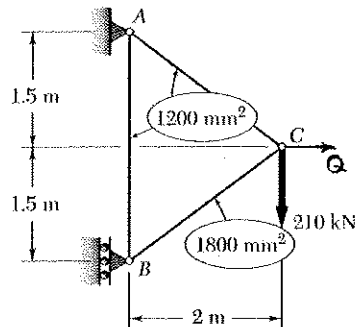
$$\delta_P = \frac{1}{E} (3036.27 P) = \frac{(3036.27)(210 \times 10^3)}{200 \times 10^9} = 3.19 \times 10^{-3} \text{ m}$$

$$\delta_P = 3.19 \text{ mm} \downarrow$$

Problem 11.100

11.99 and 11.100 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

11.100 Horizontal deflection of joint C.



Call the vertical force P . Add a horizontal force Q (positive $\rightarrow$) at joint C. The horizontal deflection of joint C is δ_Q .

$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EI} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial Q}$$

Joint C $\rightarrow \sum F_x = 0:$

$$-\frac{4}{5} F_{AC} - \frac{4}{5} F_{BC} + Q = 0$$

$\uparrow \sum F_y = 0:$

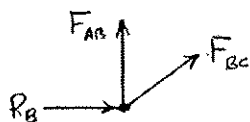
$$\frac{3}{5} F_{AC} - \frac{3}{5} F_{BC} - P = 0$$

Solving simultaneously, $F_{AC} = \frac{5}{6} P + \frac{5}{8} Q$ $F_{BC} = -\frac{5}{6} P + \frac{5}{8} Q$

Joint B $\uparrow \sum F_y = 0:$

$$F_{AB} + \frac{3}{5} F_{BC} = 0$$

$$F_{AB} = -\frac{3}{5} F_{BC} = \frac{1}{2} P - \frac{3}{8} Q$$



Member	F	$\partial F / \partial Q$	L (m)	A (10^6 m^2)	with Q = 0 $F (\partial F / \partial Q) L / A$
AB	$\frac{1}{2} P - \frac{3}{8} Q$	$-\frac{3}{8}$	3	1200	$-468.75 P$
AC	$\frac{5}{6} P + \frac{5}{8} Q$	$+\frac{5}{8}$	2.5	1200	$1085.07 P$
BC	$-\frac{5}{6} P + \frac{5}{8} Q$	$+\frac{5}{8}$	2.5	1800	$-723.38 P$
					$-107.06 P$

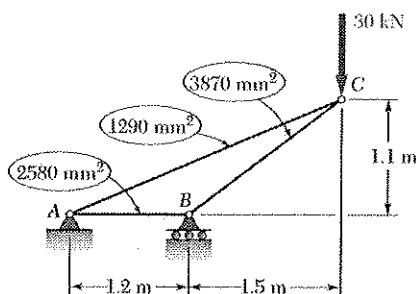
$$\delta_Q = \frac{1}{E} (-107.06 P) = -\frac{(107.06)(210 \times 10^3)}{200 \times 10^9} = -0.1124 \times 10^{-3} \text{ m}$$

$$\delta_Q = 0.1124 \text{ mm} \leftarrow$$

Problem 11.101

11.101 and 11.102 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

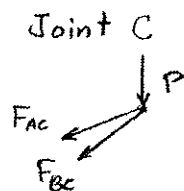
11.101 Vertical deflection of joint C.



Call the vertical load P . The vertical deflection of joint C is δ_P .

$$\delta_P = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial P}$$

Geometry $\overline{AC} = \sqrt{2.7^2 + 1.1^2} = 2.91 \text{ m}$
 $\overline{BC} = \sqrt{1.5^2 + 1.1^2} = 1.86 \text{ m}$

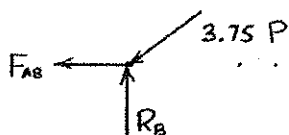


$$+\rightarrow \sum F_x = 0: -\frac{2.7}{2.91} F_{AC} - \frac{1.5}{1.86} F_{BC} = 0$$

$$+\uparrow \sum F_y = 0: -\frac{1.1}{2.91} F_{AC} - \frac{1.1}{1.86} F_{BC} - P = 0$$

Solving simultaneously, $F_{AC} = 3.3 P$, $F_{BC} = -3.8 P$

Joint B



$$+\rightarrow \sum F_x = 0: -F_{AB} - \frac{1.5}{1.86} F_{BC} = 0$$

$$F_{AB} = -3.06 P$$

Member	F	$\partial F / \partial P$	L (m)	$\frac{L}{A} \times 10^{-6}$ (m²)	$F(\partial F / \partial P) L / A$
AB	$-3.06 P$	3.06	1.2	2580	$4355 P$
AC	$3.3 P$	3.3	2.91	1290	$24566 P$
BC	$-3.8 P$	3.8	1.86	3870	$6940 P$

Σ

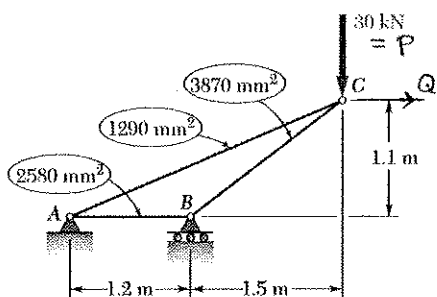
$$35861 P$$

$$\delta_P = \frac{35861 P}{E} = \frac{(35861)(30)}{200 \times 10^6} = 5.379 \times 10^{-3} \text{ m} = 5.38 \text{ mm} \downarrow$$

Problem 11.102

11.101 and 11.102 Each member of the truss shown is made of steel and has the cross-sectional area shown. Using $E = 200 \text{ GPa}$, determine the deflection indicated.

11.102 Horizontal deflection of point C.



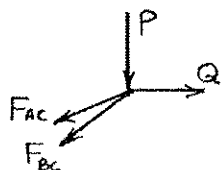
Call the vertical load P . Add horizontal dummy load Q at joint C. The horizontal deflection of joint C is δ_Q .

$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EA} = \frac{1}{E} \sum \frac{FL}{A} \frac{\partial F}{\partial Q}$$

$$\text{Geometry: } \bar{AC} = \sqrt{2.7^2 + 1.1^2} = 2.91 \text{ m}$$

$$\bar{BC} = \sqrt{1.5^2 + 1.1^2} = 1.86 \text{ m}$$

Joint C



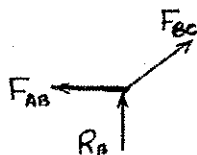
$$+\rightarrow \sum F_x = 0: -\frac{2.7}{2.91} F_{AC} - \frac{1.5}{1.86} F_{BC} + Q = 0$$

$$+\uparrow \sum F_y = 0: -\frac{1.1}{2.91} F_{AC} - \frac{1.1}{1.86} F_{BC} - P = 0$$

$$\text{Solving simultaneously, } F_{AC} = 3.3P + 2.475Q$$

$$F_{BC} = -3.8P - 1.583Q$$

Joint B



$$+\rightarrow \sum F_x = 0: \frac{1.5}{1.86} F_{AC} - F_{AB} = 0$$

$$F_{AB} = \frac{1.5}{1.86} F_{BC} = -3.06P - 1.275Q$$

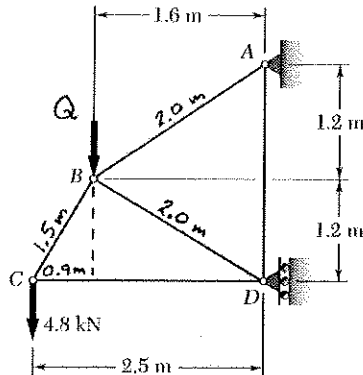
Member	F	$\partial F / \partial Q$	L (m)	A ($\times 10^{-6} \text{ m}^2$)	$F(\partial F / \partial Q)L/A$
AB	$-3.06P - 1.275Q$	-1.275	1.2	2580	$1814.7P$
AC	$3.3P + 2.475Q$	2.475	2.91	1290	$18424.4P$
BC	$-3.8P - 1.583Q$	-1.583	1.86	3870	$2891.1P$
Σ					$581.67P$

$$\delta_Q = \frac{23130.2P}{E} = \frac{(23130.2)(30)}{200 \times 10^6} = 0.00347 \text{ m} = 3.5 \text{ mm} \rightarrow$$

Problem 11.103

11.103 and 11.104 Each member of the truss shown is made of steel and has a cross-sectional area of 500mm^2 . Using $E = 200\text{ GPa}$, determine the deflection indicated.

11.103 Vertical deflection of joint B.



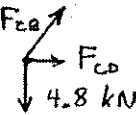
Find the length of each member as shown.

Add vertical force Q at joint B.

$$S_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial Q} L$$

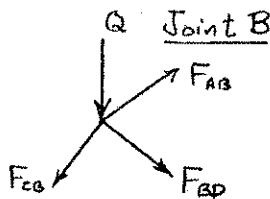
Joint C $+ \uparrow \sum F_y = 0: \frac{4}{5} F_{CB} - 4.8 = 0$

$$F_{CB} = 6.0 \text{ kN}$$



$+ \rightarrow \sum F_x = 0: \frac{3}{5} F_{CB} + F_{CD} = 0$

$$F_{CD} = -3.6 \text{ kN}$$



$+ \rightarrow \sum F_x = 0: \frac{4}{5} F_{AB} + \frac{4}{5} F_{BD} - 3.6 = 0$

$+ \uparrow \sum F_y = 0: \frac{3}{5} F_{AB} - \frac{3}{5} F_{BD} - 4.8 - Q = 0$

Solving simultaneously, $F_{AB} = 6.25 + 0.8333 Q \text{ kN}$

$$F_{BD} = -1.75 - 0.8333 Q \text{ kN}$$

Joint D



$+ \uparrow \sum F_y = 0: \frac{3}{5} F_{BD} + F_{AD} = 0$

$$F_{AD} = -\frac{3}{5} F_{BD} = 1.05 + 0.5 Q$$

Member	F (10^3 N)	$\partial F / \partial Q$	L (m)	with $Q = 0$ $F(\partial F / \partial Q) L$ ($10^3 \text{ N}\cdot\text{m}$)
AB	$6.25 + 0.8333 Q$	0.8333	2.0	10.4167
AD	$1.05 + 0.5 Q$	0.5	2.4	1.26
BD	$-1.75 - 0.8333 Q$	-0.8333	2.0	2.9167
BC	6.0	0	1.5	0
CD	-3.6	0	2.5	0
Σ				14.593

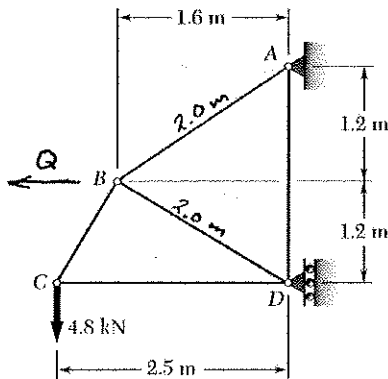
$$S_B = \frac{1}{EA} \sum F(\partial F / \partial Q) L = \frac{14.593 \times 10^3}{(200 \times 10^9)(500 \times 10^{-6})} = 145.9 \times 10^{-6} \text{ m}$$

$$S_B = 0.1459 \text{ mm} \downarrow$$

Problem 11.104

11.103 and 11.104 Each member of the truss shown is made of steel and has a cross-sectional area of 500mm^2 . Using $E = 200\text{ GPa}$, determine the deflection indicated.

11.104 Horizontal deflection of joint B.



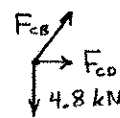
Find the length of each member as shown.

Add horizontal force Q at joint B.

$$\delta_B = \frac{\partial U}{\partial Q} = \frac{\partial}{\partial Q} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial Q} L$$

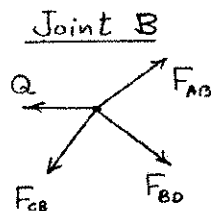
Joint C $\uparrow \sum F_y = 0: \frac{4}{5} F_{CB} - 4.8 = 0$

$$F_{CB} = 6.0 \text{ kN}$$



$\rightarrow \sum F_x = 0: \frac{3}{5} F_{CB} + F_{CD} = 0$

$$F_{CD} = -3.6 \text{ kN}$$



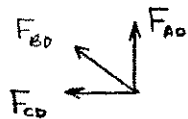
$\rightarrow \sum F_x = 0: \frac{4}{5} F_{AB} + \frac{4}{5} F_{BD} - 3.6 - Q = 0$

$\uparrow \sum F_y = 0: \frac{3}{5} F_{AB} - \frac{3}{5} F_{BD} - 4.8 = 0$

Solving simultaneously $F_{AB} = 6.25 + 0.625 Q \text{ kN}$

$$F_{BD} = -1.75 + 0.625 Q \text{ kN}$$

Joint D



$\uparrow \sum F_y = 0: \frac{3}{5} F_{BD} + F_{AD} = 0$

$$F_{AD} = -\frac{3}{5} F_{BD} = 1.05 - 0.375 Q$$

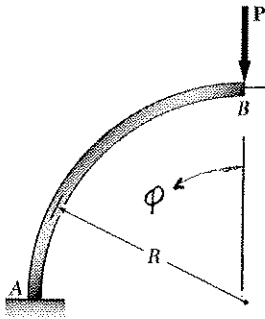
Member	F 10^3 N	$\partial F / \partial Q$	L (m)	$F(\partial F / \partial Q)L$ ($10^3 \text{ N}\cdot\text{m}$)
AB	$6.25 + 0.625 Q$	0.625	2.0	7.8125
AD	$1.05 + 0.375 Q$	-0.375	2.4	-0.9450
BD	$-1.75 + 0.625 Q$	0.625	2.0	-2.1875
BC	6.0	0	1.5	0
CD	-3.6	0	2.5	0
Σ				4.680

$$\delta_B = \frac{1}{EA} \sum F(\partial F / \partial Q)L = \frac{4.680 \times 10^3}{(200 \times 10^9)(500 \times 10^{-6})} = 46.8 \times 10^{-6} \text{ m}$$

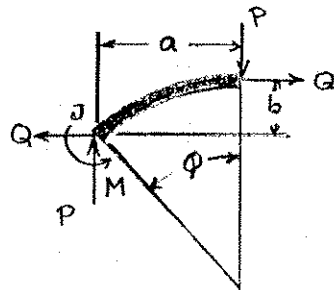
$$\delta_B = 0.0468 \text{ mm} \leftarrow$$

Problem 11.105

11.105 For the beam and loading shown and using Castigliano's theorem, determine (a) the horizontal deflection of point B, (b) the vertical deflection of point B.



Add horizontal force Q at point B.
Use polar coordinate φ .



$$U = \int_0^{\frac{\pi}{2}} \frac{M^2}{2EI} R d\varphi$$

Bending moment.

$$+\circlearrowleft \sum M_J = 0: M - Pa - Qb = 0$$

$$M = Pa + Qb \\ = PR \sin \varphi + QR(1 - \cos \varphi)$$

$$\frac{\partial M}{\partial P} = R \sin \varphi \quad \frac{\partial M}{\partial Q} = R(1 - \cos \varphi) \quad \text{Set } Q = 0$$

$$\begin{aligned} (a) \quad \delta_a &= \frac{\partial U}{\partial Q} = \frac{1}{EI} \int_0^{\frac{\pi}{2}} M \frac{\partial M}{\partial Q} R d\varphi = \frac{1}{EI} \int_0^{\frac{\pi}{2}} PR \sin \varphi R(1 - \cos \varphi) R d\varphi \\ &= \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} (\sin \varphi - \sin \varphi \cos \varphi) d\varphi = \frac{PR^3}{EI} \left(-\cos \varphi - \frac{1}{2} \sin^2 \varphi \right) \Big|_0^{\frac{\pi}{2}} \\ &= \frac{PR^3}{EI} \left(-\cos \frac{\pi}{2} + \cos 0 - \frac{1}{2} \sin^2 \frac{\pi}{2} + \frac{1}{2} \sin^2 0 \right) \\ &= \frac{PR^3}{EI} \left(0 + 1 - \frac{1}{2} + 0 \right) \end{aligned}$$

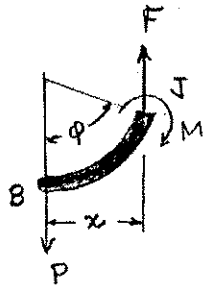
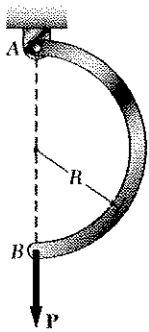
$$\delta_a = \frac{PR^3}{2EI} \rightarrow \blacktriangleleft$$

$$\begin{aligned} (b) \quad \delta_p &= \frac{\partial U}{\partial P} = \frac{1}{EI} \int_0^{\frac{\pi}{2}} M \frac{\partial M}{\partial P} R d\varphi = \frac{1}{EI} \int_0^{\frac{\pi}{2}} PR \sin \varphi R \sin \varphi R d\varphi \\ &= \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} \sin^2 \varphi d\varphi = \frac{PR^3}{EI} \int_0^{\frac{\pi}{2}} \frac{1}{2} (1 - \cos 2\varphi) d\varphi \\ &= \frac{PR^3}{EI} \left(\frac{1}{2} \varphi - \frac{1}{2} \sin 2\varphi \right) \Big|_0^{\frac{\pi}{2}} = \frac{PR^3}{EI} \left(\frac{1}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \cdot 0 - \frac{1}{2} \sin \pi + \frac{1}{2} \sin 0 \right) \\ &= \frac{PR^3}{EI} \left(\frac{\pi}{4} - 0 - 0 + 0 \right) \end{aligned}$$

$$\delta_p = \frac{\pi PR^3}{4EI} \downarrow \blacktriangleleft$$

Problem 11.106

11.106 For the uniform rod and loading shown and using Castigliano's theorem, determine the deflection of point B.



Use polar coordinate ϕ .

Calculate the bending moment $M(\phi)$ using free body B.J.

$$+\circlearrowleft \sum M_J = 0: Px - M = 0$$

$$M = Px = PR \sin \phi$$

Strain energy: $U = \int_0^L \frac{M^2}{2EI} ds$

$$U = \int_0^\pi \frac{(PR \sin \phi)^2}{2EI} (R d\phi) = \frac{P^2 R^3}{2EI} \int_0^\pi \sin^2 \phi d\phi$$

$$= \frac{P^2 R^3}{2EI} \int_0^\pi \frac{1 - \cos 2\phi}{2} d\phi$$

$$= \frac{P^2 R^3}{2EI} \left(\frac{1}{2} \phi \Big|_0^\pi - \frac{1}{4} \sin 2\phi \Big|_0^\pi \right) = \frac{\pi P^2 R^3}{4EI}$$

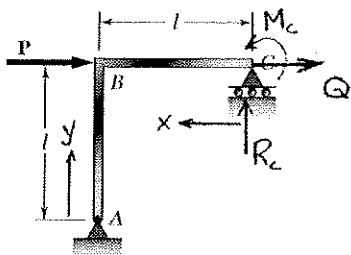
By Castigliano's theorem,

$$\delta = \frac{\partial U}{\partial P}$$

$$\delta = \frac{\pi P R^3}{2EI} \downarrow$$

Problem 11.107

11.107 Two rods AB and BC of the same flexural rigidity EI are welded together at B . For the loading shown, determine (a) the deflection of point C , (b) the slope of member BC at point C .



Add horizontal force Q and couple M_c at C .

$$+\circlearrowleft \sum M_A = 0: R_c l + M_c - (P+Q)l = 0$$

$$R_c = P + Q + \frac{M_c}{l}$$

$$+\rightarrow \sum F_x = 0: P + Q + R_{Ax} = 0 \quad R_{Ax} = P + Q \leftarrow$$

Member AB : $M = R_{Ax}y = (P+Q)y, \quad \frac{\partial M}{\partial Q} = y, \quad \frac{\partial M}{\partial M_c} = 0$

$$U_{AB} = \int_0^l \frac{M^2}{2EI} dy$$

Set $Q = 0$ and $M_c = 0$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dy = \frac{1}{EI} \int_0^l (Py)(y) dy = \frac{1}{3} \frac{Pl^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial M_c} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dx = 0$$

Member BC : $M = M_c + R_c x = M_c + (P + Q + \frac{M_c}{l})x$

$$\frac{\partial M}{\partial Q} = x, \quad \frac{\partial M}{\partial M_c} = 1 - \frac{x}{l}$$

$$U_{BC} = \int_0^l \frac{M^2}{2EI} dx$$

Set $Q = 0$ and $M_c = 0$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^l (Px)x dx = \frac{1}{3} \frac{Pl^3}{EI}$$

$$\begin{aligned} \frac{\partial U}{\partial M_c} &= \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dx = \frac{1}{EI} \int_0^l (Px) \left(1 - \frac{x}{l}\right) dx = \frac{P}{EI} \int_0^l \left(x - \frac{x^2}{l}\right) dx \\ &= \frac{P}{EI} \left(\frac{1}{2}l^2 - \frac{1}{3}l^2\right) = \frac{1}{6} \frac{Pl^3}{EI} \end{aligned}$$

(a) Deflection at C : $\delta_c = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q}$

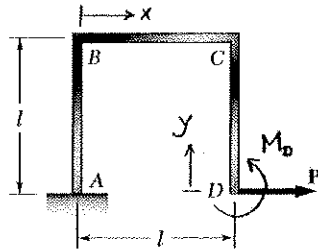
$$\frac{2Pl^3}{3EI} \rightarrow \blacktriangleleft$$

(b) Slope at C : $\theta_c = \frac{\partial U_{AB}}{\partial M_c} + \frac{\partial U_{BC}}{\partial M_c}$

$$\frac{Pl^2}{6EI} \rightarrow \blacktriangleleft$$

Problem 11.108

11.108 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (a) the horizontal deflection of point D, (b) the slope at point D.



Add couple M_D at point D.

Reactions at A: $R_{Ay} = 0$, $R_{Ax} = P \leftarrow$, $M_A = M_D \curvearrowright$

Member AB: $M = M_A + R_{Ay} = M_D + Py$ $\frac{\partial M}{\partial P} = y$, $\frac{\partial M}{\partial M_D} = 1$
 $U_{AB} = \int_0^l \frac{M^2}{2EI} dy$ Set $M_D = 0$.

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dy = \frac{1}{EI} \int_0^l (Py)(y) dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{AB}}{\partial M_D} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_D} dy = \frac{1}{EI} \int_0^l (Py)(1) dy = \frac{Pl^2}{2EI}$$

Member BC: $M = M_A + R_A l = M_D + Pl$ $\frac{\partial M}{\partial P} = l$, $\frac{\partial M}{\partial M_D} = 1$
 $U_{BC} = \int_0^l \frac{M^2}{2EI} dx$ Set $M_D = 0$.

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dx = \frac{1}{EI} \int_0^l (Pl)(l) dx = \frac{Pl^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial M_D} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_D} dx = \frac{1}{EI} \int_0^l (Pl)(1) dx = \frac{Pl^2}{EI}$$

Member CD: $M = M_D + Py$ $\frac{\partial M}{\partial P} = y$, $\frac{\partial M}{\partial M_D} = 1$
 $U_{CD} = \int_0^l \frac{M^2}{2EI} dy$ Set $M_D = 0$.

$$\frac{\partial U_{CD}}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dy = \frac{1}{EI} \int_0^l (Py)(y) dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{CD}}{\partial M_D} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_D} dy = \frac{1}{EI} \int_0^l (Py)(1) dy = \frac{Pl^2}{2EI}$$

(a) Horizontal deflection of point D:

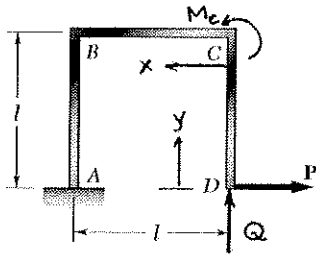
$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} + \frac{\partial U_{CD}}{\partial P} = \left(\frac{1}{3} + 1 + \frac{1}{3}\right) \frac{Pl^3}{EI} \quad \delta_P = \frac{5Pl^3}{3EI} \rightarrow \leftarrow$$

(b) Slope at point D:

$$\theta_D = \frac{\partial U_{AB}}{\partial M_D} + \frac{\partial U_{BC}}{\partial M_D} + \frac{\partial U_{CD}}{\partial M_D} = \left(\frac{1}{2} + 1 + \frac{1}{2}\right) \frac{Pl^2}{EI} \quad \theta_D = \frac{3Pl^2}{EI} \curvearrowright \leftarrow$$

Problem 11.109

11.109 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (a) the vertical deflection of point D, (b) the slope of BC at point C.



Add vertical force Q at point D and couple M_c at point C.

Reactions at A: $R_{Ax} = P$, $R_{Ay} = Ql$
 $M_A = Ql + M_c$

Member AB: $M = M_A + R_{Ay}y = Ql + M_c + Py$, $\frac{\partial M}{\partial Q} = l$, $\frac{\partial M}{\partial M_c} = 1$
 $U_{AB} = \int_0^l \frac{M^2}{2EI} dy$ Set $Q = 0$, and $M_c = 0$.

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dy = \frac{1}{EI} \int_0^l (Py)(l) dy = \frac{Pl^3}{2EI}$$

$$\frac{\partial U_{AB}}{\partial M_c} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dy = \frac{1}{EI} \int_0^l (Py)(1) dy = \frac{Pl^2}{2EI}$$

Member BC: $M = M_c + Pl + Qx$ $\frac{\partial M}{\partial Q} = x$ $\frac{\partial M}{\partial M_c} = 1$

$$U_{BC} = \frac{1}{EI} \int_0^l \frac{M^2}{2EI} dx$$
 Set $Q = 0$, and $M_c = 0$.

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^l (Pl)(x) dx = \frac{Pl^3}{2EI}$$

$$\frac{\partial U_{BC}}{\partial M_c} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial M_c} dx = \frac{1}{EI} \int_0^l (Pl)(1) dx = \frac{Pl^2}{EI}$$

Member CD: $M = Py$ $\frac{\partial M}{\partial Q} = 0$ $\frac{\partial M}{\partial M_c} = 0$

$$\frac{\partial U_{CD}}{\partial Q} = 0 \quad \frac{\partial U_{CD}}{\partial M_c} = 0$$

(a) Vertical deflection of point D:

$$S_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} + \frac{\partial U_{CD}}{\partial Q} = \left(\frac{1}{2} + \frac{1}{2} + 0\right) \frac{Pl^3}{EI}$$

$$S_Q = \frac{Pl^3}{EI} \uparrow$$

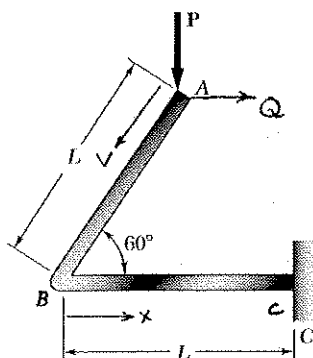
(b) Slope of BC at C:

$$\theta_c = \frac{\partial U_{AB}}{\partial M_c} + \frac{\partial U_{BC}}{\partial M_c} + \frac{\partial U_{CD}}{\partial M_c} = \left(\frac{1}{2} + 1 + 0\right) \frac{Pl^2}{EI}$$

$$\theta_c = \frac{3Pl^2}{2EI} \nearrow$$

Problem 11.110

11.110 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (a) the vertical deflection of point A, (b) the horizontal deflection of point A.



Add horizontal force Q at point A.

Over AB: $M = \frac{1}{2}Pv + \frac{\sqrt{3}}{2}Qv$

$$\frac{\partial M}{\partial P} = \frac{1}{2}v \quad \frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}v$$

$$U_{AB} = \int_0^L \frac{M^2}{2EI} dv \quad \text{Set } Q = 0$$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dv = \frac{1}{EI} \int_0^L \left(\frac{1}{2}Pv\right)\left(\frac{1}{2}v\right) dv$$

$$= \frac{1}{12} \frac{PL^3}{EI}$$

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dv = \frac{1}{EI} \int_0^L \left(\frac{1}{2}Pv\right)\frac{\sqrt{3}}{2} dv$$

$$= \frac{\sqrt{3}}{12} \frac{PL^3}{EI}$$

Over BC: $M = -P\left(x - \frac{L}{2}\right) + \frac{\sqrt{3}}{2}QL$, $\frac{\partial M}{\partial P} = -\left(x - \frac{L}{2}\right)$, $\frac{\partial M}{\partial Q} = \frac{\sqrt{3}}{2}L$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx \quad \text{Set } Q = 0.$$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dx = \frac{1}{EI} \int_0^L P\left(x - \frac{L}{2}\right)^2 dx = \frac{P}{3EI} \left(x - \frac{L}{2}\right)^3 \Big|_0^L = \frac{1}{12} \frac{PL^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial Q} dx = \frac{1}{EI} \int_0^L P\left(x - \frac{L}{2}\right)\left(\frac{\sqrt{3}}{2}\right)L dx = -\frac{\sqrt{3}P}{4EI} \left(x - \frac{L}{2}\right)^2 \Big|_0^L = 0$$

(a) Vertical deflection of point A.

$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P}$$

$$\delta_P = \frac{PL^3}{6EI} \downarrow$$

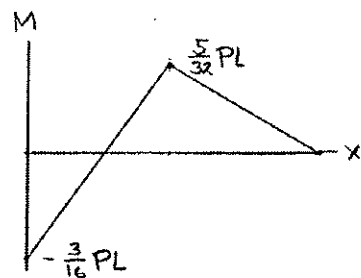
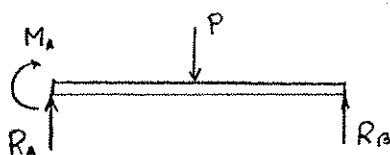
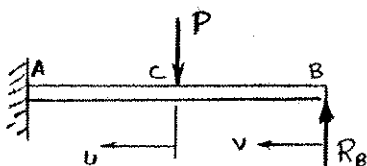
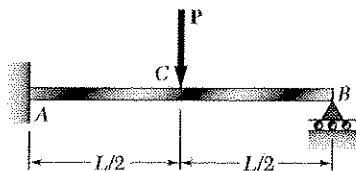
(b) Horizontal deflection of point A.

$$\delta_Q = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q} = \frac{\sqrt{3}}{12} \frac{PL^3}{EI}$$

$$\delta_Q = 0.1443 \frac{PL^3}{EI} \rightarrow$$

Problem 11.111

11.111 through 11.114 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Remove support B and add reaction R_B as a load:

$$U = U_{AC} + U_{CB} = \int_0^{L/2} \frac{M^2}{2EI} du + \int_0^{L/2} \frac{M^2}{2EI} dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{\partial U_{AB}}{\partial R_B} + \frac{\partial U_{CB}}{\partial R_B} = 0$$

Over AC: $M = R_B(u + \frac{L}{2}) - Pu$, $\frac{\partial M}{\partial R_B} = (u + \frac{L}{2})$

$$\frac{\partial U_{AB}}{\partial R_B} = \frac{1}{EI} \int_0^{L/2} [R_B(u + \frac{L}{2}) - Pu](u + \frac{L}{2}) du$$

$$= \frac{R_B}{EI} \int_0^{L/2} (u + \frac{L}{2})^2 du - \frac{P}{EI} \int_0^{L/2} u(u + \frac{L}{2}) du$$

$$= \frac{R_B}{3EI} [L^3 - (\frac{L}{2})^3] - \frac{P}{EI} [\frac{1}{3}(\frac{L}{2})^3 + \frac{L}{2} \cdot \frac{1}{2}(\frac{L}{2})^2]$$

$$= \frac{7}{24} \frac{R_B L^3}{EI} - \frac{5}{48} \frac{PL^3}{EI}$$

Over CB: $M = R_B v$, $\frac{\partial M}{\partial R_B} = v$

$$\frac{\partial U_{CB}}{\partial R_B} = \frac{1}{EI} \int_0^{L/2} (R_B v) v dv = \frac{R_B}{3EI} (\frac{L}{2})^3 = \frac{1}{24} \frac{R_B L^3}{EI}$$

$$y_B = (\frac{7}{24} + \frac{1}{24}) \frac{R_B L^3}{EI} - \frac{5}{48} \frac{PL^3}{EI} = 0$$

$$R_B = \frac{5}{16} P \quad \uparrow$$

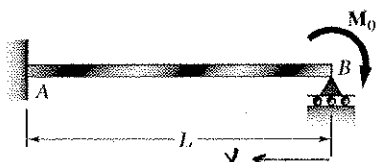
$$M_C = R_B \frac{L}{2} = \frac{5}{32} PL$$

$$M_A = R_B L - P \frac{L}{2} = (\frac{5}{16} - \frac{1}{2}) PL = -\frac{3}{16} PL$$

$$M_B = 0$$

Problem 11.112

1.111 through 11.114 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.

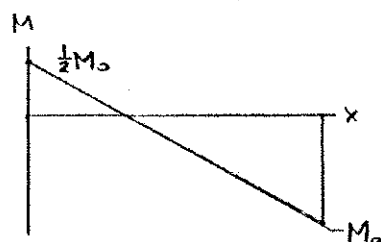
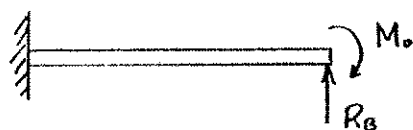


Remove support B and add reaction R_B as a load.

$$U = \int_0^L \frac{M^2}{2EI} dv$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_B} dv = 0$$

$$M = R_B v - M_0 \quad \frac{\partial M}{\partial R_B} = v$$



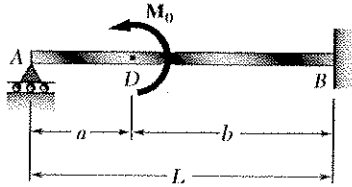
$$\begin{aligned} y_B &= \frac{1}{EI} \int_0^L (R_B v - M_0) v dv \\ &= \frac{R_B}{EI} \int_0^L v^2 dv - \frac{M_0}{EI} \int_0^L v dv \\ &= \frac{R_B L^3}{3EI} - \frac{M_0 L^2}{2EI} = 0 \end{aligned}$$

$$R_B = \frac{3}{2} \frac{M_0}{L} \uparrow$$

$$M_A = R_B - M_0 = \frac{3}{2} M_0 - M_0 = \frac{1}{2} M_0$$

Problem 11.113

11.111 through 11.114 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Remove support A and add reaction R_A as a load.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_A} dx = 0$$

Portion AD: ($0 < x < a$) $M = R_A x$ $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AD}}{\partial R_A} = \frac{1}{EI} \int_0^a (R_A x)(x) dx = \frac{R_A a^3}{3EI}$$

Portion DB: ($a < x < L$) $M = R_A x - M_0$ $\frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI} \int_a^L (R_A x - M_0)(x) dx = \frac{1}{EI} \left\{ \frac{1}{3} R_A (L^3 - a^3) - \frac{1}{2} M_0 (L^2 - a^2) \right\}$$

$$S_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \frac{1}{EI} \left\{ R_A \left(\frac{1}{3} a^3 + \frac{1}{3} L^3 - \frac{1}{3} a^3 \right) - \frac{1}{2} M_0 (L^2 - a^2) \right\} = 0$$

$$R_A = \frac{3}{2} \frac{M_0 (L^2 - a^2)}{L^3}$$

$$R_A' = \frac{3 M_0 b (L + a)}{2 L^3} \quad \blacktriangleleft$$

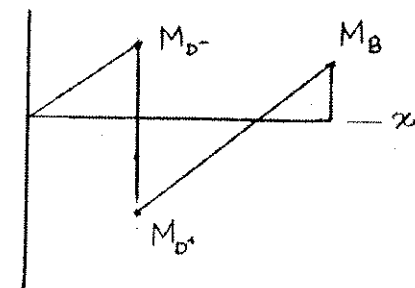
$$M_A = 0$$

$$M_{D-} = R_A a = \frac{3}{2} \frac{M_0 a b (L + a)}{L^3}$$

$$M_{D+} = M_{D-} - M_0 = \frac{3}{2} \frac{M_0 a b (L + a)}{L^3} - M_0$$

$$M_B = R_A L - M_0 = \frac{3}{2} \frac{M_0 b (L + a)}{L^2} - M_0 \quad \blacktriangleleft$$

Bending moment diagram drawn to scale for $a = \frac{1}{3}L$.

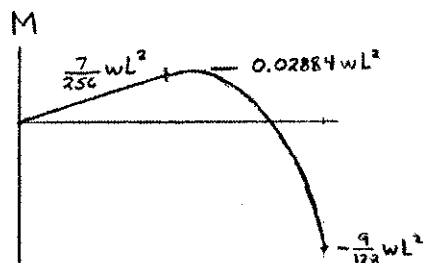
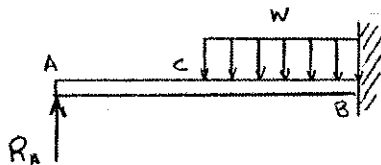
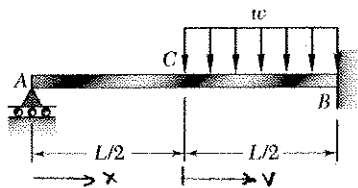


By singularity functions,

$$M = 3 M_0 b (L + a) x / 2 L^3 - M_0 \langle L - a \rangle^0 \quad \blacktriangleleft$$

Problem 11.114

11.111 through 11.114 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and loading shown.



Remove support A and add reaction R_A as a load.

$$U = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx + \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dv$$

$$\delta_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dx + \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dv = 0$$

Portion AC: $(0 < x < \frac{L}{2}) \quad M = R_A x \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AC}}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} (R_A x)(x) dx = \frac{R_A L^3}{24 EI}$$

Portion CB: $(0 < v < \frac{L}{2})$

$$M = R_A (v + \frac{L}{2}) - \frac{1}{2} w v^2 \quad \frac{\partial M}{\partial R_A} = (v + \frac{L}{2})$$

$$\begin{aligned} \frac{\partial U_{CB}}{\partial R_A} &= \frac{1}{EI} \int_0^{\frac{L}{2}} [R_A (v + \frac{L}{2}) - \frac{1}{2} w v^2] (v + \frac{L}{2}) dv \\ &= \frac{1}{EI} \left\{ R_A \int_0^{\frac{L}{2}} (v + \frac{L}{2})^2 dv - \frac{1}{2} w \int_0^{\frac{L}{2}} (v^3 + \frac{L}{2} v^2) dv \right\} \\ &= \frac{R_A}{EI} \left[\frac{1}{3} L^3 - \frac{1}{3} (\frac{L}{2})^3 \right] - \frac{w}{2EI} \left[\frac{1}{4} (\frac{L}{2})^4 + \frac{L}{2} \frac{1}{3} (\frac{L}{2})^3 \right] \\ &= (\frac{1}{3} - \frac{1}{24}) \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI} \end{aligned}$$

$$\delta_A = \frac{\partial U_{AC}}{\partial R_A} + \frac{\partial U_{CB}}{\partial R_A} = \frac{1}{3} \frac{R_A L^3}{EI} - \frac{7}{384} \frac{w L^4}{EI} = 0$$

$$R_A = \frac{7}{128} w L \quad \uparrow$$

Bending moments:

Over AC: $M = \frac{7}{128} w L x$

$$M_c = \frac{7}{256} w L^2 = 0.02734 w L^2$$

Over CB: $M = \frac{7}{128} w L (v + \frac{L}{2}) - \frac{1}{2} w v^2$

$$\begin{aligned} M_B &= \frac{7}{128} w L^2 - \frac{1}{2} w (\frac{L}{2})^2 = -\frac{9}{128} w L^2 \\ &= -0.07031 w L^2 \end{aligned}$$

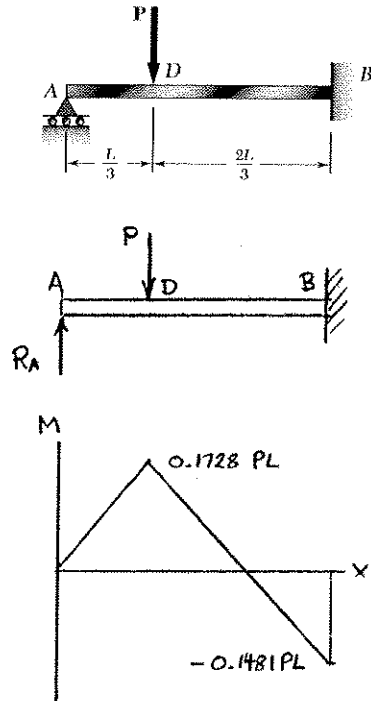
$$\frac{dM}{dv} = \frac{7}{128} w L - w v_m = 0 \quad v_m = \frac{7}{128} L$$

$$\begin{aligned} M_m &= \frac{7}{128} w L \left(\frac{7}{128} L + \frac{L}{2} \right) - \frac{1}{2} w \left(\frac{7}{128} L \right)^2 \\ &= \frac{945}{32768} w L^2 = 0.02884 w L^2 \end{aligned}$$

on $M = \frac{7}{128} w L x - w \langle x - L/2 \rangle^2 / 2$

Problem 11.115

11.115 Determine the reaction at the roller support and draw the bending-moment diagram for the beam and load shown.



Remove support A and add reaction R_A as a load.

$$U = \int_0^L \frac{M^2}{2EI} dx$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial R_A} dx$$

Portion AD: ($0 < x < \frac{L}{3}$) $M = R_A x$ $\frac{\partial M}{\partial R_A} = x$

$$\begin{aligned} \frac{\partial U_{AD}}{\partial R_A} &= \frac{1}{EI} \int_0^{\frac{L}{3}} M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_0^{\frac{L}{3}} (R_A x)(x) dx \\ &= \frac{R_A}{3EI} \left(\frac{L}{3}\right)^3 = \frac{1}{81} \frac{R_A L^3}{EI} \end{aligned}$$

Portion DB: ($\frac{L}{3} < x < L$) $M = R_A x - P(x - \frac{L}{3})$
 $\frac{\partial M}{\partial R_A} = x$

$$\begin{aligned} \frac{\partial U_{DB}}{\partial R_A} &= \frac{1}{EI} \int_{\frac{L}{3}}^L M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_{\frac{L}{3}}^L [R_A x - P(x - \frac{L}{3})] x dx \\ &= \frac{R_A}{EI} \int_{\frac{L}{3}}^L x^2 dx - \frac{P}{EI} \int_{\frac{L}{3}}^L (x^2 - \frac{L}{3} x) dx \\ &= \frac{R_A}{3EI} \left[L^3 - \left(\frac{L}{3}\right)^3 \right] - \frac{P}{EI} \left[\frac{1}{3} \left(L^3 - \left(\frac{L}{3}\right)^3 \right) - \frac{L}{6} \left(L^2 - \left(\frac{L}{3}\right)^2 \right) \right] \\ &= \left(\frac{1}{3} - \frac{1}{81} \right) \frac{R_A L^3}{EI} - \left(\frac{1}{3} - \frac{1}{81} - \frac{1}{6} + \frac{1}{54} \right) \frac{PL^3}{EI} \end{aligned}$$

$$S_A = \frac{\partial U_{AD}}{\partial R_A} + \frac{\partial U_{DB}}{\partial R_A} = \left(\frac{1}{81} + \frac{1}{3} - \frac{1}{81} \right) \frac{R_A L^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = \frac{1}{3} \frac{R_A L^3}{EI} - \frac{14}{81} \frac{PL^3}{EI} = 0$$

$$R_A = \frac{14}{27} P \uparrow$$

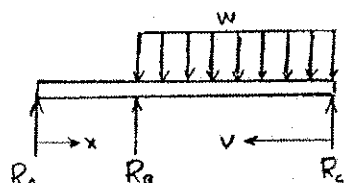
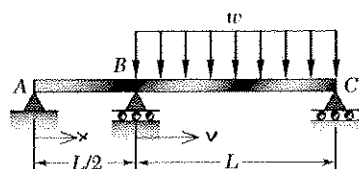
Bending moments: $M_D = R_A \left(\frac{L}{3}\right) = \frac{14}{81} PL = 0.1728 PL$

$$M_B = R_A L - P\left(\frac{2L}{3}\right) = -\frac{4}{27} PL = -0.1481 PL$$

By singularity functions, $M = \frac{14}{27} Px - P\langle x - L/3 \rangle'$

Problem 11.116

11.116 For the uniform beam and loading shown, determine the reaction at each support.



Remove support A and add reaction R_A as a load.

$$\sum M_B = 0 \quad -R_A \frac{L}{2} - \frac{1}{2} w L^2 + R_C L = 0$$

$$R_C = \frac{1}{2} R_A + \frac{1}{2} w L$$

$$U = U_{AB} + U_{BC} = \int_0^{\frac{L}{2}} \frac{M^2}{2EI} dx + \int_0^L \frac{M^2}{2EI} dv$$

$$S_A = \frac{\partial U}{\partial R_A} = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = 0$$

Portion AB: $M = R_A x, \quad \frac{\partial M}{\partial R_A} = x$

$$\frac{\partial U_{AB}}{\partial R_A} = \frac{1}{EI} \int_0^{\frac{L}{2}} M \frac{\partial M}{\partial R_A} dx = \frac{1}{EI} \int_0^{\frac{L}{2}} (R_A x)(x) dx = \frac{R_A}{3EI} \left(\frac{L}{2} \right)^3 = \frac{1}{24} \frac{R_A L^3}{EI}$$

Portion BC: $M = R_C v - \frac{1}{2} w v^2 = \frac{1}{2} R_A v + \frac{1}{2} w L v - \frac{1}{2} w L v^2$

$$\frac{\partial M}{\partial R_A} = \frac{1}{2} v$$

$$\frac{\partial U_{BC}}{\partial R_A} = \frac{1}{EI} \int_0^L \left[\frac{1}{2} R_A v + \frac{1}{2} w (L v - v^2) \right] \left(\frac{1}{2} v \right) dv = \frac{1}{4EI} \int_0^L [R_A v^2 + w (L v^2 - v^3)] dv$$

$$= \frac{1}{4EI} \left[R_A \frac{L^3}{3} + w \left(\frac{L^4}{3} - \frac{L^4}{4} \right) \right] = \frac{R_A L^3}{12EI} + \frac{w L^4}{48EI}$$

$$S_A = \frac{\partial U_{AB}}{\partial R_A} + \frac{\partial U_{BC}}{\partial R_A} = \left(\frac{1}{24} + \frac{1}{12} \right) \frac{R_A L^3}{EI} + \frac{w L^4}{48EI} = 0$$

$$R_A = -\frac{1}{6} w L =$$

$$R_A = \frac{1}{6} w L \downarrow \quad \blacktriangleleft$$

$$R_C = \frac{1}{2} \left(-\frac{1}{6} w L \right) + \frac{1}{2} w L$$

$$R_C = \frac{5}{12} w L \uparrow \quad \blacktriangleleft$$

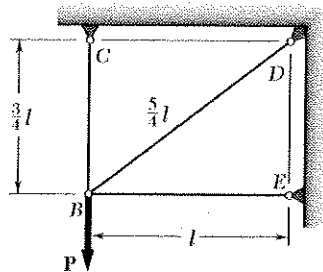
$$+\uparrow \sum F_y = 0: \quad R_A + R_B + R_C - w L = 0$$

$$-\frac{1}{6} w L + R_B + \frac{5}{12} w L - w L = 0$$

$$R_B = \frac{3}{4} w L \uparrow \quad \blacktriangleleft$$

Problem 11.117

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .

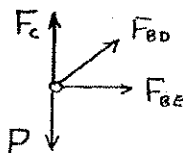


Detach member BC from support C . Add reaction F_c as a load.

$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2EA} \sum F^2 L$$

$$S_c = \frac{\partial U}{\partial F_c} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_c} L$$

Joint B



$$+\uparrow \sum F_y = 0: F_c - P + \frac{3}{5} F_{BD} = 0 \quad F_{BD} = \frac{5}{3} P - \frac{5}{3} F_c$$

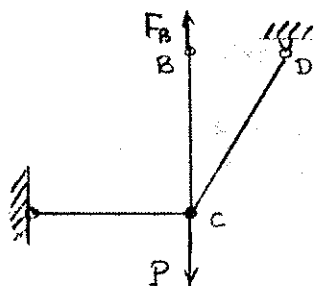
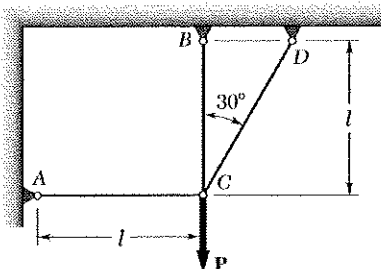
$$+\rightarrow \sum F_x = 0: F_{BE} + \frac{4}{5} F_{BD} = 0 \quad F_{BE} = -\frac{4}{3} P + \frac{4}{3} F_c$$

Member	F	$\partial F / \partial F_c$	L	$F(\partial F / \partial F_c) L$
BC	F_c	1	$\frac{3}{4} l$	$\frac{3}{4} F_c l$
BD	$\frac{5}{3} P - \frac{5}{3} F_c$	$-\frac{5}{3}$	$\frac{5}{4} l$	$-\frac{125}{36} P l + \frac{125}{36} F_c l$
BE	$-\frac{4}{3} P + \frac{4}{3} F_c$	$\frac{4}{3}$	l	$-\frac{16}{9} P l + \frac{16}{9} F_c l$
Σ				$-\frac{21}{4} P l + 6 F_c l$

$$S_c = \frac{1}{EA} \left(-\frac{21}{4} P l + 6 F_c l \right) = 0 \quad F_c = \frac{7}{8} P \quad F_{BC} = F_c \quad F_{BC} = \frac{7}{8} P$$

Problem 11.118

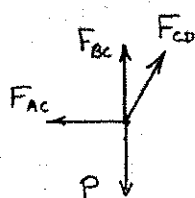
11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .



Cut member BC at end B and replace member force F_{BC} by load F_B acting on member BC at B .

$$S_B = \frac{\partial U}{\partial F_B} = \frac{\partial}{\partial F_B} \sum \frac{F^2 L}{EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_B} L$$

Joint C



$$+\uparrow \Sigma F_y = 0: \frac{\sqrt{3}}{2} F_{CD} + F_{BC} - P = 0$$

$$F_{CD} = \frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$$

$$\pm \Sigma F_x = 0: F_{AC} - \frac{1}{2} F_{CD} = 0$$

$$F_{AC} = \frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$$

Member	F	$\partial F / \partial F_B$	L	$F(\partial F / \partial F_B) L$
AC	F_B	1	l	$F_B l$
BC	$\frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$	$-\frac{1}{\sqrt{3}}$	l	$-\frac{1}{3} P l + \frac{1}{3} F_B l$
CD	$\frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$	$-\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}} l$	$-\frac{8}{3} P l + \frac{8}{3} F_B l$
Σ				$-(\frac{1}{3} + \frac{8}{3}) P l + (\frac{4}{3} + \frac{8}{3}) F_B l$

$$S_B = -(\frac{1}{3} + \frac{8}{3}) \frac{P l}{EA} + (\frac{4}{3} + \frac{8}{3}) \frac{F_B l}{EA}$$

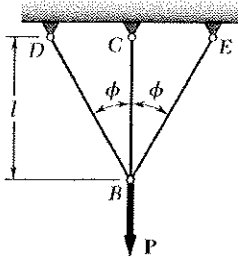
$$F_B = \frac{\frac{1}{3} + \frac{8}{3}}{\frac{4}{3} + \frac{8}{3}} P = \frac{8 + \sqrt{3}}{8 + 4\sqrt{3}} P = 0.652 P$$

$$F_{BC} = F_B$$

$$F_{BC} = 0.652 P \quad \blacktriangleleft$$

Problem 11.119

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .



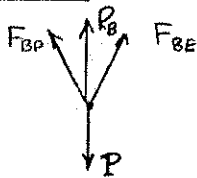
Detach member BC at support C .

Add reaction R_c as a load.

$$U = \sum \frac{F^2 L}{2EA} \quad y_c = \frac{\partial U}{\partial R_c} = \sum \frac{FL}{EA} \frac{\partial F}{\partial R_c} = 0$$

Joint C $F_{BC} = R_c$

Joint B



$$+\rightarrow \sum F_x = 0: \quad F_{BE} \sin \phi - F_{BD} \sin \phi = 0 \quad F_{BE} = F_{BD}$$

$$+\uparrow \sum F_y = 0: \quad F_{BD} \cos \phi + F_{BE} \cos \phi + R_B - P$$

$$F_{BD} = F_{BE} = \frac{P - R_B}{2 \cos \phi}$$

Member	F	$\partial F / \partial R_B$	L	$(FL/EA) \partial F / \partial R_B$
BD	$(P - R_B) / 2 \cos \phi$	$-1 / 2 \cos \phi$	$l / \cos \phi$	$(R_B - P) l / 4EA \cos^3 \phi$
BE	$(P - R_B) / 2 \cos \phi$	$-1 / 2 \cos \phi$	$l / \cos \phi$	$(R_B - P) l / 4EA \cos^3 \phi$
BC	R_B	1	l	$R_B l / EA$

$$y_B = -Pl / 2EA \cos^3 \phi + R_B l / 2EA \cos^3 \phi + R_B l / EA = 0$$

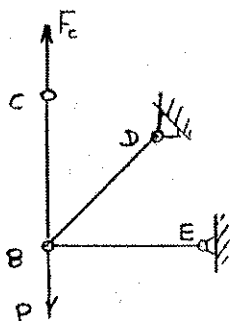
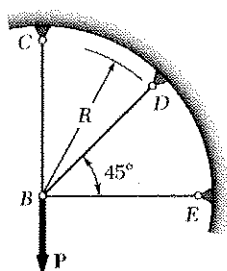
$$R_B = \frac{P}{1 + 2 \cos^3 \phi}$$

$$F_{BC} = R_B$$

$$F_{BC} = \frac{P}{1 + 2 \cos^3 \phi}$$

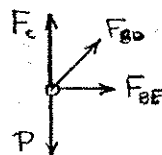
Problem 11.120

11.117 through 11.120 Three members of the same material and same cross-sectional area are used to support the load P . Determine the force in member BC .



Detach member BC from its support at point C . Add reaction F_c as a load.

Joint B



$$\uparrow \sum F_y = 0 :$$

$$\frac{\sqrt{2}}{2} F_{BD} + F_c - P = 0$$

$$F_{BD} = \sqrt{2} P - \sqrt{2} F_c$$

$$\rightarrow \sum F_x = 0 :$$

$$\frac{\sqrt{2}}{2} F_{BD} + F_{BE} = 0$$

$$F_{BE} = -P + F_c$$

$$\delta_c = \frac{R}{EA} (-3P + 4F_c) = 0$$

$$F_c = \frac{3}{4} P$$

$$F_{BC} = F_c \quad F_{BC} = \frac{3}{4} P \quad \blacktriangleleft$$

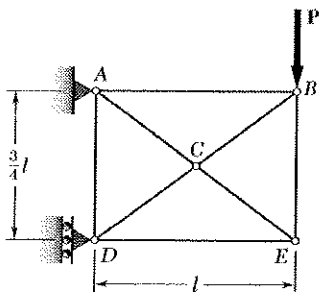
$$U = \sum \frac{F^2 R}{2EA} = \frac{R}{2EA} \sum F^2$$

$$\delta_c = \frac{\partial U}{\partial F_c} = \frac{R}{EA} \sum F \frac{\partial F}{\partial F_c} = 0$$

Member	F	$\partial F / \partial F_c$	$F(\partial F / \partial F_c)$
BC	F_c	1	F_c
BD	$\sqrt{2} P - \sqrt{2} F_c$	$-\sqrt{2}$	$-2P + 2F_c$
BE	$-P + F_c$	1	$-P + F_c$
Σ			$-3P + 4F_c$

Problem 11.121

11.121 and 11.122 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member AB.



Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

$$S_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A} \sum \frac{F^2 L}{EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_A} L = 0$$

Joint B

$$+\rightarrow \Sigma F_x = 0: -F_A - \frac{4}{5} F_{BD} = 0 \quad F_{BD} = -\frac{5}{4} F_A$$

$$+\uparrow \Sigma F_y = 0: -P - F_{BE} - \frac{3}{5} F_{BD} = 0 \quad F_{BE} = -P + \frac{3}{4} F_A$$

Joint E

$$+\uparrow \Sigma F_y = 0: F_{BE} + \frac{3}{5} F_{AE} = 0 \quad F_{AE} = \frac{5}{3} P - \frac{5}{4} F_A$$

$$+\rightarrow \Sigma F_x = 0: -\frac{4}{5} F_{AE} - F_{DE} = 0 \quad F_{DE} = -\frac{4}{3} P + F_A$$

Joint D

$$+\uparrow \Sigma F_y = 0: F_{AD} + \frac{3}{5} F_{BD} = 0 \quad F_{AD} = \frac{3}{4} F_A$$

Member	F	$\partial F / \partial F_A$	L	$F(\partial F / \partial F_A) L$
AB	F_A	1	l	$F_A l$
AD	$\frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$\frac{27}{64} F_A l$
AE	$\frac{5}{3} P - \frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{48} P l + \frac{125}{64} F_A l$
BD	$-\frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$\frac{125}{64} F_A l$
BE	$-P + \frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$-\frac{9}{16} P l + \frac{27}{64} F_A l$
DE	$-\frac{4}{3} P + F_A$	1	l	$-\frac{4}{3} P l + F_A l$
Σ				$-\frac{9}{2} P l + \frac{27}{4} F_A l$

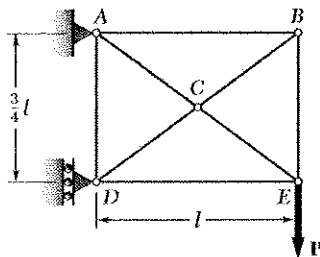
$$S_A = \frac{1}{EA} \left(-\frac{9}{2} P l + \frac{27}{4} F_A l \right) = 0 \quad F_A = \frac{2}{3} P$$

$$F_{AB} = F_A$$

$$F_{AB} = \frac{2}{3} P \quad \blacktriangleleft$$

Problem 11.122

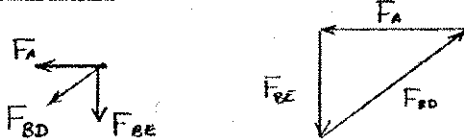
11.121 and 11.122 Knowing that the eight members of the indeterminate truss shown have the same uniform cross-sectional area, determine the force in member AB.



Cut member AB at end A and replace member force F_{AB} by load $F_A \leftarrow$ acting on member AB at end A.

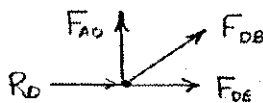
$$S_A = \frac{\partial U}{\partial F_A} = \frac{\partial}{\partial F_A} \sum \frac{F^2 L}{2EA} = \frac{1}{EA} \sum F \frac{\partial F}{\partial F_A} L = 0$$

Joint B



$$F_{BD} = -\frac{5}{4} F_A \quad F_{BE} = \frac{3}{4} F_A$$

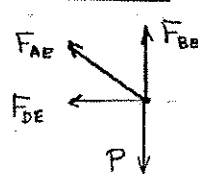
Joint D



$$+\uparrow \sum F_y = 0: F_{AD} + \frac{3}{5} F_{DB} = 0$$

$$F_{AD} = -\frac{3}{5} F_{DB} = -\frac{3}{4} F_A$$

Joint E



$$+\uparrow \sum F_y = 0:$$

$$F_{BE} - P + \frac{3}{5} F_{AE}$$

$$F_{AE} = \frac{5}{3} P - \frac{5}{3} F_{BE} \\ = \frac{5}{3} P - \frac{5}{4} F_A$$

$$+\rightarrow \sum F_x = 0: -\frac{4}{5} F_{AE} - F_{DE} = 0$$

$$F_{DE} = -\frac{4}{5} F_{AE} = -\frac{4}{3} P + F_A$$

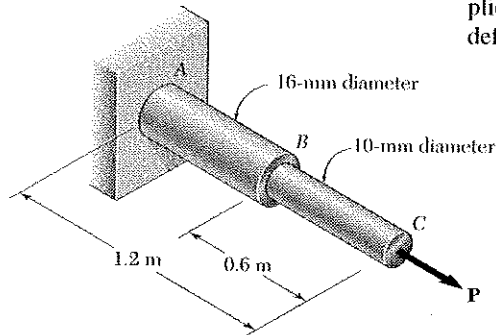
Member	F	$\partial F / \partial F_A$	L	$F(\partial F / \partial F_A) L$
AB	F_A	1	l	$F_A l$
AD	$-\frac{3}{4} F_A$	$-\frac{3}{4}$	$\frac{3}{4} l$	$-\frac{27}{64} F_A l$
AE	$\frac{5}{3} P - \frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{48} P l + \frac{125}{64} F_A l$
BD	$-\frac{5}{4} F_A$	$-\frac{5}{4}$	$\frac{5}{4} l$	$-\frac{125}{64} F_A l$
BE	$\frac{3}{4} F_A$	$\frac{3}{4}$	$\frac{3}{4} l$	$\frac{27}{64} F_A l$
DE	$-\frac{4}{3} P + F_A$	1	l	$-\frac{4}{3} P l + F_A l$
Σ				$-\frac{63}{16} P l + \frac{27}{4} F_A l$

$$S_A = \frac{1}{EA} \left(-\frac{63}{16} P l + \frac{27}{4} F_A l \right) = 0 \quad F_A = \frac{7}{12} P$$

$$F_{AB} = F_A =$$

$$F_{AB} = \frac{7}{12} P \quad \leftarrow$$

Problem 11.123



11.123 Rod AB is made of a steel for which the yield strength is $\sigma_Y = 450$ MPa and the modulus of elasticity is $E = 200$ GPa. Knowing that a strain energy of $7 \text{ N} \cdot \text{m}$ must be acquired by the rod as the axial load P is applied, determine the factor of safety of the rod with respect to permanent deformation.

$$A_{AB} = \frac{\pi}{4} (16)^2 = 201.06 \text{ mm}^2$$

$$A_{BC} = \frac{\pi}{4} (10)^2 = 78.54 \text{ mm}^2$$

$$P_Y = \sigma_Y A_{\min} = (450)(78.54) = 35,343 \text{ N}$$

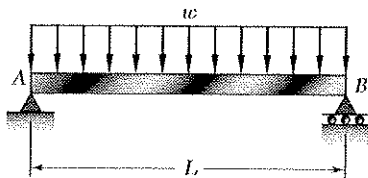
$$U_r = \sum \frac{P_r^2 \ell}{EA} = \frac{P_Y^2}{2E} \sum \frac{\ell}{A}$$

$$U_r = \frac{(35,343)^2}{2(200 \times 10^9)} \left[\frac{1.2 - 0.6}{201.07 \times 10^{-6}} + \frac{0.6}{78.54 \times 10^{-6}} \right] = 33.18 \text{ Nm}$$

$$F.S. = \frac{U_r}{U_{\text{design}}} = \frac{33.18}{7} = 4.74$$

Problem 11.124

11.124 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam AB for the loading shown.



$$\sum M_B = 0: -R_A L + (wL)\left(\frac{L}{2}\right) = 0 \quad R_A = \frac{wL}{2}$$

$$\text{Bending moment: } M = R_A x - \frac{1}{2} wx^2 \\ = \frac{w}{2} (Lx - x^2)$$

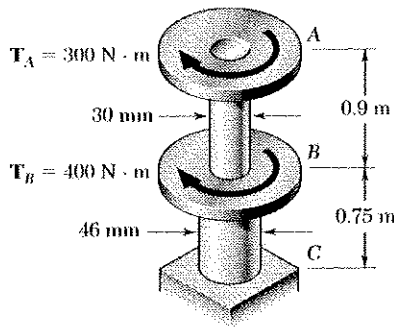
$$U = \int_0^L \frac{M^2}{2EI} dx = \frac{w^2}{8EI} \int_0^L (Lx - x^2)^2 dx$$

$$= \frac{w^2}{8EI} \int_0^L (L^2 x^2 - 2Lx^3 + x^4) dx = \frac{w^2}{8EI} \left[\frac{L^2 x^3}{3} - \frac{2Lx^4}{4} + \frac{x^5}{5} \right] \Big|_0^L$$

$$= \frac{w^2 L^5}{8EI} \left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right]$$

$$U = \frac{w^2 L^5}{240EI}$$

Problem 11.125



11.125 In the assembly shown torques T_A and T_B are exerted on disks A and B , respectively. Knowing that both shafts are solid and made of aluminum ($G = 73$ GPa), determine the total strain energy acquired by the assembly.

Over portion AB:

$$T_{AB} = T_A = 300 \text{ N}\cdot\text{m}$$

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2} \left(\frac{30}{2}\right)^4 = 79.52 \times 10^3 \text{ mm}^4 = 79.52 \times 10^{-9} \text{ m}^4$$

$$L_{AB} = 0.9 \text{ m}$$

$$U_{AB} = \frac{T_{AB}^2 L_{AB}}{2 G J_{AB}} = \frac{(300)^2 (0.9)}{(2)(73 \times 10^9)(79.52 \times 10^{-9})} = 6.977 \text{ J}$$

Over portion BC: $T_{BC} = T_A + T_B = 300 + 400 = 700 \text{ N}\cdot\text{m}$, $L_{BC} = 0.75 \text{ m}$

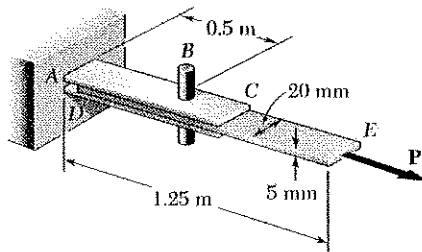
$$J_{BC} = \frac{\pi}{2} \left(\frac{46}{2}\right)^4 = 439.57 \times 10^3 \text{ mm}^4 = 439.57 \times 10^{-9} \text{ m}^4$$

$$U_{BC} = \frac{T_{BC}^2 L_{BC}}{2 G J_{BC}} = \frac{(700)^2 (0.75)}{(2)(73 \times 10^9)(439.57 \times 10^{-9})} = 5.726 \text{ J}$$

Total: $U = U_{AB} + U_{BC} = 6.977 + 5.726$

$$U = 12.70 \text{ J} \quad \blacktriangleleft$$

Problem 11.126



11.126 A single 6-mm-diameter steel pin B is used to connect the steel strip DE to two aluminum strips, each of 20-mm width and 5-mm thickness. The modulus of elasticity is 200 GPa for the steel and 70 GPa for the aluminum. Knowing that for the pin at B the allowable shearing stress is $\tau_{all} = 85$ MPa, determine, for the loading shown, the maximum strain energy that can be acquired by the assembled strips.

$$A_{pin} = \frac{\pi}{4} d^2 = \frac{\pi}{4} (6)^2 = 28.274 \text{ mm}^2 = 28.274 \times 10^{-6} \text{ m}^2$$

$$\tau_{all} = 85 \times 10^6 \text{ Pa}$$

$$\text{Double shear: } P = 2A\tau = (2)(28.274 \times 10^{-6})(85 \times 10^6) = 4.8066 \times 10^3 \text{ N}$$

For strips AB, DB, BE , $A = (20)(5) = 100 \text{ mm}^2 = 100 \times 10^{-6} \text{ m}^2$

$$F_{AB} = F_{DB} = \frac{1}{2} P = 2.4033 \times 10^3 \text{ N}$$

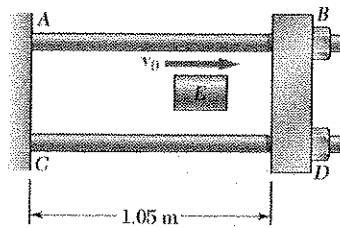
$$U_{AB} = U_{DB} = \frac{F_{AB}^2 L_{AB}}{2 E_A A_{AB}} = \frac{(2.4033 \times 10^3)^2 (0.5)}{(2)(70 \times 10^9)(100 \times 10^{-6})} = 0.2063 \text{ J}$$

$$U_{BE} = \frac{F_{BE}^2 L_{BE}}{2 E_S A_{BE}} = \frac{(4.8066 \times 10^3)^2 (1.25 - 0.5)}{(2)(200 \times 10^9)(100 \times 10^{-6})} = 0.4332 \text{ J}$$

Total: $U = U_{AB} + U_{DB} + U_{BE} = 0.846 \text{ J}$

$$U = 0.846 \text{ J} \quad \blacktriangleleft$$

Problem 11.127



11.127 The cylindrical block E has a speed $v_0 = 4.8$ m/s when it strikes squarely the yoke BD that is attached to the 22-mm-diameter rods AB and CD . Knowing that the rods are made of a steel for which $\sigma_Y = 350$ MPa and $E = 200$ GPa, determine the weight of block E for which the factor of safety is 5 with respect to permanent deformation of the rods.

At the onset of yielding the force in each rod is

$$F = \sigma_Y A$$

Corresponding strain energy.

$$U_{AB} = \frac{F_{AB}^2 L_{AB}}{2EA_{AB}} = \frac{\sigma_Y^2 A^2 L}{2EA} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_{CD} = \text{same} = \frac{\sigma_Y^2 AL}{2E}$$

$$U_m = U_{AB} + U_{CD} = \frac{\sigma_Y^2 AL}{E}$$

$$U_m = \left(\frac{1}{2} m v_0^2\right) (\text{F.S.}) = \left(\frac{1}{2} \frac{W}{g} v_0^2\right) (\text{F.S.})$$

$$\text{Solving for } W; \quad W = \frac{2g U_m}{v_0^2 (\text{F.S.})} = \frac{2g \sigma_Y^2 AL}{v_0^2 (\text{F.S.}) E}$$

$$\text{Data: } g = 9.81 \text{ m/s}^2, \quad \sigma_Y = 350 \text{ MPa},$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (22)^2 = 380 \text{ mm}^2$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$L = 1.05 \text{ m}$$

$$\text{F.S.} = 5$$

$$v_0 = 4.8 \text{ m/s}$$

$$W = \frac{(2)(9.81)(350 \times 10^6)^2 (380 \times 10^{-6})(1.05)}{(4.8)^2 (5)(200 \times 10^9)} = 41.6 \text{ N}$$

Problem 11.128

11.128 A block of weight W is placed in contact with a beam at some given point D and released. Show that the resulting maximum deflection at point D is twice as large as the deflection due to a static load W applied at D .

Consider dropping the weight from a height h above the beam. The work done by the weight is

$$\text{Work} = W(h + y_m)$$

$$\text{Strain energy: } U = \frac{1}{2} P_m y_m = \frac{1}{2} k y_m^2$$

where k is the spring constant of the beam for loading at point D .

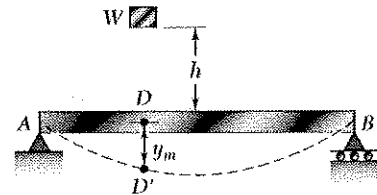
$$\text{Equating work and energy, } W(h + y_m) = \frac{1}{2} k y_m^2$$

$$\text{Setting } h = 0, \quad W y_m = \frac{1}{2} k y_m^2, \quad y_m = \frac{2W}{k}$$

The static deflection at point D due to weight applied at D is

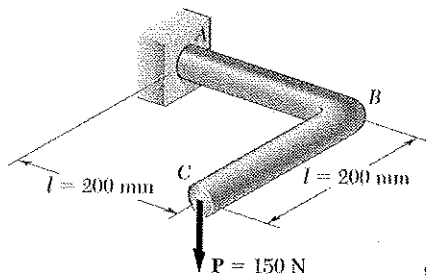
$$S_{st} = \frac{W}{k}$$

$$y_m = 2 S_{st} \quad \blacktriangleleft$$



Problem 11.129

11.129 The 12-mm-diameter steel rod ABC has been bent into the shape shown. Knowing that $E = 200$ GPa and $G = 77.2$ GPa, determine the deflection of end C caused by the 150-N force.

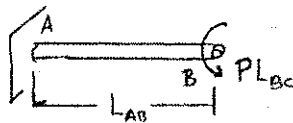
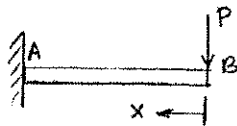


$$J = \frac{\pi}{2} C^4 = \frac{\pi}{2} \left(\frac{12}{2} \right)^4 = 2.0358 \times 10^3 \text{ mm}^4 = 2.0358 \times 10^{-9} \text{ m}^4$$

$$I = \frac{1}{2} J = 1.0179 \times 10^{-9} \text{ m}^4$$

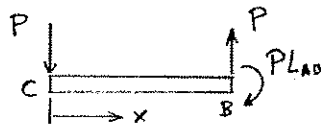
Portion AB: bending $M = -Px$

$$U_{AB,b} = \int_0^{L_{AB}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{L_{AB}} x^2 dx = \frac{P^2 L_{AB}^3}{6EI} = \frac{(150)^2 (200 \times 10^{-3})^3}{(6)(200 \times 10^9)(1.0179 \times 10^{-9})} = 0.14736 \text{ J}$$



torsion $T = PL_{BC}$

$$U_{AB,t} = \frac{T^2 L_{AB}}{2GJ} = \frac{P^2 L_{BC}^2 L_{AB}}{2GJ} = \frac{(150)^2 (200 \times 10^{-3})^2 (200 \times 10^{-3})}{(2)(77.2 \times 10^9)(2.0358 \times 10^{-9})} = 0.57265 \text{ J}$$



Portion BC: $M = -Px$

$$U_{BC} = \int_0^{L_{BC}} \frac{M^2}{2EI} dx = \frac{P^2}{2EI} \int_0^{L_{BC}} x^2 dx = \frac{P^2 L_{BC}^3}{6EI} = \frac{(150)^2 (200 \times 10^{-3})^3}{(6)(200 \times 10^9)(1.0179 \times 10^{-9})} = 0.14736 \text{ J}$$

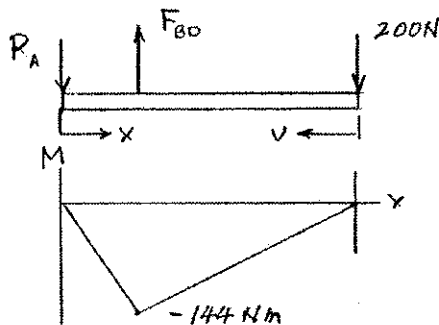
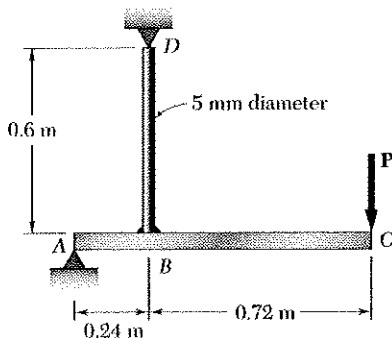
Total: $U = U_{AB,b} + U_{AB,t} + U_{BC} = 0.86737 \text{ J}$

Work-energy: $\frac{1}{2} PS = U \quad S = \frac{2U}{P} = \frac{(2)(0.86737)}{150}$

$$= 11.57 \times 10^{-3} \text{ m} \quad \delta = 11.57 \text{ mm} \downarrow$$

Problem 11.130

11.130 The steel bar ABC has a square cross section of side 18 mm and is subjected to a 200-N load P . Using $E = 200$ GPa, determine the deflection of point C .



Assume member BD is a two-force member.

$$\sum M_A = 0: 0.24 F_{BD} - (0.96)(200) = 0 \quad F_{BD} = 800 \text{ N}$$

$$A_{BD} = \frac{\pi}{4} (0.005)^2 = 19.635 \times 10^{-6} \text{ m}^2$$

$$U_{BD} = \frac{F_{BD}^2 L_{BD}}{2EA} = \frac{(800)^2 (0.6)}{(2)(200 \times 10^9)(19.635 \times 10^{-6})} = 0.04889 \text{ J}$$

Member ABC :

$$I = \frac{1}{12} (0.018)(0.018)^3 = 8.748 \times 10^{-9} \text{ m}^4$$

Portion AB : $M = -144 \frac{x}{0.24} = -600x$

$$U_{AB} = \int_0^{0.24} \frac{M^2}{2EI} dx = \frac{600^2}{2EI} \int_0^{0.24} x^2 dx = \frac{(600)^2 (0.24^3)}{(2)(200 \times 10^9)(8.748 \times 10^{-9})(3)} = 0.47407 \text{ J}$$

Portion BC : $M = -200v$

$$U_{BC} = \int_0^{0.72} \frac{M^2}{2EI} dv = \frac{200^2}{2EI} \int_0^{0.72} v^2 dv = \frac{(200)^2 (0.72^3)}{(2)(200 \times 10^9)(8.748 \times 10^{-9})(3)} = 1.42222 \text{ J}$$

Total: $U = U_{BD} + U_{AB} + U_{BC} = 1.94516 \text{ J}$

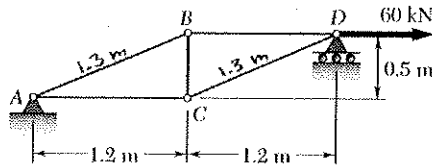
$$\frac{1}{2} P \delta_c = U$$

$$\delta_c = \frac{2U}{P} = \frac{(2)(1.94516)}{200} = 0.0194516$$

$$\delta_c = 19.5 \text{ mm} \quad \downarrow \quad \blacktriangleleft$$

Problem 11.131

11.131 Each member of the truss shown is made of steel; the cross-sectional area of member BC is 800 mm² and for all other members the cross-sectional area is 400 mm². Using $E = 200$ GPa, determine the deflection of point D caused by the 60-kN load.



Entire truss $\sum M_A = 0$:

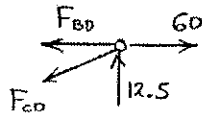
$$2.4 R_D - (0.5)(60) = 0 \quad R_D = 12.5 \text{ kN}$$

Joint D

$$+\uparrow \sum F_y = 0$$

$$12.5 - \frac{0.5}{1.3} F_{CD} = 0$$

$$F_{CD} = 32.5 \text{ kN}$$



$$+\rightarrow \sum F_x = 0$$

$$60 - F_{BD} - \frac{1.2}{1.3} F_{CD} = 0$$

$$F_{BD} = 30 \text{ kN}$$

Joint B

$$+\rightarrow \sum F_x = 0$$

$$30 - \frac{1.2}{1.3} F_{AB} = 0$$

$$F_{AB} = 32.5 \text{ kN}$$



$$+\uparrow \sum F_y = 0$$

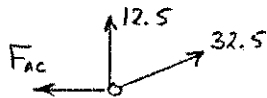
$$-\frac{0.5}{1.3} F_{AB} + F_{BC} = 0$$

$$F_{BC} = 12.5 \text{ kN}$$

Joint C

$$+\rightarrow \sum F_x = 0: -F_{AC} + \frac{1.2}{1.3}(32.5) = 0$$

$$F_{AC} = 30 \text{ kN}$$



$$U = \sum \frac{F^2 L}{2EA} = \frac{1}{2E} \sum \frac{F^2 L}{A}$$

Member	F (kN)	L (m)	A (10 ⁶ m ²)	F ² L/A (N ² /m)
CD	32.5	1.3	400	3.4328 × 10 ¹²
BD	30	1.2	400	2.7 × 10 ¹²
AB	32.5	1.3	400	3.4328 × 10 ¹²
BC	12.5	0.5	800	0.0977 × 10 ¹²
AC	30	1.2	400	2.7 × 10 ¹²
Σ				12.3633 × 10 ¹²

$$E = 200 \times 10^9 \text{ Pa}$$

$$U = \frac{12.3633 \times 10^{12}}{(2)(200 \times 10^9)} = 30.908 \text{ J}$$

Work-energy:

$$\frac{1}{2} P \Delta = U$$

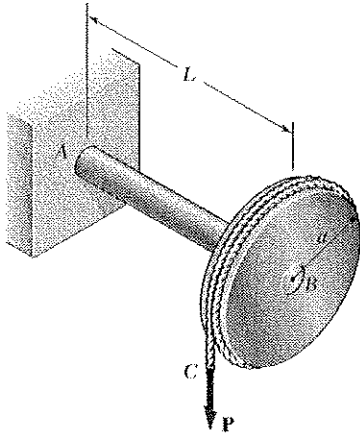
$$\Delta = \frac{2U}{P} = \frac{(2)(30.908)}{60 \times 10^3} = 1.030 \times 10^{-3} \text{ m}$$

$$\Delta = 1.030 \text{ mm} \rightarrow$$

Problem 11.132

11.132 A disk of radius a has been welded to end B of the solid steel shaft AB . A cable is then wrapped around the disk and a vertical force P is applied to end C of the cable. Knowing that the radius of the shaft is r and neglecting the deformations of the disk and of the table, show that the deflection of point C caused by the application of P is

$$\delta_c = \frac{PL^3}{3EI} \left(1 + 1.5 \frac{Er^2}{GL^2} \right)$$



Torsion: $T = Pa$

$$U_t = \frac{T^2 L}{2GJ} = \frac{P^2 a^2 L}{2GJ}$$

Bending: $M = Pv$

$$U_b = \int_0^L \frac{M^2 dv}{2EI} = \int_0^L \frac{P^2 v^2 dv}{2EI}$$

$$= \frac{P^2 L^3}{6EI}$$



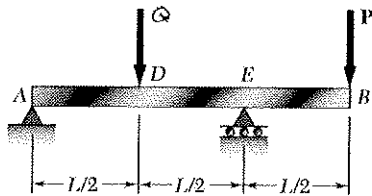
Total: $U = \frac{P^2 a^2 L}{2GJ} + \frac{P^2 L^3}{6EI} = \frac{1}{2} P \delta_c$

$$\delta_c = \frac{Pa^2 L}{GJ} + \frac{PL^3}{3EI} = \frac{PL^3}{3EI} \left(1 + \frac{3EIa^2}{GJL^2} \right)$$

Since $J = 2I$, $\delta_c = \frac{PL^3}{3EI} \left(1 + 1.5 \frac{Er^2}{GL^2} \right)$

Problem 11.133

11.133 For the prismatic beam shown, determine the deflection of point D.



Change force at D from P to Q.

Reactions: $R_A = \frac{1}{2}(Q-P) \uparrow$, $R_E = \frac{1}{2}(3P+Q) \uparrow$

Strain energy: $U = U_{AD} + U_{DE} + U_{EB}$

Deflection of point D (Formula).

$$\delta_D = \frac{\partial U}{\partial Q} = \frac{\partial U_{AD}}{\partial Q} + \frac{\partial U_{DE}}{\partial Q} + \frac{\partial U_{EB}}{\partial Q}$$

Portion AD: $U_{AD} = \int_0^{L/2} \frac{M^2}{2EI} dx$

$$M = R_A x = \frac{1}{2}(Q-P)x$$

$$\frac{\partial M}{\partial Q} = \frac{1}{2}x \quad \text{With } Q=P, M=0$$

$$\frac{\partial U_{AD}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dx = 0$$

Portion DE: $U_{DE} = \int_{L/2}^L \frac{M^2}{2EI} dx$

$$M = R_A x - Q(x - \frac{L}{2}) = \frac{1}{2}(Q-P)x - Q(x - \frac{L}{2}) = \frac{1}{2}Q(L-x) - \frac{1}{2}Px$$

$$\frac{\partial M}{\partial Q} = \frac{1}{2}(L-x)$$

$$\text{With } Q=P, M = -\frac{1}{2}P(2x-L)$$

$$\frac{\partial U_{DE}}{\partial Q} = \int_{L/2}^L \frac{M}{EI} \frac{\partial M}{\partial Q} dx = -\frac{P}{4EI} \int_{L/2}^L (2x-L)(L-x) dx$$

$$= -\frac{P}{4EI} \int_{L/2}^L (3Lx - 2x^2 - L^2) dx$$

$$= -\frac{P}{4EI} \left(\frac{3}{2}Lx^2 - \frac{2}{3}x^3 - L^2x \right) \Big|_{L/2}^L$$

$$= -\frac{PL^3}{4EI} \left[\frac{3}{2} - \frac{2}{3} - 1 - \left(\frac{3}{2} \right) \left(\frac{1}{4} \right) + \left(\frac{2}{3} \right) \left(\frac{1}{8} \right) + \frac{1}{2} \right] = -\frac{PL^3}{96EI}$$

Portion EB: $U_{EB} = \int_0^{L/2} \frac{M^2}{2EI} dv$

$$M = -Pv$$

$$\frac{\partial M}{\partial Q} = 0$$

$$\frac{\partial U_{EB}}{\partial Q} = \int_0^{L/2} \frac{M}{EI} \frac{\partial M}{\partial Q} dv = 0$$

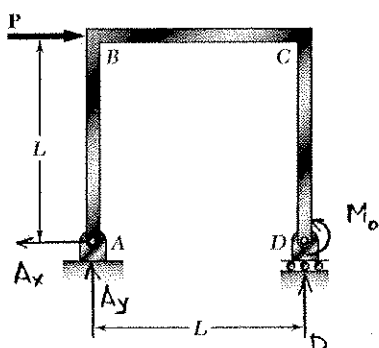
Deflection of Point D.

$$\delta_D = 0 - \frac{PL^3}{96EI} + C$$

$$\delta_D = \frac{PL^3}{96EI} \uparrow$$

Problem 11.134

11.134 Three rods, each of the same flexural rigidity EI , are welded to form the frame $ABCD$. For the loading shown, determine the angle formed by the frame at point D .



Add couple M_0 at point D .

Statics $+\circlearrowleft M_A = 0: M_0 + DL - PL = 0$

$$D = P - \frac{M_0}{L} \uparrow$$

$$+\rightarrow \Sigma F_x = 0: -A_x + P = 0 \quad A_x = P \leftarrow$$

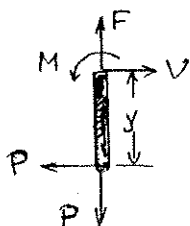
$$+\uparrow \Sigma F_y = 0: A_y + D = 0 \quad A_y = P - \frac{M_0}{L} \downarrow$$

$$U = U_{AB} + U_{BC} + U_{CD}$$

By Castigliano's theorem, $\theta_D = \frac{\partial U}{\partial M_0}$

$$\theta_D = \frac{\partial U_{AB}}{\partial M_0} + \frac{\partial U_{BC}}{\partial M_0} + \frac{\partial U_{CD}}{\partial M_0}$$

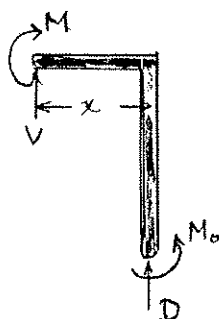
Member AB:



$$M = Py \quad \frac{\partial M}{\partial M_0} = 0 \quad U_{AB} = \int_0^L \frac{M^2}{2EI} dy$$

$$\frac{\partial U_{AB}}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dy = 0$$

Member BC:



$$M = M_0 + Dx = M_0 + Px - \frac{Mx}{L}$$

$$\text{Set } M_0 = 0 \quad M = Px$$

$$U_{BC} = \int_0^L \frac{M^2}{2EI} dx$$

$$\frac{\partial U_{BC}}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = \frac{P}{EI} \int_0^L x \left(1 - \frac{x}{L}\right) dx = \frac{PL^2}{6EI}$$

Member CD: $M = M_0 \quad \frac{\partial M}{\partial M_0} = 1$

$$\text{Set } M_0 = 0 \quad M = 0$$

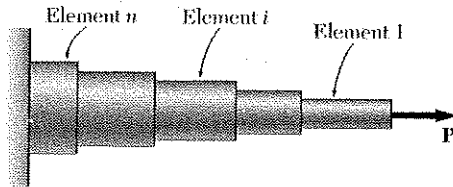
$$U = \int_0^L \frac{M^2}{2EI} dy$$

$$\frac{\partial U}{\partial M_0} = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial M_0} dx = 0$$

$$\theta_D = 0 + \frac{PL^2}{6EI} + 0$$

$$\theta_D = \frac{PL^2}{6EI} \curvearrowright \blacktriangleleft$$

PROBLEM 11.C1



11.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to a load P applied at its free end. The length of element i is denoted by L_i and its diameter by d_i . (a) Denoting by E the modulus of elasticity of the material used in the rod, write a computer program that can be used to determine the strain energy acquired by the rod and the deformation measured at the free end. (b) Use this program to determine the strain energy and deformation of the rods of Probs. 11.9 and 11.10.

SOLUTION

ENTER: P AND E

FOR EACH ELEMENT

ENTER A_i AND D_i

COMPUTE: NORMAL STRESS: $\sigma_i = \frac{P}{A_i}$

STRAIN ENERGY: $U_i = \frac{P^2 L_i}{2 A_i E}$

STRAIN ENERGY DENSITY: $u = \frac{\sigma_i^2}{2E}$

TOTAL STRAIN ENERGY

UPDATE THROUGH n ELEMENTS

$U = U + U_i$

TOTAL DEFORMATION

$\frac{1}{2} P \Delta = U \quad : \quad \Delta = \frac{2U}{P}$

PROGRAM OUTPUT

Problem 11.9

Axial load = 35 kN

Modulus of elasticity = 200 GPa

Element	Length m	delta L mm	Stress MPa	Strain Energy J	Strain Energy Density kJ/m ³
1	0.6	0.52	174.1	9.093	75.777
2	1.5	0.836	111.4	14.622	31.025

Total Strain Energy = 23.715 J

Total Deformation = 1.356 mm

Problem 11.10

Axial load = 25.000 kN

Modulus of elasticity = 200 GPa

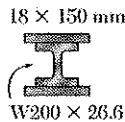
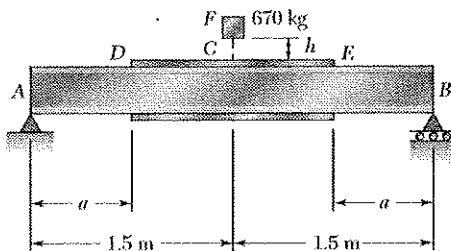
Element	Length m	delta L mm	Stress MPa	Strain Energy J	Strain Energy Density kJ/m ³
1	0.80	0.497	124.34	6.22	38.65
2	1.20	0.477	79.58	5.97	15.83

Total Strain Energy = 12.1853 J

Total Deformation = 0.9748 mm

PROBLEM 11.C2

11.C2 Two 18×150 -mm cover plates are welded to a $W200 \times 26.6$ rolled-steel beam as shown. The 670-kg block is to be dropped from a height $h = 50$ mm onto the beam. (a) Write a computer program to calculate the maximum normal stress on transverse sections just to the left of D and at the center of the beam for values of a from 0 to 1.5 m using 125-mm increments. (b) From the values considered in part a, select the distance a for which the maximum normal stress is as small as possible. Use $E = 200$ GPa.



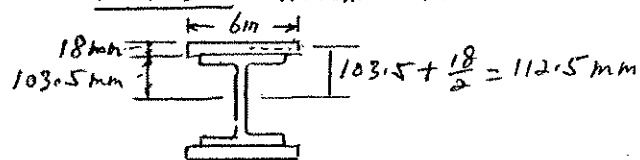
SOLUTION

COMPUTE AND ENTER MOMENTS OF INERTIA AND SECTION MODULI

FOR AD AND EB: $W200 \times 26.6$.

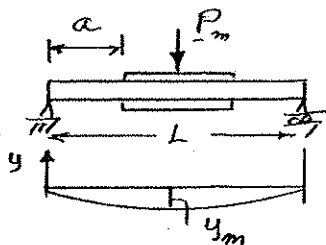
$$I_1 = 25.8 \times 10^6 \text{ mm}^4 \quad S_1 = 249 \times 10^3 \text{ mm}^3 \quad \triangleleft$$

FOR DCE: $W200 \times 26.6$ PLUS COVER PLATES



$$I_2 = 25.8 \times 10^6 + 2(18 \times 150)(112.5)^2 = 94.144 \times 10^6 \text{ mm}^4 \quad \triangleleft$$

$$S_2 = \frac{I_2}{(103.5 + 18)} = \frac{94.144 \times 10^6}{121.5} = 774.85 \times 10^3 \text{ mm}^3 \quad \triangleleft$$



$y_m = P_m \alpha$ WHERE α = INFLUENCE COEFFICIENT
SEE NEXT PAGE FOR DETERMINATION OF α

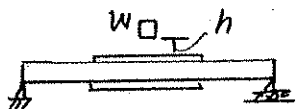
P_m = EQUIVALENT STATIC LOAD

$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{P_m^2}{\alpha}$$

WORK DONE BY W IS $W(h + y_m)$

$$\frac{1}{2} \frac{P_m^2}{\alpha} = Wh + W y_m$$

$$\text{OR: } y_m^2 - 2W\alpha y_m - 2Wh\alpha \quad \textcircled{A}$$



POSITION 1



POSITION 2

PROGRAM SOLUTION OF $\textcircled{A}$ FOR y_m

ENTER $L = 3$ m, $h = 50$ mm, $W = 670 \times 9.81 = 6573$ N, $E = 200$ GPa.

FOR $a = 0$ TO 1.5 m, STEP 125 mm:

SOLVE $\textcircled{A}$ FOR y_m , $P_m = y_m / \alpha$, $y_{ST} = W\alpha$

$$\sigma_D = \sigma_1 = \frac{1}{2} P_m a / S_1 \quad ; \quad \sigma_C = \sigma_2 = \frac{1}{4} P_m L / S_2$$

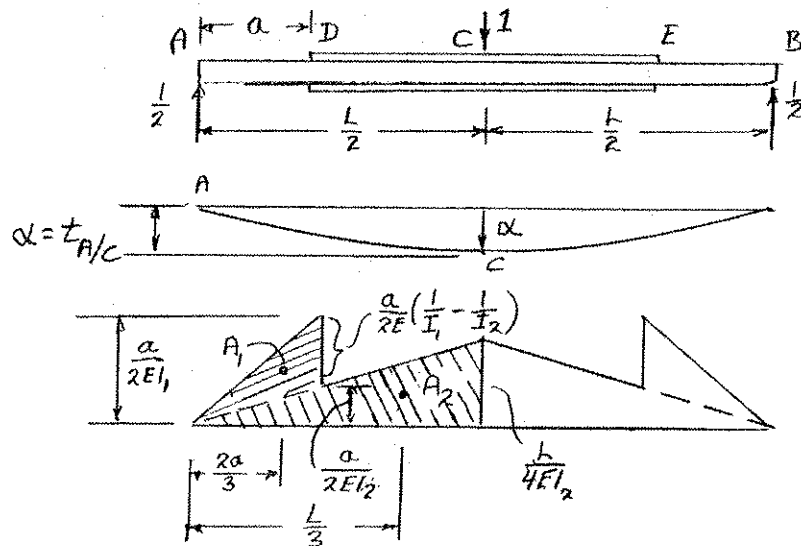
PRINT: a , y_{ST} , y_m , P_m , σ_1 , σ_2 , AND $(\sigma_1 - \sigma_2)$

REPEAT WITH SMALLER INTERVALS TO FIND a FOR $(\sigma_1 - \sigma_2) = 0$
THIS IS THE DISTANCE a FOR τ_{max} AS SMALL AS POSSIBLE

CONTINUED

PROBLEM 11.C2 - CONTINUED

DETERMINATION OF Δ : Δ IS DEFLECTION AT C FOR A UNIT LOAD AT C.



$$\Delta = \Delta_{A/C} = A_1 \left(\frac{2a}{3} \right) + A_2 \left(\frac{L}{3} \right) = \left[\frac{1}{2EI_1} \left(\frac{1}{I_1} - \frac{1}{I_2} \right) \frac{a}{2} \right] \frac{2a}{3} + \left[\frac{1}{2} \cdot \frac{L}{4EI_2} \cdot \frac{L}{2} \right] \frac{L}{3}$$

$$\Delta = \left[\left(\frac{1}{I_1} - \frac{1}{I_2} \right) a^3 + \frac{1}{8I_2} L^3 \right] / 6E$$

PROGRAM OUTPUT

Beam = W 200x26.6 with two 18 by 158 mm cover plates
 h = 50 mm W = 6573 N L = 3 m.

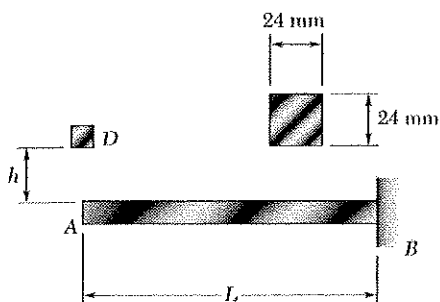
a m	y _{stat} mm	y _{max} mm	P _{max} kN	σ ₁ MPa	σ ₂ MPa	σ ₁ - σ ₂ MPa
0	0.1943	4.605	1510	0	150.22	-150.22
0.5	0.2148	4.855	1525	156.1	143.15	12.95
0.75	0.264	5.4075	1383	212.31	129.78	82.53
1.0	0.3595	6.365	1195	244.65	112.14	132.51
1.5	0.752	9.4575	848.8	260.61	79.66	180.95

Use smaller increments to seek the smallest maximum normal stress

0.4582	0.21	4.7975	145.43	144.60	144.66	-0.06
0.4585	0.21	4.8	145.42	144.67	144.65	0.02
0.4588	0.2103	4.8	145.41	144.74	144.64	0.1

Max stress small as possible for a = 0.4585 m
 Smallest max stress = 144.6 MPa

PROBLEM 11.C3



11.C3 The 16-kg block *D* is dropped from a height *h* onto the free end of the steel bar *AB*. For the steel used $\sigma_{all} = 120 \text{ MPa}$ and $E = 200 \text{ GPa}$. (a) Write a computer program to calculate the maximum allowable height *h* for values of the length *L* from 100 mm to 1.2 m, using 100-mm increments. (b) From the values considered in part a, select the length corresponding to the largest allowable height.

SOLUTION

ENTER $\sigma_{all} = 120 \text{ MPa}$, $E = 200 \text{ GPa}$, $d = 0.024 \text{ m}$

$m = 16.8 \text{ kg}$, $g = 9.81 \text{ m/s}^2$

$I = d^4/12$ $S = \frac{I}{c} = \frac{I}{d/2} = \frac{d^3}{6}$

FOR $L = 100 \text{ mm}$ TO 1200 mm STEP 100 mm

$L = L/1000$

$y_{st} = mgL^3/3EI$

$M_{max} = J_{all} S$

$P_{max} = M_{max}/L$

$y_{max} = P_{max} L^3/3EI$

FROM PROB. 11.69, page 705

$y_m = y_{st} \left[1 + \sqrt{1 + \frac{2h}{y_{st}}} \right]$ SOLVE FOR $h = \left[\left(\frac{y_{max}}{y_{st}} - 1 \right)^2 - 1 \right] \frac{y_{st}}{2}$

PRINT: L , y_{st} , y_{max} , P_{max} , M_{max} , h

RETURN

PROGRAM OUTPUT

Problem 11.C3

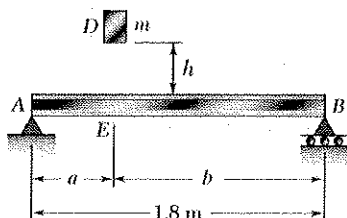
$m = 16.0 \text{ kg}$ $d = 24 \text{ mm}$ $\sigma = 120 \text{ MPa}$ $E = 200 \text{ GPa}$

L mm	y _{st} mm	y _{max} mm	P _{max} N	M _{max} N·m	h mm
100	0.00946	0.167	2764.8	276.48	1.301
200	0.07569	0.667	1382.4	276.48	2.269
300	0.25547	1.500	921.6	276.48	2.904
400	0.60556	2.667	691.2	276.48	3.205
500	1.18273	4.167	553.0	276.48	3.173
600	2.04375	6.000	460.8	276.48	2.807
700	3.24540	8.167	395.0	276.48	2.109
800	4.84445	10.667	345.6	276.48	1.076
900	6.89766	13.500	307.2	276.48	-0.289
1000	9.46181	16.667	276.5	276.48	-1.988
1100	12.59367	20.167	251.3	276.48	-4.020
1200	16.35000	24.000	230.4	276.48	-6.385

Use smaller increments to seek the largest height *h*

435	0.77883	3.154	635.6	276.48	3.2316
440	0.80599	3.227	628.4	276.48	3.2320
445	0.83378	3.300	621.3	276.48	3.2317

PROBLEM 11.C4



11.C4 The block D of mass $m = 8 \text{ kg}$ is dropped from a height $h = 750 \text{ mm}$ onto the rolled-steel beam AB . Knowing that $E = 200 \text{ GPa}$, write a computer program to calculate the maximum deflection of point E and the maximum normal stress in the beam for values of a from 100 to 900 mm, using 100-mm increments.

SOLUTION

ENTER: $L = 1.8 \text{ m}$, $E = 200 \text{ GPa}$, $h = 0.75 \text{ m}$
 $m = 8 \text{ kg}$, $g = 9.81 \text{ m/s}^2$
 $I = 6.87 \times 10^{-6} \text{ m}^4$
 $S = 91.6 \times 10^{-6} \text{ m}^3$

FOR $a = 100 \text{ mm}$ TO 900 mm STEP 100 mm

$$a = a/1000$$

$$b = L - a$$

$$y_{st} = m g a^2 b^2 / 3 E I L$$

$$\alpha = a^2 b^2 / 3 E I L$$

$$y_m = y_{st} \left[1 + \sqrt{1 + \frac{2h}{y_{st}}} \right]$$

$$P_{max} = y_m / \alpha$$

$$M_{max} = P_{max} a b / L$$

$$\tau_{max} = M_{max} / S$$

PRINT: α , y_{st} , y_m , P_{max} , τ_{max}

RETURN

SEE PROB. 11.71, page 705 →

INFLUENCE COEFFICIENT FOR Δ_E } →
 FOR UNIT LOAD AT E

SEE PROB. 11.69, page 705 →

Problem 11.C4

Beam: W 150 x 13.5

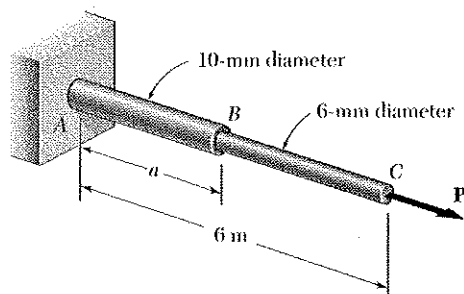
$$I = 6.87 \times 10^{-6} \text{ m}^4 \quad S = 91.6 \times 10^{-6} \text{ m}^3$$

$$L = 1.8 \text{ m} \quad h = 750 \text{ mm} \quad m = 8 \text{ kg} \quad g = 9.81 \text{ m/s}^2$$

a mm	y _{stat} mm	y _{max} mm	P _{max} N	σ _{max} MPa
100	0.0003	0.6775	173.93	179.33
200	0.0011	1.2757	92.43	179.40
300	0.0021	1.7946	65.75	179.46
400	0.0033	2.2339	52.85	179.51
500	0.0045	2.5936	45.55	179.55
600	0.0055	2.8734	41.13	179.59
700	0.0063	3.0734	38.46	179.61
800	0.0068	3.1934	37.02	179.63
900	0.0069	3.2334	36.56	179.63

NOTE: THE SMALL VARIATION IN τ_{max} . THIS IS DUE TO THE ENERGY ACQUIRED BY THE MASS AS IT FALLS THROUGH y_{max} . SEE PROB. 11.147, page 731, FOR A CASE WHERE ENERGY DELIVERED IS CONSTANT AND τ_{max} IS ALSO CONSTANT.

PROBLEM 11.C5



11.C5 The steel rods AB and BC are made of a steel for which $\sigma_Y = 300$ MPa and $E = 200$ GPa. (a) Write a computer program to calculate, for values of a from 0 to 6 m, using 1-m increments, the maximum strain energy that can be acquired by the assembly without causing any permanent deformation. (b) For each value of a considered, calculate the diameter of a uniform rod of length 6 m and of the same mass as the original assembly, and the maximum strain energy that could be acquired by this uniform rod without causing permanent deformation.

SOLUTION

$$\text{ENTER: } \sigma_Y = 300 \text{ MPa}, E = 200 \text{ GPa}, L = 6 \text{ m}$$

$$\text{AREA}_{AB} = \frac{\pi}{4} (0.010 \text{ m})^2, \text{ AREA}_{BC} = \frac{\pi}{4} (0.006 \text{ m})^2$$

$$P_m = \sigma_Y \text{ AREA}_{BC}$$

FOR $a = 0$ TO 6 m STEP 1 m

$$U = \frac{P_m^2}{2E} \left(\frac{a}{\text{AREA}_{AB}} + \frac{L-a}{\text{AREA}_{BC}} \right)$$

FOR UNIFORM ROD OF SAME VOLUME

$$\text{VOL} = a (\text{AREA}_{AB}) + (L-a) (\text{AREA}_{BC})$$

$$d = \sqrt{\frac{4 \text{ VOL}}{\pi L}}$$

$$\text{AREA}_{\text{NEW}} = \frac{\pi}{4} d^2$$

$$P_{\text{NEW}} = \sigma_Y (\text{AREA}_{\text{NEW}})$$

$$U_{\text{NEW}} = \frac{P_{\text{NEW}}^2 L}{2E (\text{AREA}_{\text{NEW}})}$$

PRINT $a, U, \text{VOL}, d, P_{\text{NEW}}, U_{\text{NEW}}$

RETURN

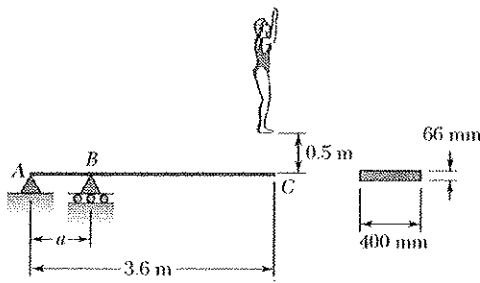
PROGRAM OUTPUT

Problem 11C5

sigmaY = 300 MPa, Pm = 8482 N, L = 6 m, E = 200 GPa

a m	U J	Vol m ³	d mm	New P N	newU J
0.00	38.17	169.65	6.00	8482.30	38.17
1.00	34.10	219.91	6.83	10995.58	49.48
2.00	30.03	270.18	7.57	13508.85	60.79
3.00	25.96	320.44	8.25	16022.12	72.10
4.00	21.88	370.71	8.87	18535.40	83.41
5.00	17.81	420.97	9.45	21048.67	94.72
6.00	13.74	471.24	10.00	23561.95	106.03

PROBLEM 11.C6



11.C6 A 72-kg diver jumps from a height of 0.5 m onto end C of a diving board having the uniform cross section shown. Write a computer program to calculate for values of a from 250 to 1250 mm, using 250-mm increments, (a) the maximum deflection of point C, (b) the maximum bending moment in the board, (c) the equivalent static load. Assume that the diver's legs remain rigid and use $E = 12 \text{ GPa}$.

SOLUTION

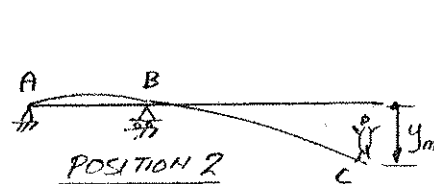
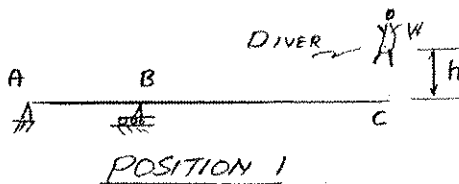
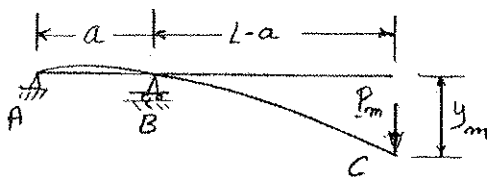
ENTER: $L = 3.6 \text{ m}$, $h = 500 \text{ mm}$, $W = (72 \times 9.81) = 706 \text{ N}$
 $E = 12 \text{ GPa}$.

$$I = (0.4)(0.066)^3 / 12$$

$$S = (0.4)(0.066)^2 / 6$$

$y_m = P_m \alpha$ WHERE $\alpha = \text{INFLUENCE COEFFICIENT}$
 SEE BELOW FOR DETERMINATION OF α

WHERE $P_m = \text{EQUIVALENT STATIC LOAD}$



$$U_2 = \frac{1}{2} P_m y_m = \frac{1}{2} \frac{y_m^2}{\alpha}$$

$$\text{WORK} = W(h + y_m)$$

$$\text{WORK} = U_2$$

$$W(h + y_m) = \frac{1}{2} \frac{y_m^2}{\alpha}$$

PROGRAM SOLUTION OF a FOR y_m . ENTER α

FOR $a = 10 \text{ in. TO } 50 \text{ in. STEP } 10 \text{ in.}$

SOLVE A FOR y_m , $P_m = y_m / \alpha$

$$M_{\max} = M_B = P_m (L - a)$$

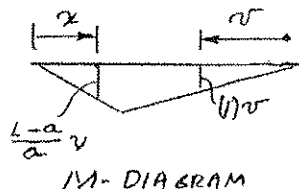
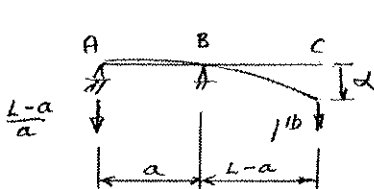
$$T = M_{\max} / S$$

PRINT α , y_m , P_m , M_m , T

PROGRAM OUTPUT

$a(\text{in})$	$y_m(\text{mm})$	$P_m(\text{kN})$	Max M (kNm)	Sigma (MPa)
0.25	365.55	3.370	11.473	37.954
0.5	331.55	3.570	11.246	37.198
0.75	298.75	3.806	11.022	36.456
1.0	267.08	4.088	10.801	35.728
1.25	236.55	4.432	10.584	35.007

DETERMINATION OF INFLUENCE COEFFICIENT α



$$U = \frac{1}{2} (1/b) \alpha = \sum \int \frac{M^2}{2EI} dv$$

$$\frac{\alpha}{2} = \frac{1}{2EI} \left[\int_0^{L-a} \left(\frac{L-a}{a} \right)^2 x^2 dv + \int_{L-a}^L x^2 dv \right]$$

$$\alpha = \frac{1}{EI} \left[\frac{(L-a)^2 a^3}{3} + \frac{(L-a)^3}{3} \right]$$

$$\alpha = \frac{1}{3EI} \left[(L-a)^2 a + (L-a)^3 \right]$$

